**Introduction to Quantum Mechanics**
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**Lecture – 10**
**Bohr's model of hydrogen spectrum**

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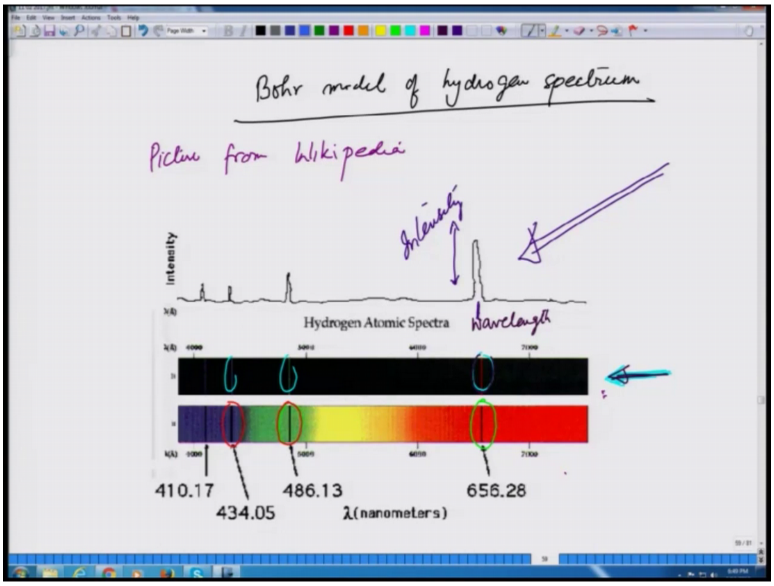


In this lecture, we are now going to discuss application of quantum theory to explain the spectrum of hydrogen atom what is known as Bohr model of hydrogen spectrum. Let me first explain what do we mean by spectrum and for that I will show you a picture from Wikipedia. So, picture from Wikipedia and acknowledge this here. So, when you look at a hydrogen spectrum, this is what it is going to look like. So, let me explain the upper portion where I am showing this by red the green arrow is the emission spectrum.

So, you see line; a line at red here, a green line and some other lines, the lower portion is the absorption spectrum where you see that certain wavelengths are missing and this is where the gas when light is passing through it has absorbed these particular colors; whatever it absorbs, the same kind of line same kind of frequency, whatever it absorbs, same wavelength it emits when it is emitting and what this shows here is the intensity, the height here gives intensity and this position here the position here gives wavelength.

So, what we are seeing here is that suppose, I take the light coming out of hydrogen gas when it is excited and measure the intensity of particular wavelengths, I am going to see certain wavelengths coming out. For example, in this emission spectrum, I see a wavelength that corresponds to red corresponds to green and so on and in the absorption spectrum, if I let light pass through it at the same positions where the particle wavelengths are coming; there is an absorption how is this spectrum taken.

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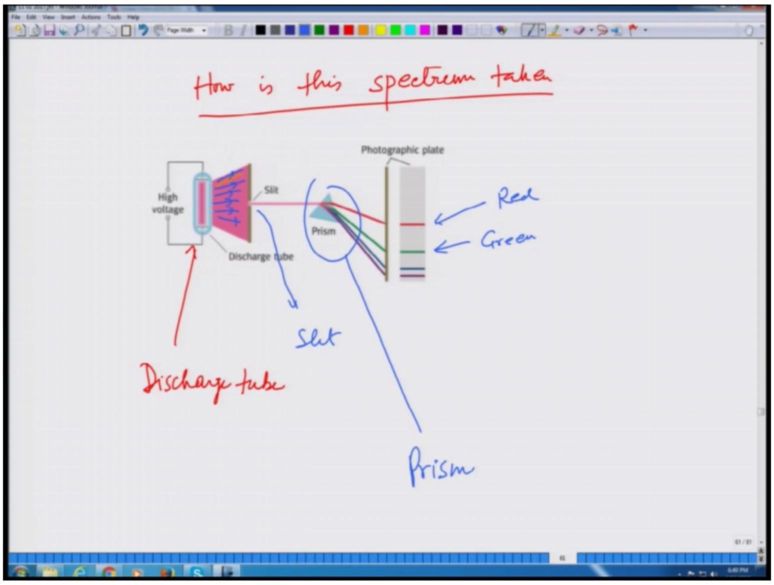


So, what I want to show in house how is this spectrum taken and for that symbolically, I show you this picture where the light is coming from a; this is discharge tube where electricity is discharged so that the atoms get excited and light comes out, the light is taken through a slit. So, this light is coming out in all directions here let me make it with blue in all directions here and here is a slit.

From which it comes out and then, the light is made to pass through a prism, here is a prism; what does the prism do? The prism separates all the lines all the wavelengths. So, red color comes here green goes here. So, all the light that is coming out its wavelengths is separated by this prism and that is taken on a photographic plate that is how a spectrum is taken. So, what I showed you earlier, the hydrogen spectrum when the light coming out of hydrogen was passed through this prism once saw these different colors, different wavelengths that is how you know only these wavelengths come in this is spectrum.

Under the same wavelengths, when you let up light pass through gas at the same wavelengths, the light is absorbed and therefore, that was missing from the spectrum as you saw in the previous slide that when the light was passed through gas and then again pass through prism certain wavelengths. So, missing these are shown here, I am just showing them again, these were shown here, they were missing and these are the absorption lines. So, at the same wavelength, it gives out light at the same wavelength, it absorbs light, alright and this had to be explained.

Why is it that only certain wavelengths are absorbed? Why is it that only certain wavelengths are emitted and for this Bohr model was given based on the quantum theory.

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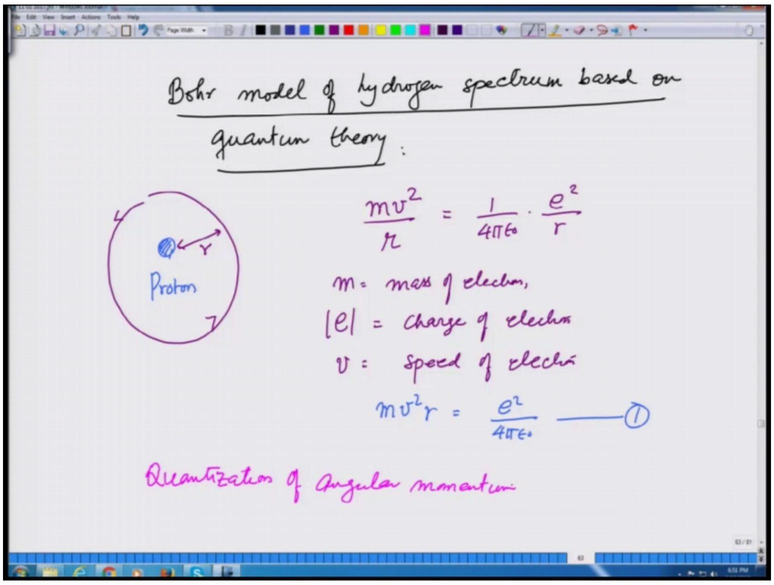


So, Bohr model of hydrogen spectrum based on quantum theory; what he said is that in an atom, there is a hydrogen atom, this is a proton around which an electron is going around in orbits and they follow classical law; that means, if I were to take the centripetal force m v square over r, it should be equal to 1 over 4 pi epsilon 0 e square over r where m is the mass of electron e; mod e is charge of electron, r is the radius of this orbit and 4 pi epsilon 0 represents that constant 9 times 10 raise to 9 v, the speed of electron and this gives you m v square r equals e square over 4 pi epsilon 0 that is equation 1.

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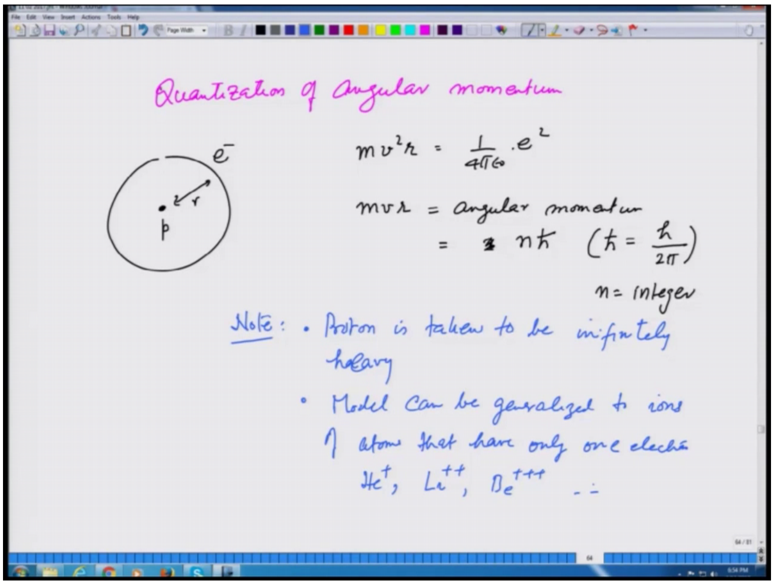


What was dramatic about Bohr's model this is this is classical physics nothing new what is dramatic was the quantization of angular momentum. So, dramatic about Bohr's model was quantization of angular momentum. So, what Bohr said is here is this proton, here is this electron going around and this radius r. So, first law was very clear m v square, r is equal to 1 over 4 pi epsilon 0 times e square. The angular momentum cannot be arbitrary angular momentum or m v r which is equal to the angular momentum of electron going around this orbit can take only those values which are integer multiples of h cross, let me write h cross equals h Planck's constant divided by 2 pi.

And n is an integer. So, he brought this quantum idea into the atomic domain and he said just like an oscillator cannot take arbitrary values of energy, it has to be in the quant of h nu angular momentum of electron going around in hydrogen atom can be in multiples of h cross only, just I wish to note that in this a proton is taken to be infinitely heavy number 1. Number 2: the model can be generalized to ions of atoms that have only one electron; that means, helium plus lithium plus plus beryllium plus plus plus and so on that have only one electron that is a note which will talk about later, but for the time being this is what is.

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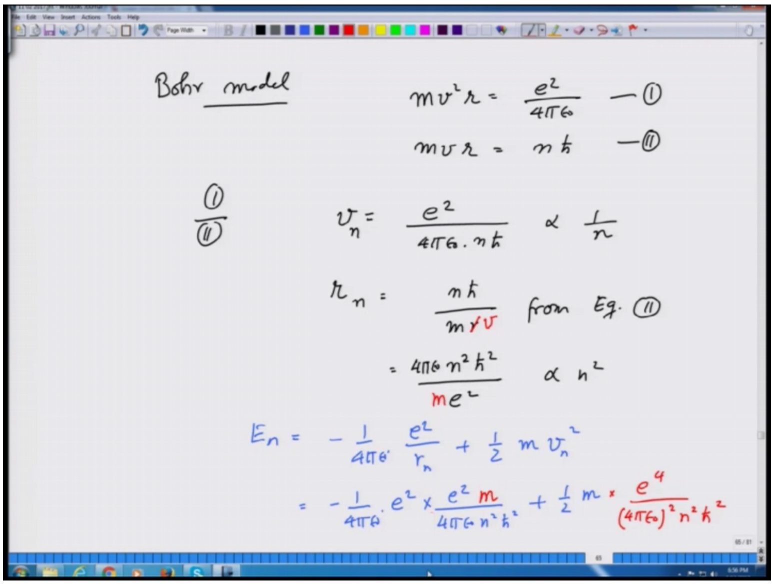


So, according to Bohr model, I have m v square r equals e square over 4 pi epsilon 0 and m v r equals n h cross this is equation 1; equation 2.

If I divide 1 by 2, I get v equals e square over 4 pi epsilon 0 n h cross. So, velocity is r proportional to 1 over n in each arbitrary and the radius. So, let me call this v n radius n is going to be n h cross over m r from equation 2 above and that is going to be n square h cross square times 4 pi epsilon 0 over e square proportional to n square and therefore, the energy of the system depends on n is going to be equal to E n is going to be equal to minus 1 over 4 pi epsilon 0 e square over r in the nth orbital plus one half m v n square which is the kinetic energy and that will come out to be that is it substitute the numbers minus 1 over 4 pi epsilon 0 e square 1 over r n is going to be e square over 4 pi epsilon 0 n square h cross square plus 1 half m v n square which is equal to; there should be an m here this should be m v.

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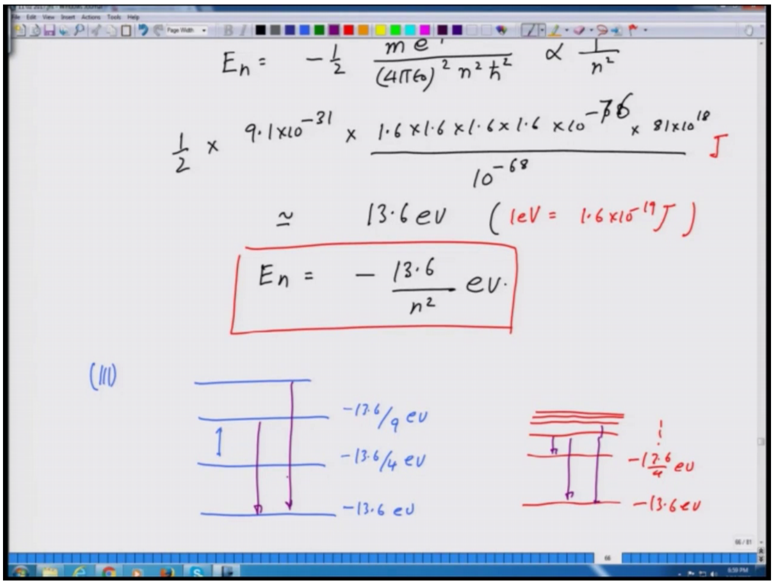


So, there should be m square with E m here and there should be half v n square is nothing, but e raise to 4 divided by 4 pi epsilon 0 whole square n square h cross square that is the energy and this if you collect together comes out to be E n is equal to minus half m e raise to 4 divided by 4 pi epsilon 0 square n square h cross square. So, this is proportional to 1 over n square.

Let us just quickly calculate the value of this. So, this is going to be 1 half times m is 9.1 times 10 raise to minus 31 kgs e raise to 4 is 1.6 times; 1.6 times; 1.6 times; 1.6 times 10 raise to minus 76 divided by 4 pi epsilon 0 4 pi epsilon 0 is 9 times 10 raise to 9. So that 81 times 10 raise to 18 divided by h cross square h cross is roughly 1 times 10 raise to minus 34. So, that is 68.

And that roughly comes out to be 13.6 electron volts. So, E n is minus 13.6 over n square e v electron volts where let me just terrify 1 electron volt is 1.6 times 10 raise to minus 19 joules and this is a joules. So, this is the energy of an electron in hydrogen atom and then the third thing that Bohr said is that these electrons that have energies which are different, so minus 13.6 e v minus 13.6 over 4 e v minus 13.6 over 9 e v. These distance are shown to be roughly equal, but in reality as you go to higher and higher levels, they become smaller. So, I should actually be plotting at I minus 13.6 minus 13.6 0.4, 13.6 by 9; 13.6 by 16 and so, they come closer and closer and closer.

So minus 13.6 e v minus 13.6 over 4 e v and so on what he said is when electrons emit light they jump from one orbit to the other in doing. So, they emit one photon. So, what Bohr said is that when.

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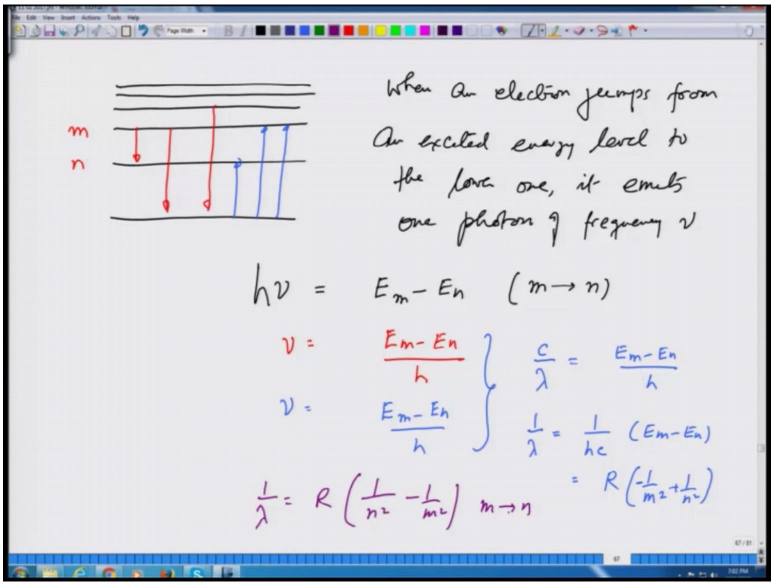


So, these are the levels when an electron jumps from an excited energy level to lower one, it emits 1 photon of frequency nu. So, this h nu which is the energy is going to be equal to E m minus E n when m to n jump is made. So, suppose this level is m, this level is n and this electron jumps from here to here or here to here or here to here, it gives a frequency nu which is E m minus E n over h and the reverse process when it absorbs energy exactly same frequency is going to be observed E m minus E n over h.

So, those frequencies are going to be equal and they are going to have very specific values. This is sometimes expressed in terms of frequency this is C over lambda is going to be E m minus E n over h and therefore, 1 over lambda is going to be 1 over h c E m minus E n, sometimes this is written as Rydberg constant 1 over m square minus 1 over n square and this is to have a positive sign, this is going to be negative like this, all right. So, 1 over lambda comes out to be Rydberg constant 1ne over n square minus 1 over m square when m to n transition is made and that explained why the spectrum was discrete and that was a success of Bohr model.

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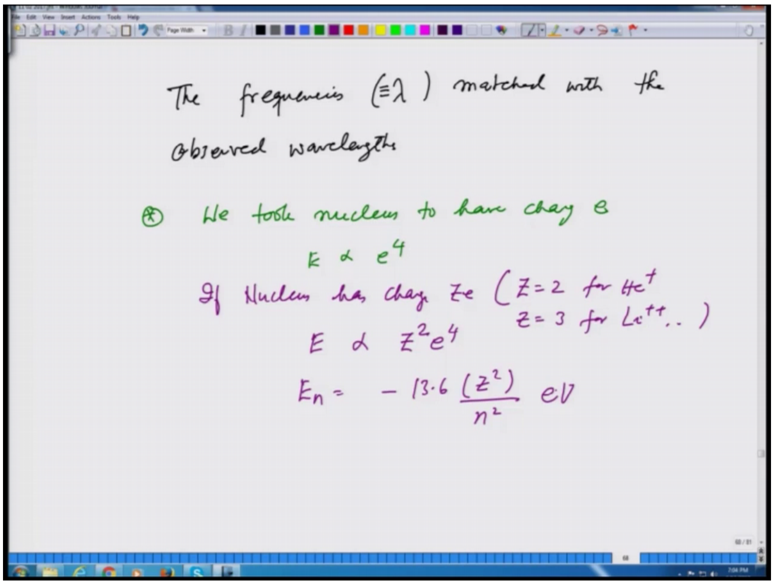


So, let us just say this that the frequencies or lambda equivalently matched with the observed wavelengths. So, we develop quantum theory applied it to explain specific heat of solids and now applied it to explain the hydrogen spectrum theory must be write alright. Now I just want to make 2 points; number 1; we took nucleus in this case to have charge E. So, my energy was proportional to e raise to 4, if nucleus has charge z e; that means, z equals 2 for helium plus z equals 3 for lithium plus plus and so on, then E would be proportional to z square e raise to 4. So, E n then comes out to be minus 13.6 times z square over n square e v that is point number one.

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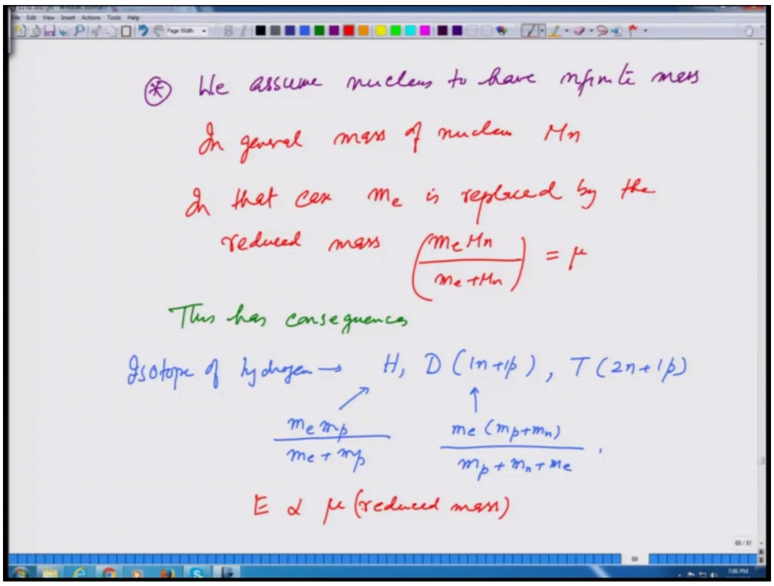

Point number two; we assumed nucleus to have infinite mass in general mass of nucleus is not infinite, but m n in that case m e electron mass is replaced by the reduced mass which is m E m n divided by m e plus m n and this has consequences what are the consequences suppose I take isotope, let us say isotope of hydrogen which are hydrogen deuterium which has 1 neutron plus 1 proton tritium which has 2 neutrons plus 1 protons.

So, the nuclear charge is still 1, but they are more neutrons and therefore, mass changes this would have a reduced mass of m e m proton divided by m e plus m proton, this would have a reduced mass of m e times m proton plus m neutron divided by m proton plus m neutron plus m e and this would have some other reduced mass and that would affect the spectrum because energy is proportional to the mu this is by the way denoted mu or the reduced mass.

And therefore, there will be slightly slight difference in the energy of hydrogen deuterium and therefore, is spectrum would shift a bit and that would. So, wavelengths for hydrogen and deuterium are going to be slightly different and in fact, that is how deuterium was discovered if the quantity is deuterium is very small, then the line strength or the intensity of line is going to be very small, but it is going to be there. So, keep this in mind that 2 things if we have hydrogen like ions where z is higher energy goes up as z square and energy also depends on the nucleus mass.

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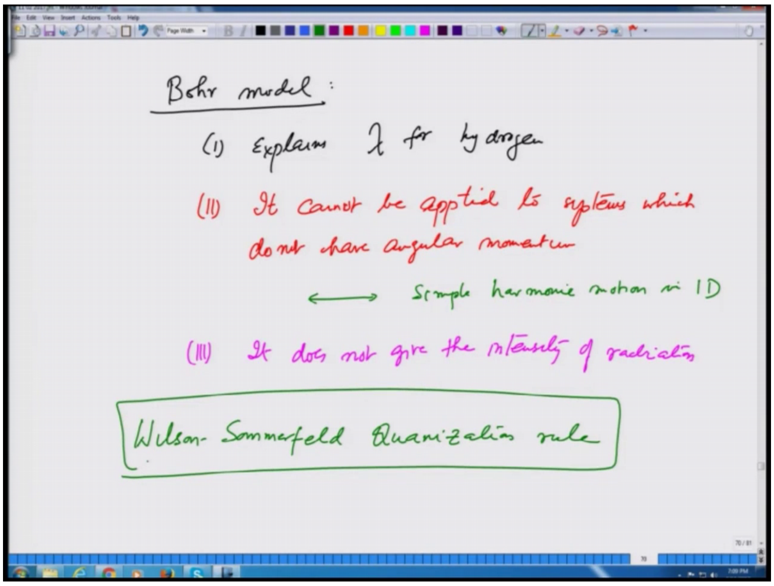


So, to conclude this lecture, Bohr model explains lambda for hydrogen.

However, I must point out that it cannot be applied to systems which do not have angular momentum. For example, I leave you with this thought, suppose there is a particle that is performing simple harmonic motion in one day, how do I apply Bohr model to it, number 3 does not give the intensity of radiation. So, to conclude Bohr model was a step in the right direction, it applied quantum theory and it could explain why they only certain wavelengths are observed. However, there was much more to be done did not explain it did not tell you how to quantize one dimensional motion for example, or other systems and it did not gave the intensity.

So, this was the build up to quantum theory. Next week I am going to generalize Bohr model to, so that it can be applied to other systems also and this is going to be done through what is known as Wilson Sommerfeld quantization that is going to be done next week.

Thank you.