Introduction to Quantum Mechanics
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Lecture - 06
Stimulated emission and amplification of light in a LASER

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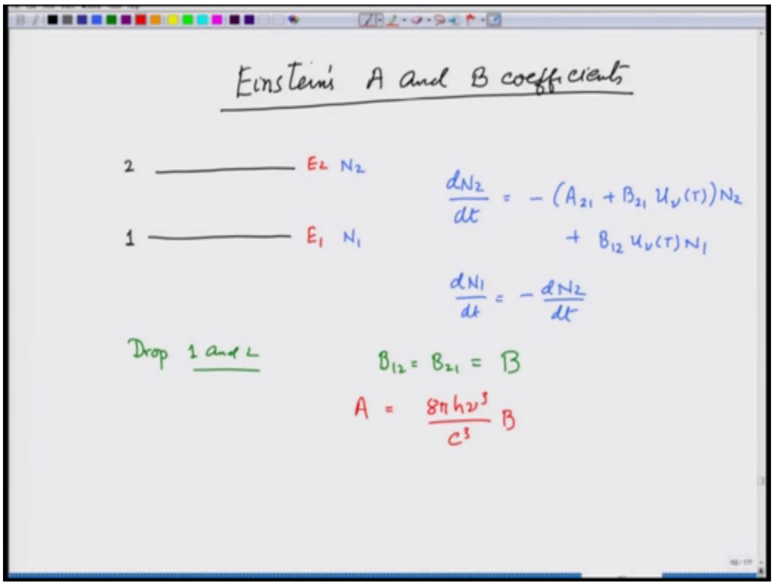

In the previous lecture we discussed Einstein's A and B coefficients, and learnt that if there are two energy levels 1 and 2 with energies E 1 and E 2. And number of atoms in the lower level being N 1 and the upper level being N 2. Then the rate at which the atoms come down to lower level is equal to minus A 21 plus B 21 u nu at that temperature; N 2 plus B 12 u nu T N 1. And d N 1 over dt is equal to minus d N 2 by dt, because the total number of atoms remains the same.

From now on I am going to drop 1 and 2; it is understood that we are talking about two levels. And we also saw that B 12 was same as B 21 so I am going to call this B. And the A coefficient A was equal to 8 pi h nu cubed over C cubed times B.

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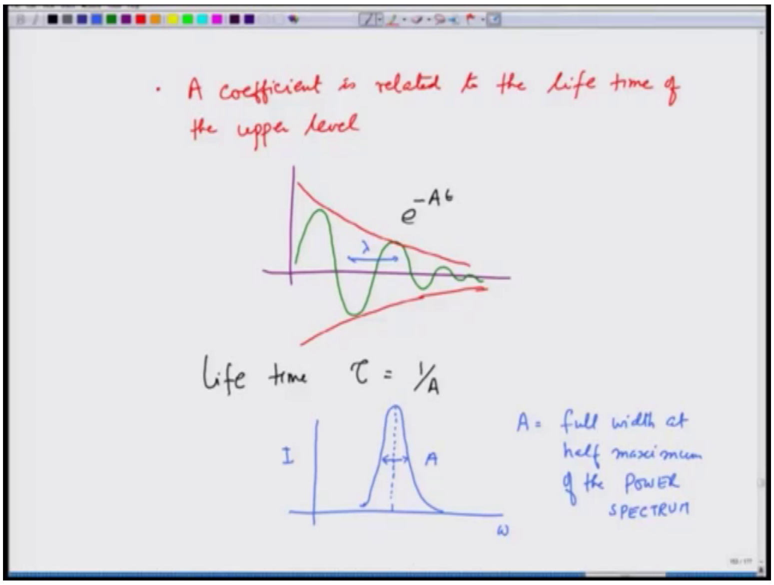

We also learnt that A coefficient is related to the lifetime of the upper level. That means, on an average this is how long the lifetime is, this is how long the atom is going to remain in the upper level. And this arose from the fact that as time progresses a radiating atom would give out signals or light which goes down in amplitude exponentially. And this exponential is like e raise to minus A t. So, it decreases by a factor of E in time 1 over A. And this is what we call the lifetime tau is going to be 1 over A this how the two are related.

And this also gives then because this a wave and that has decaying amplitude, if I plot the intensity versus omega for this wave or versus new this wave it comes out to be peak that certain frequency which is related to the wavelength, given here lambda. But it also has the frequencies in this width full width at half maximum is A. So, A is also full width at half maximum of what you known as power spectrum. That means, what intensity is coming out at what frequency.

So, this is how this kind of going up and coming down this kind of peak I have shown in blue is the kind of peak that you see is kind of intensity you measure when light is coming out of an atom that is emitting, alright.

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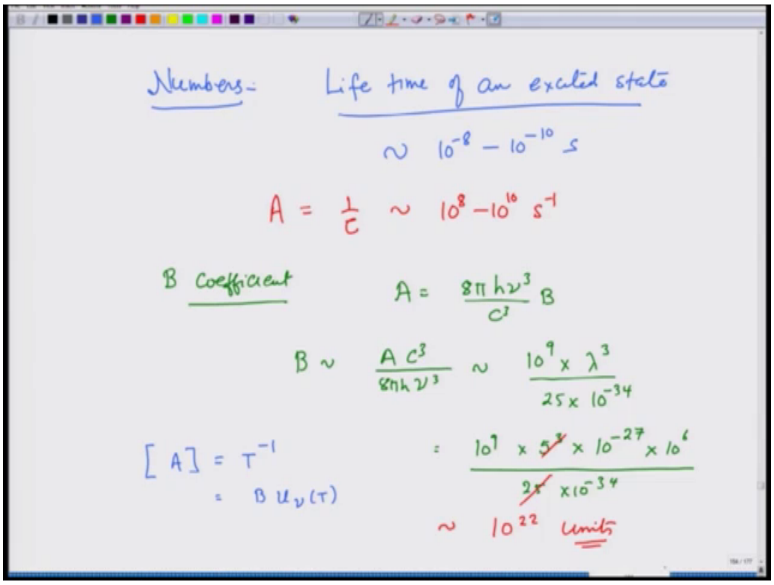

So, now let us get a feeling for some numbers. Now life time of an exited state or the upper state of an atom is of the order of 10 raise to minus 8 to 10 raise to minus let say 10 seconds. And therefore, the A coefficient A is goes as 1 over tau will be of the order of 10 raise to 8 to 10 raise to 10 second inverse. This is the kind of number that we are talking about A.

How about B coefficient? A is related to B as 8 pi h nu cubed over C cubed times B. So, B is A C cubed over nu cubed 8 pi h of the order of the 10 raise to 8 to 10 raise to 10. So, let us take a 10 raise to 9 C cube over nu cubed was lambda cube divided by 8 pi is of the order of 25 times h is the of order of 10 raise to minus 34. This is equal to 10 raise to 9 lambda is 500 nano meters. So, that is 5 cubed nano is 10 raise to minus 9 meters or 10 raise to minus 27 times 10 raise to 5 that is 500 cubed divided by 25 times 10 raise to minus 34.

So, we are talking about of the order of let say this cancels roughly; this cancels and you get 10 raise to 7; 10 raise to 7 plus 6 10 raise to 13 10 raise to 22 whatever units. I will let you work out these units and give this as an assignment problem. Just keep in mind in working out these units that unit of A are T inverse and that same as B u nu T.

So, that is the kind of orders we talking about.

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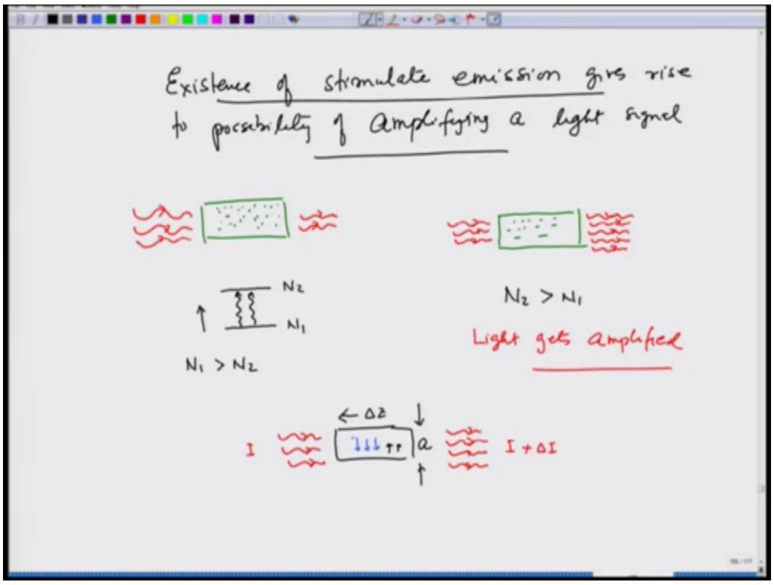

Now this gives raise to the existence of stimulated emission gives raise to possibility of amplifying a light signal, and let me explain. Suppose I have a cube in which some gas is filled and you pass light through it; what would happen is this light will be get absorbed and what comes out will be light of lower intensity. And why does that happen? That happens because there are atoms in the upper state N 2, there are atoms in the lower state N 1, and the frequency light gets absorbed. And normally, if N 1 is greater than N 2 and I will show this mathematically soon then what would happen is that atoms that are getting absorbed would be larger in number then the atoms that are coming down. Tnd therefore, light will get absorbed.

Consider the possibility however, that this gas has atoms such that N 2 is greater than N 1 then light which is going in will get amplified; more light will come out. That is because now more atoms will be coming down and giving out radiation than those which are absorbing light. So therefore, light gets amplified.

Let us now see this relationship between N 1 and N 2 and light getting amplified or diminished mathematically. So, for this what I will consider is- let us take this tube and let this length be delta set and let this cross sectional area be a. And suppose light of intensity I is coming in of wall frequencies light of intensities (Refer Time: 10:13) is coming in. And light of intensity I plus delta I, I am taking it to be positive so that mathematics curves work out in more convenient wave. Light of I plus delta I goes out.

And why does that happen that happens because there are atoms that will be giving out light, and there are atoms that will be absorbing light, and the difference of the emission and absorption gives this light.

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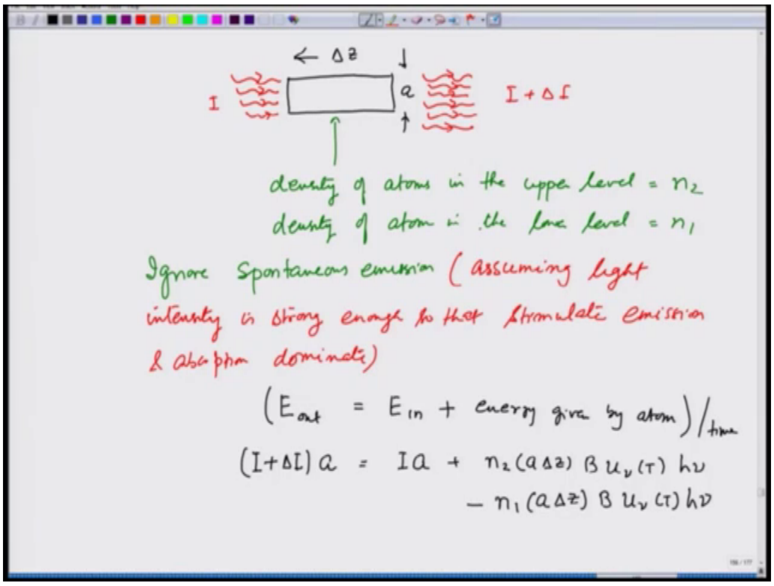

So, I am taking this tube of length delta Z cross sectional area a and light of intensity I is coming in and light of intensity I plus delta I goes out, and the difference arises from the absorption and emission taking place inside. Let the density of atoms in the upper level be N 2, density of atoms in the lower level be N 1. Then what do I have? I have an ignore spontaneous emission; and when can I do that I am assuming light intensity is strong enough so that stimulated emission and absorption of course is there nominate.

So, from the energy balance equation I am going to have going have E going out is going to be equal to E coming N plus whatever has been observed, whatever has been given by the atoms energy given by atoms. Energy which is going out can be written as the intensity which is energy per unit per area per unit time. And this is all this is per unit time.

So, per unit time energy going out is going to be E intensity I plus delta I going across the area a is going to be equal to I a plus the number of atoms which are coming from upper to lower level is going to be N 2 times the volume a delta Z; that is a total number of atoms that are in the upper level and the rate of emission is u nu T. And each one every time an atom makes a transition from upper to lower level it gives out energy h nu.

Minus the number of atoms that are making transitions from lower to upper level, and that is going to be N 1 a delta Z B u nu T h nu from the balance equation that I wrote at the beginning of this lecture.

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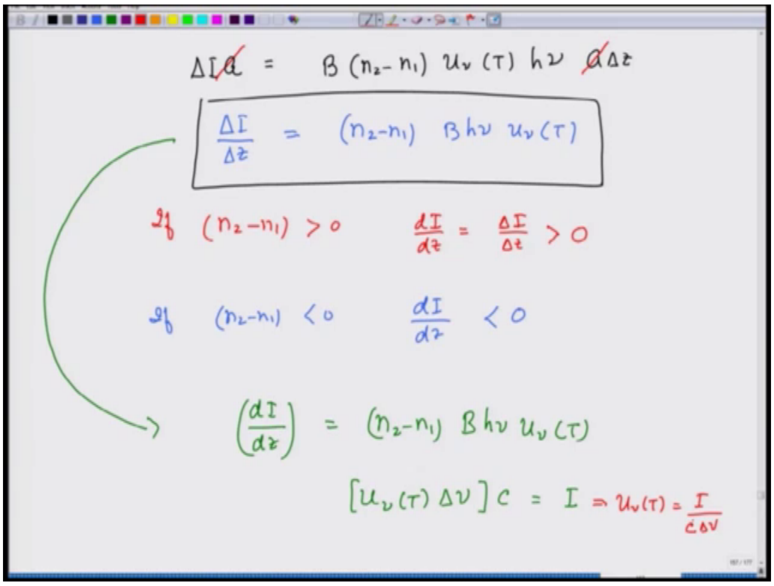

And therefore, what I get am I right this in the next page- is that delta I times a that is all taking place per time is equal to B n 2 minus n 1 u nu T h nu times a delta Z. I have collected all the terms. And therefore, delta I over delta Z is equal to a cancels, this is equal to n 2 minus n 1 B h nu u nu T.

From this it is clear if n 2 minus n 1 is greater than 0; that means the number of atoms in the upper level is more than the number of atoms in the lower level d I by d Z which I am writing as delta I over delta S. So, far going to greater than 0, intensity is going to be whatever I like sent inside is coming out amplified. On the other hand if n 2 minus n 1 is less than 0 d I by d Z is going to be less than 0 and the light gets diminished it gets absorbed.

Lets us write this equation in a slightly better form now. So, this is d I by d Z is equal to n 2 minus n 1 B h nu u nu T; I would like to write u nu T in terms of the frequent the intensity itself. So, recall that I am sending light over A frequency range delta nu, so this is the total energy contained and this is like a plane wave so this time C is intensity. Therefore, I can write this implies that u nu T is I over C delta nu. I am going to substitute that in the equation and I get.

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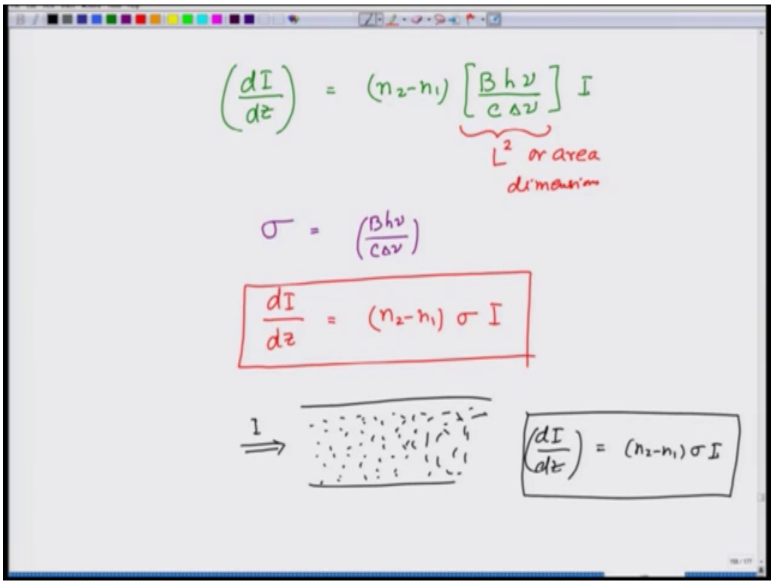

D I over d Z is equal to n 2 minus n 1 B h nu over C delta nu times I. You can easily show that this has dimensions of length square or area dimensions.

So, I am going to define something called the cross section and this is transition cross section if you like which is going to B h nu over C delta nu. And therefore, write the equation in the form that d I over d Z when the light travels over length in a medium this going to change as n 2 minus n 1 times sigma I. Let me again make a picture what we are considering. We are considering now I can make a general statement that a container in which there are the these atoms with N 2 being the upper level and N 1being in the lower level and I am sending light of frequency close to the transition frequency with intensity I as it travels it satisfies the equation d I by d Z equals n 2 minus n 2 sigma I.

So if N 2 is greater than N 1 that is the density of the atoms in the upper level is larger than the density of atoms in lower level intensity is going to increase as like travels and it gets amplified.

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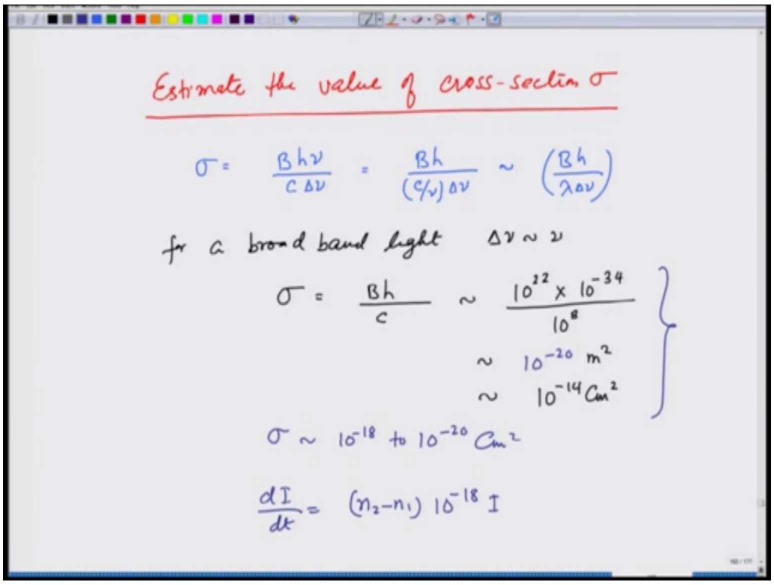

Let us now estimate the value of cross section sigma. And for this I have just define this sigma which is B h nu over C delta nu, which I can write as B h over C over nu delta nu of the order of B h over lambda delta nu. When I am shining white light or a broad band light right. So, for a broad band light I can take delta nu of the same order of as nu. And therefore, I can write sigma as equal to B h over C.

Let us estimate the number I have just shown you few minutes suppose B is of the order 22 units h is of the order of 10 raise to minus 34 divided by C is of the order of 10 raise to 8. So, this comes out be 10 raise to be minus 20 meter square or of the order of 10 raise to minus 14 centimeter square. What we find usually in atoms sigma is of the order of 10 raise to minus 18 to 10 raise to minus 20 centimeter square. But what I have done here is just made an estimate.

So, what you are going to have is that d I by d T is going to be order of n 2 minus n 1 times 10 raise to minus let us say 18 I. So, n 2 minus n 1 has to be really large if you want to have any significant change on the intensity.

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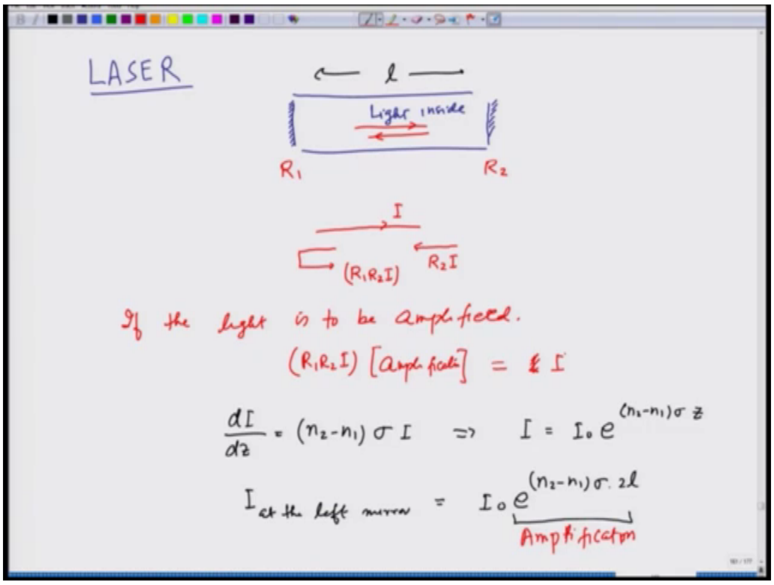

Now, how is this used into formalize to make lasers. So, let us see now what happens in a laser. What one does in a laser is puts two mirrors, these are mirrors and generates a light inside. And this light goes back and forth. Let the reflectivity of mirror on the left be R 1, the reflectivity of the mirror on the right be R 2. So, what happens is when light of intensity I goes to the right what comes back is R 2 I. And after its get reflected from the right side mirror I get R 1 times R 2 times I.

If the light is to we amplify it then R 1, R 2, I times the amplification factor must be at least one that is the critical. Amplification factor actually should be larger than R 1 and R 2. And how much is the amplification factor let us check that. So, I have d I over d Z is equal to n 2 minus n 1 sigma I and this gives me that I is equal to I 0 e raise to n 2 minus n 1 sigma Z at the light travels distance Z. Now in the cavity that we just consider between the two mirror if the length is l, by the light comes back to the original mirror I at the left mirror after travels 2 l distance is going to be equal to I 0 e raise to n 2 minus n 1 sigma times 2 l.

And this is the amplification factor. And this times R 1 R 2 should be at least 1. I should write on the top they should be I. So, amplification factor times.

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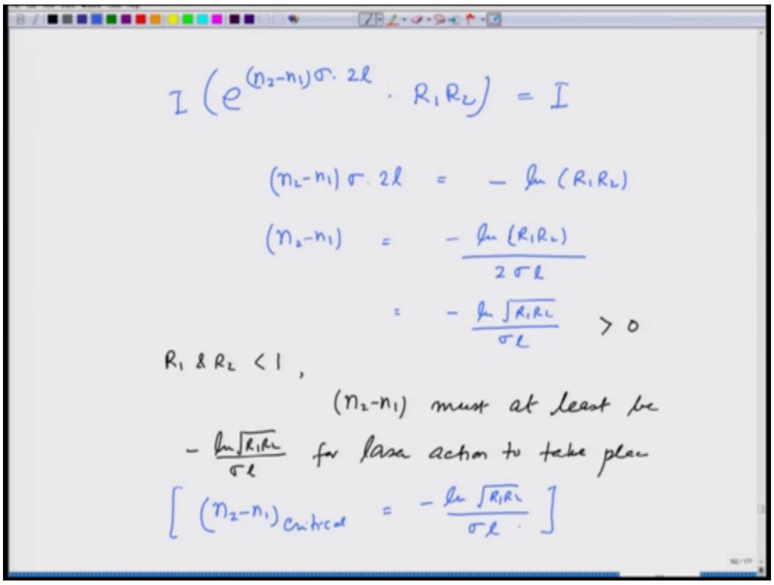

So, e raise to n 2 minus n 1 sigma times 2 l times R 1 R 2 times I should be at least I, so that the light sustained in reality it should be more if the lattice to be amplified. And therefore, I have n 2 minus n 1 sigma times 2 l is equal to minus log of R 1 R 2. So, n 2 minus n 1 should be equal to minus log of R 1 R 2 divided by 2 sigma l, which is same as minus log of square root of R 1 R 2 over sigma l.

Since R 1 and R 2 are less than 1, therefore this number is greater than 0. So, n 2 minus n 1 must at least be minus log of square root of R 1 R 2 over sigma l for laser action to take place. And this is known as a the critical population inversion n 2 minus n 1 critical is equal to minus log of square root of R 1 R 2 over sigma l.

So, this is what we have come to the conclusion that the upper level should have number of atoms larger than those in lower level. And in order to sustain laser action I should have minimum difference between them which is given by the reflectivity of the mirror and the length of this cavity in which I am holding these atoms, and which work as the amplifiers.

Now what exactly is done to achieve this n 2 minus n 1 greater than 0, I will discuss in the next lecture.