Computational Science and Engineering using Python
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Lecture – 19B
PDE Solver: Wave Equation using Finite Difference Method

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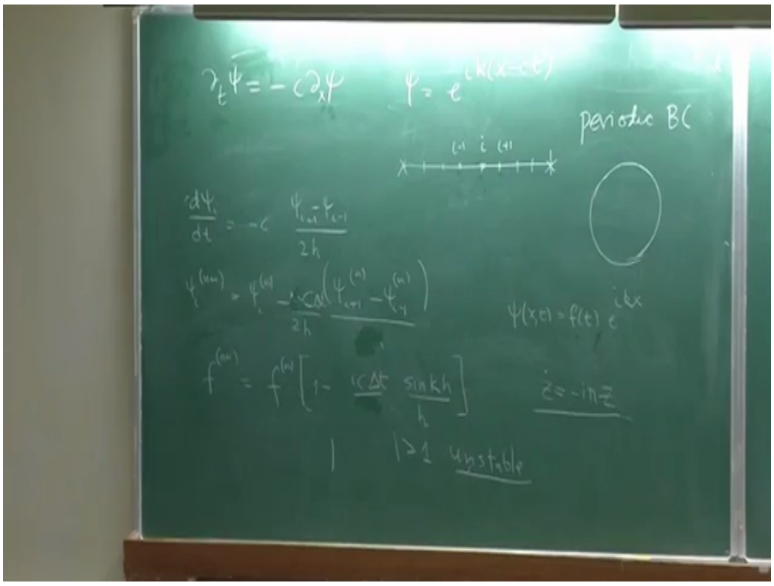

Now, the next equation, I want to solve in fact, there will be equation, but its first one in time and first fall in x. So, it can be written as super position of many waves, it is exact solution for so that is the exact solution plug it in. So, if I want to solve it using finite difference, so again I do the time discretization edges, now these are wave. So, it is not a good idea to put 0s at the ends; no, is it good idea if we put the wave will move. So, 0 will not remain 0. So, one important another boundary condition is called periodic boundary condition. So, since this is a wave, just think that whatever wave comes out; comes back here is a very commonly used boundary condition in numerics, but also in physics. So, if something is moving on a circle is a periodic.

So, these are waves moving in a cylinder. So, that will be a periodic wave. So, here do you things those lines, let us do the time stepping of this. So, again please remember that I have i here i plus 1 i minus 1 and so, I am writing equation for one of the points, but at the n of this point. So, I need to write for all of them, it will be carefully boundary condition only things you had; one important point is boundary condition you have to be careful. If you at the same point; if you do from I mean periodic waves, this point is same as this point. So, you should not time step it both the points anyway. Let us try to write down now equation for i. So, d psi i d t or c. Now I can use Euler's scheme for this first order.

So, I will write this as psi i minus psi i minus 1 by h. So or let us make a better accurate scheme for i plus 1 minus by 2 h. So, this central difference; so, derivative at this point is psi i plus 1 minus psi minus 1 by 2 h; happy. Now i time step. So, psi i n plus 1 is psi i n minus c psi i plus 1 n minus psi i minus 1 n by 2 h. So, this is Euler explicit scheme. Now I am not sure whether I am community with you or not, but this is straightforward Euler explicit scheme a delta t see this delta t it has to be there now given time give us psi n; I can go to psi n plus 1 for every i so; that means, I am got the value at next time step, then next time step next time step, I can go from t equal to 0 to p final.

Now question is this scheme stable; now stability you will find if you start solving it you expect a way, but you will not it is the way will not come, I will need to check that this system is unstable, I use the same idea, I try the function psi x t (Refer Time: 04:38); I will look for f t; f t should not blow up or ft should not go plus minus plus minus if you plug it in. So, I will get. So, shall I write the answer instead doing the algebra? So, this part you can do it yourself and what comes out. So, for n, I can write equation for f the exactly what I did here this one this one. So, this equation comes out to be f n plus 1. So, this equation for f; now is f stable or unstable; now you remember what we did for ODS; is this solution stable; yes or no, yes, yeah.

Student: (Refer Time: 06:16)

Yes why not. So, modulus of this greater than 1; exactly like what I was trying to solve in the last in one of the classes minus I pi z this was unstable for Euler explicit the ideas are very simple ideas are identical stuff, but here for f, I get this; the mod of this is greater than 1. So, it is unstable. So, we are expecting a sin wave, but it will not sin wave, but it will the wave amplitude will keep going is a moving wave know like moving wave it will just keep going. So, numerals scheme will not give you the correct result. So, we need to try to find another scheme which overcomes this problem. So, that I will do we will do till in the next class. So, we will gain focus on this equation we will try to solve it. So, that it is I get stability; otherwise your solution is not well behaved and then we will try to solve buzzer fluid equation.