Introduction to Quantum Mechanics
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Lecture - 02
General nature of the correspondence principle

Let us take a harmonic oscillator.

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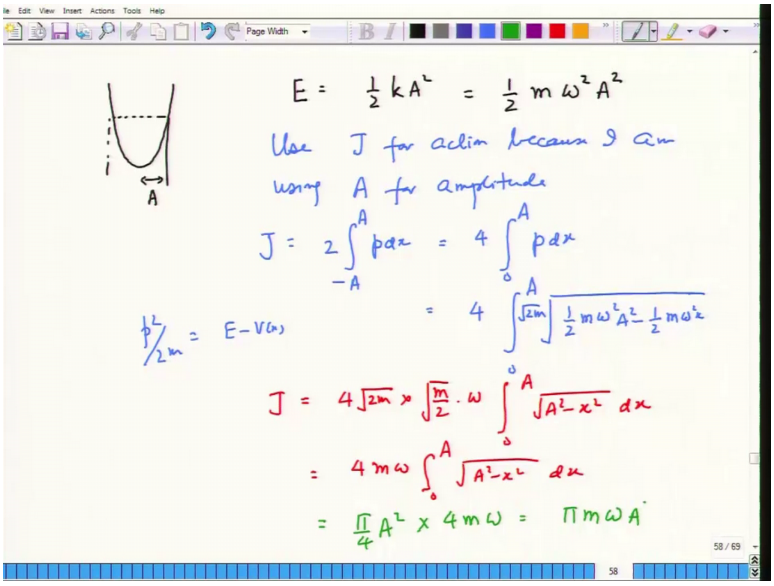

The energy for this is suppose this is oscillating with amplitude A, this is A, energy is given as one half k A square which is one half mass of the particle omega square a square. Let us write this in terms of action, let me use J for action because I am using A for amplitude. So, J is nothing but integral minus A to A p d x times 2 which is same as 4 time 0 to A p d x which is nothing but 4 times 0 to A square root of total energy one half m omega square A square minus potential energy one half m omega square x square thus the kinetic energy divided by. So, p square over 2 m, p square over two m is nothing but E minus V x. So, I am going to have square root of 2 m also here does the action.

So, action J comes out to be 4 times square root of 2 m times what will come out is m square root of m over 2 times omega and inside I have 0 to a square root of a square minus x square d x. So, this is nothing but 4 m omega time 0 to A square root of a square minus x square d x. This integral is easy to calculate and its value comes out to be pi by 4

A square times 4 m omega which is then become therefore, pi m omega A square that is the value.

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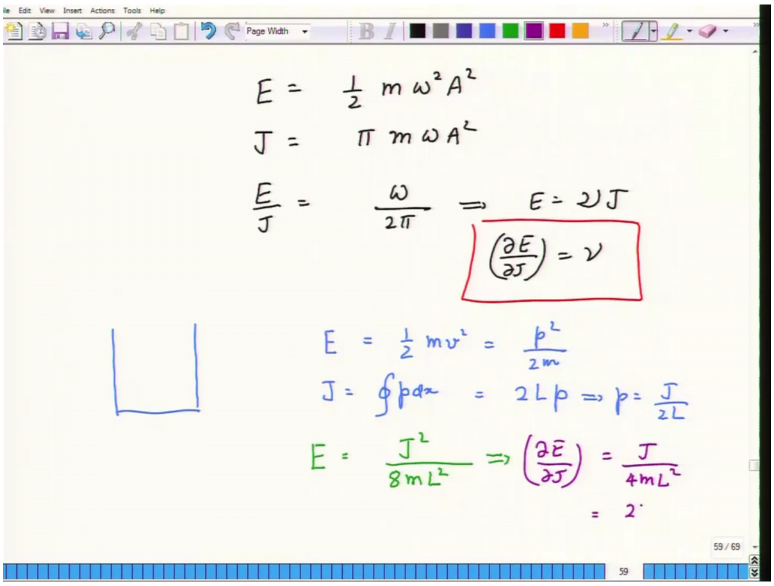

The recall that energy E is one half m omega square A square, J has come out to be pi m omega A square. So, this gives you E by J equals omega over 2 pi or E equals omega over 2 pi is a frequency of oscillation times J.

So, d E d J is nu I shown you through one example. Let us take another example let us take particle in a box. Particle in a box the energy classical is one half m v square or also p square over 2 m, and the action is p d x over the entire period and this comes out to be 2 LP. So, P is nothing but J over 2L and therefore, the energy comes out to be J square over it m L square, which gives me d E d J to be equal to J over 4 m L square which is 2L J is 2L p; so 2L p over 4 m L square.

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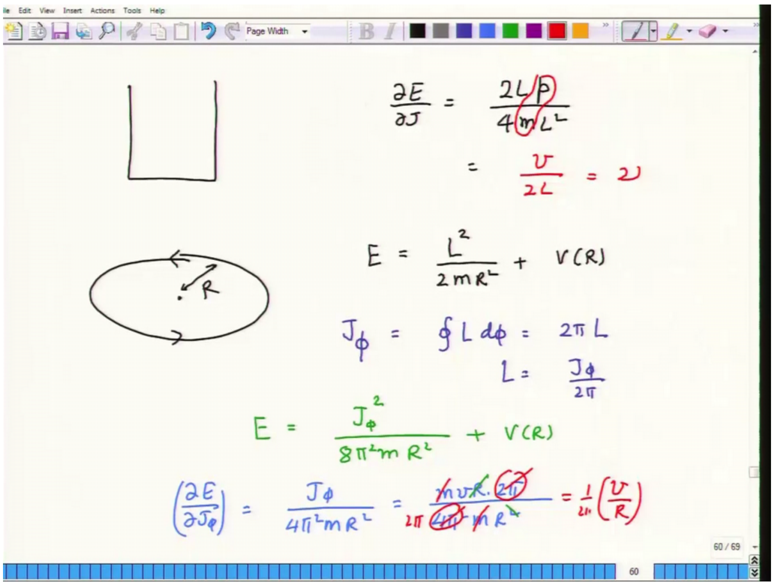

So, what we have found in particle in a box d E d J is nothing but 2L p over 4 m L square which is nothing but p over m, p over m is the velocity divided by 2L which is again the frequency of oscillation nu.

It is the two examples; third example let us take example number 3 where a particle is going around the central potential. In this case the energy a suppose this radius is fixed energy has nothing but L square where L is the angular momentum divided by 2 m R square if the radius is R plus whatever potential energy R, and recall that action J for this motion let us call it J phi is equal to integral L d phi which is 2 phi L. So, L is nothing but J phi over 2 pi and therefore, I can write the energy E as L square which is J phi square divided by 8 pi square m R square plus whatever V R s.

Since there is no V, so V R remains fixed and therefore, d E over d J phi is nothing but J phi over 4 pi square m R square and lets right J phi in terms of the angular momentum which is m v R times 2 pi divided by 4 pi square m R square, and lets cancel a few terms this m cancels this cancel one of the R, so this r cancels and let us cancel 2 pi. So, this 2 pi cancels with this 4 pi square and you get two pi which is nothing but v over R 1 over 2 pi which is omega over 2 pi which is nu again.

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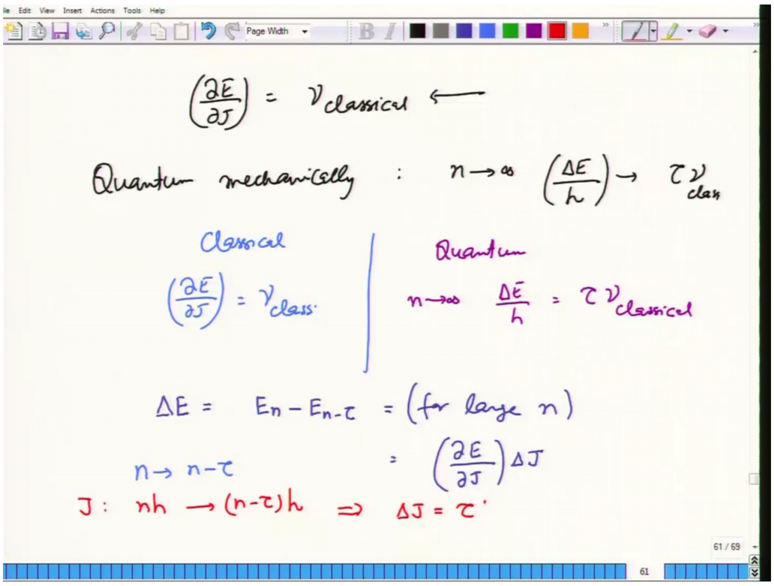

So, what I have shown you is that this result d E by d J is equal to nu classical through examples I have shown you that this is true and quantum mechanically, what we have shown is that as n approaches to large value delta E over h goes to tau times nu classical.

Some two results we have two results classical d E d J equals nu classical, and here quantum n tending to infinity delta E over h is equal to tau nu classical. Now you are going to (Refer Time: 08:15) two the wave we do it is like this, I know delta E is E n minus E n minus tau which can be written for large n, I can write this as partial E over partial J delta J, because J is going to be very large if you change tau by very small number you can effectively write as if the whole thing has gone in to a continue.

Change one or two in presence of an infinitely large number is very small, and when I making a jump from n th level to n minus tau level, my J goes from n h two n minus tau, h which implies that delta J is tau h.

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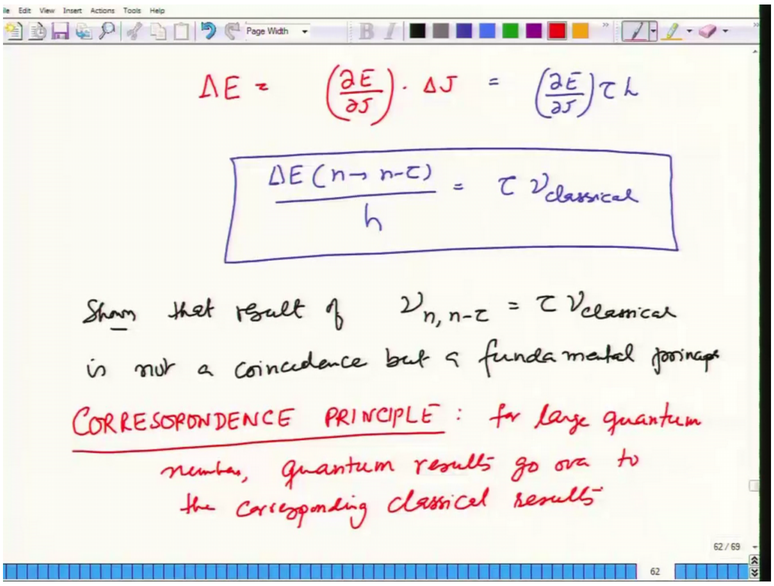

So, I have delta E is equal to d E d J times delta J, which goes over to d E d J times tau h or delta e form n two n minus tau over h is nothing but tau times new classical. So, what we have shown is that the that result of nu n to n minus tau equals tau nu classical is not a coincidence, but a fundamental result principle and this is known as the correspondence principle let me write this the correspondence principle as stated by board. This is observed in terms of frequency, but he is saying now that for large quantum numbers quantum results go over to the corresponding classical results.

Therefore I can hope if I know the classic results well, that I can go back and go for a small n values and anticipate what would happen in quantum theory. Let me just end this lecture by telling what implications does it have.

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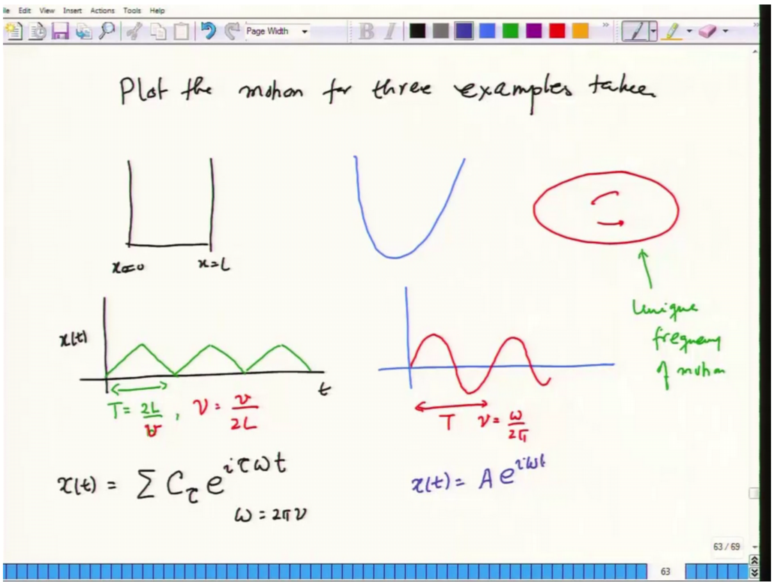

So, if I plot the motion for three examples taken. So, I will take this particle in a box, I will take the harmonic oscillator and I will take this electron going around in a circle here. If I plot x t versus t between 0 and x equals L the motion looks like this, where this time is equal to 2L over h or the frequency corresponding frequency 2L over v corresponding frequencies v over 12. In the second case motion looks harmonic and this time is the time period and frequency is omega over 2 pi it contains one frequency, and here in the third case also this is the circular motion this is the unique frequency of motion.

Let me explain this through Fourier series. In the first case x t would be given as summation some coefficient for tau e raise to i tau omega t, where omega is 2 pi nu that is the fundamental frequency, but because it is not purely harmonic, it contains many more harmonics. In the second case x t is purely harmonics. So, this is some amplitude right e raise to i omega t, plus another sub amplitude star e raise to minus I omega t.

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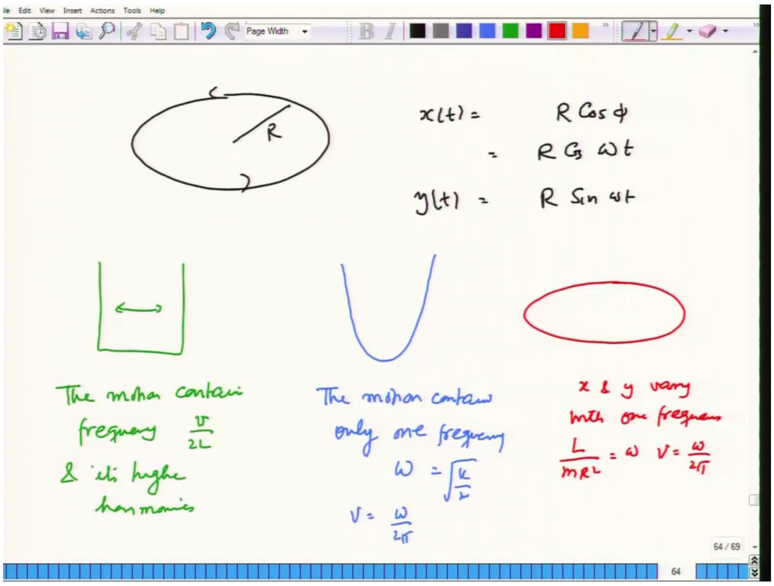

In the third case where this is the circular motion x t would be given as the radius R cosine of phi which is R cosine of omega t this is one frequency, and y t will be given as r sin of omega t.

So, in the first case where the particle is doing a particle in a box moving back and forth, the motion contains frequency v over 2L and its higher harmonics. So, if this particle what to radiate it will contain all these frequencies v over 2L and higher harmonics. On the other hand in this case the motion contains only one frequency. So, if it (Refer Time: 15:10) to radiate we will radiate that particular frequency omega which is square root of k over m or nu equals omega over 2 pi, and in the third case again x and y vary with one frequency, which we can write as L the angular momentum over m R square equals omega or nu equals omega over 2 pi and in this case also the particle classically radiates only one frequency.

Using these and making correspondence with the quantum results I will show you in next lecture how this leads to selection rules. So, to conclude this lecture I will just again write the correspondence.

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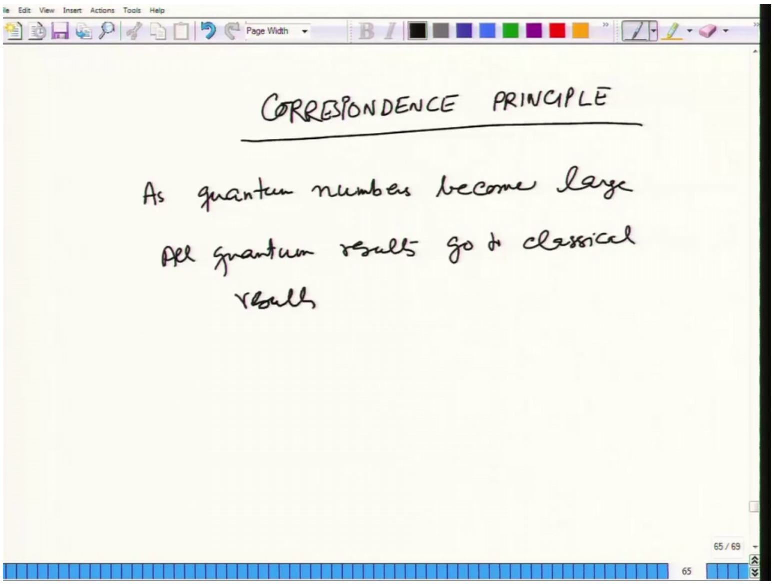

Principle which will be guiding principle for or next two or three lectures, is that as quantum numbers become large all quantum results go to a classical results.