Engineering Mechanics
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Module 1
Lecture No 09
Introduction to vectors Solved Examples-II

In this lecture, we will conclude the vector review of vector algebra by solving a few problems.

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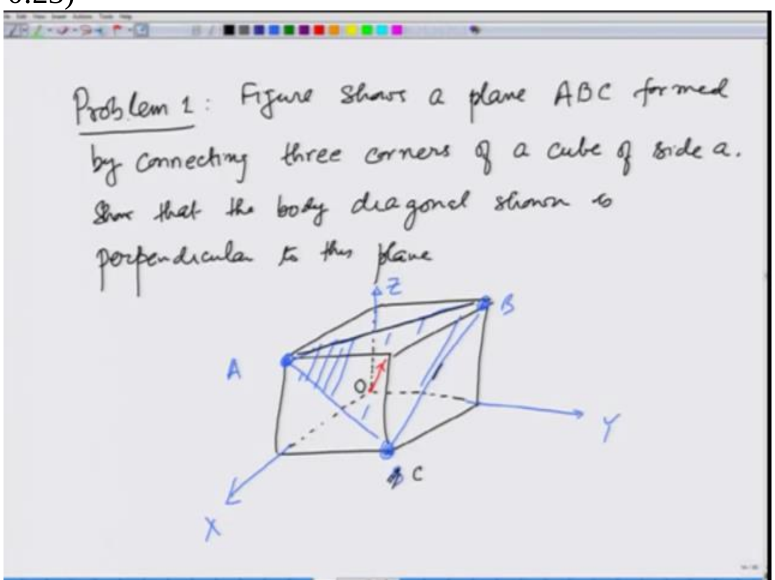

So I am going to solve problem number 1 which is this figure shows a plane ABC formed by connecting 3 corners of a cube a side A. Show that the body diagonal shown is perpendicular to the plane. And the figure we are talking about is this cube of site A and 3 points A, B, C are these. A, B, and C and the plane formed is this triangle ABC. And the body diagonal that we are talking about is going from this point to this point and we want to show that this is perpendicular to this blue plane formed by these 3 lines, AB, BC.

For convenience what I am going to do is take this access to be the X axis, this axis to be the Y axis and this axis to be the Z axis and this corner to be the origin. Any plane you will recall is defined by 2 lines. Right? So we can take AB and B BC. Sorry, this is C. BC to be the 2 lines defining this plane. And we want to use vector algebra.

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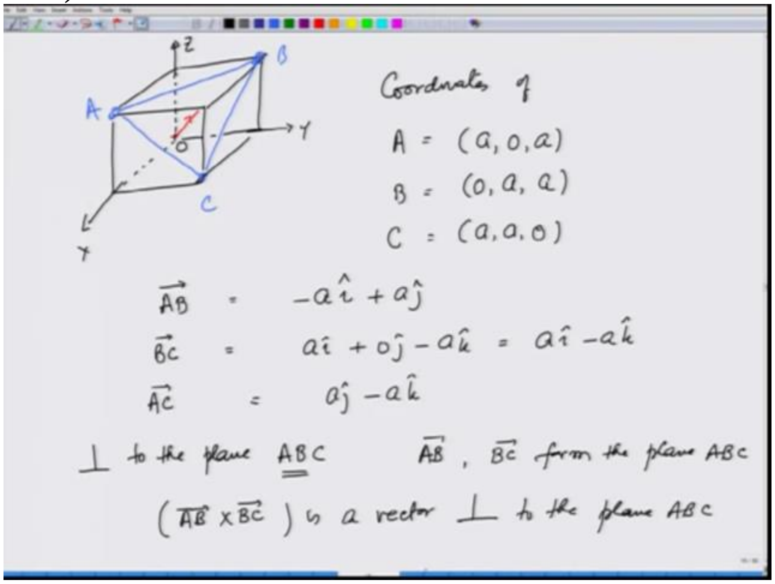

So I will draw the plane, the cube and the plane again. This is the cube. This is the plane we are talking about, ABC and this is the diagonal. Right? This is my origin. This is my X axis, this is my Y axis, this is my Z axis. So the coordinates of A are nothing but A in direction X, 0 in direction Y and A in direction Z. So this is A 0 A.

Coordinates of B are 0 in direction X, A in direction Y and A in direction Z. Coordinates of C are 0 in direction Z. But in X and Y direction I have to move by A and A. And therefore vector AB recall from how we formed vectors in the previous problem-solving sessions, is going to be coordinates of B - coordinates of A. And therefore - AI + AJ. Vector BC is going to be equal to C coordinates of C - coordinates of B and this is going to be AI + 0J - AK.

So I can write this as AI - AK. And vector AC from A to C is going to be coordinates of C - coordinates of A and that is going to be 0 in the A - A is 0 AJ - AK. So these are the 3 vectors we have formed. Now a perpendicular to the plane ABC is going to be given by cross products of any of these 3 vectors. Remember, these vectors AB, BC from the plane ABC, two vectors I have to form plane ABC.

And A cross B cross BC is going to be a vector perpendicular to the plane ABC because ABC form a plane and cross product is always perpendicular to that the plane formed by those 2 vectors. So let us calculate AB cross BC.

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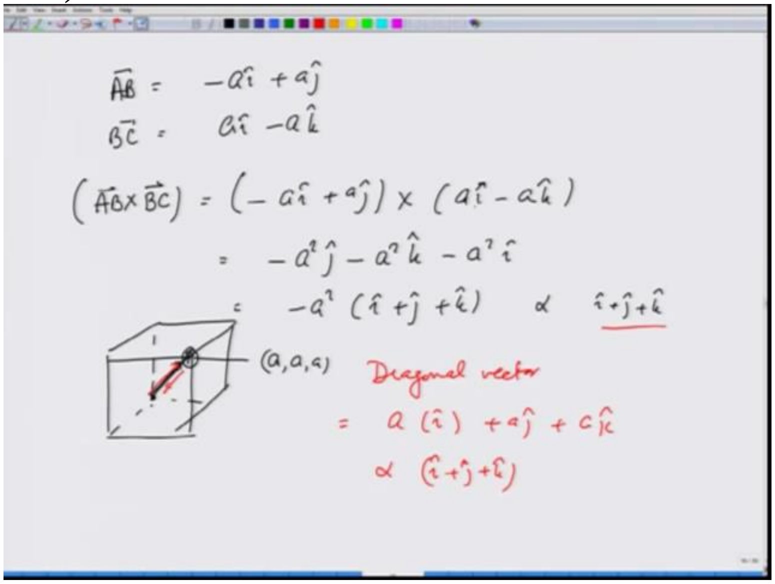

AB is nothing but - AI + AJ. BC is nothing but let us see what BC is. BC is nothing but AI - AK. So AB cross BC is nothing but - AI + AJ Cross AI - AK which is equal to I cross I is 0. I cross K is - J. So - A square J. J cross I is - A square K and J cross K is I. And therefore this is - A square I which I can write as - A square I + J + K.

So this vector AB cross BC is proportional to vector I + J + K. And how about a vector in the diagonal direction? Let us see that. Diagonal direction is from here, the origin to this point. The coordinate of this point is nothing but A, A, A. So this diagonal vector is nothing but AI + AJ + AK. Again proportional to I + J + K. So notice, AB cross BC was proportional to AI + J + K although in the negative direction, that is okay.

And the diagonal is in this direction, both are parallel. That immediately tells you that diagonal is perpendicular to the plane.

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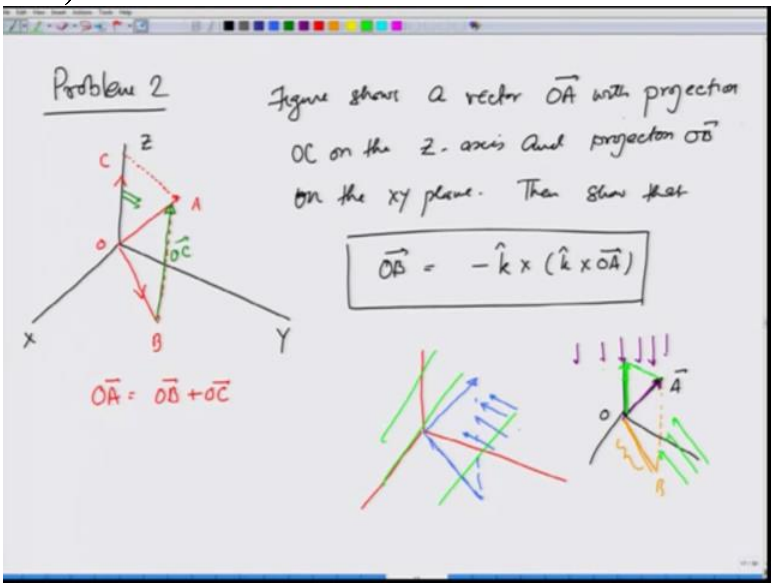

Problem number 2. Figure shows, so let me draw the figure 1st. X, Y, Z shows a vector OA with projection on Z axis OC and projection in XY plane OB. So here is a vector OA and its projection on the Z axis is OC and projection in the XY plane is OB. This is OB and this is OC. So you can see immediately that OA is nothing but OB + OC because if this vector is transported parallel to itself, this is also OC.

So I am drawing these pictures so that your imagination in three-dimension becomes better and better. So the problem is, figure shows a vector OA with projection OC on the Z axis and projection OB on the XY plane. Then show that OB is equal to - K cross K cross OA. This is what we are supposed to show. Let me give you a little idea about when we say projection, always think of projection like this.

Suppose I have this axis, X, Y, Z, and the vector OA, think of light falling parallel to the XY plane on Z along its projection or its component in the XY plane. So better would be, I will change the order. If I look at XYZ and this vector OA if I want to see its projection on the XY plane, think of light falling parallel to the Z axis on this vector. Okay? Think of this vector as a stick.

The shadow that this light makes on the XY plane, the shadow is going to be the projection in the XY plane. Next, think of light falling along this shadow perpendicular to the Z axis. Then the shadow that it makes on the Z axis is going to be the projection on the Z axis. So I am giving you a physical way of thinking, how to think of these projections? Projection is nothing but shadows made if you think of that vector as a stick and let light come from other direction.

So 1st if you want to see the projection in the XY plane, let light come parallel to the Z axis and the shadow in the XY plane gives you the projection on the XY plane. And then, parallel to that shadow you shine light perpendicular to the Z axis and the shadow it makes on the Z axis is going to be the projection on the Z axis.

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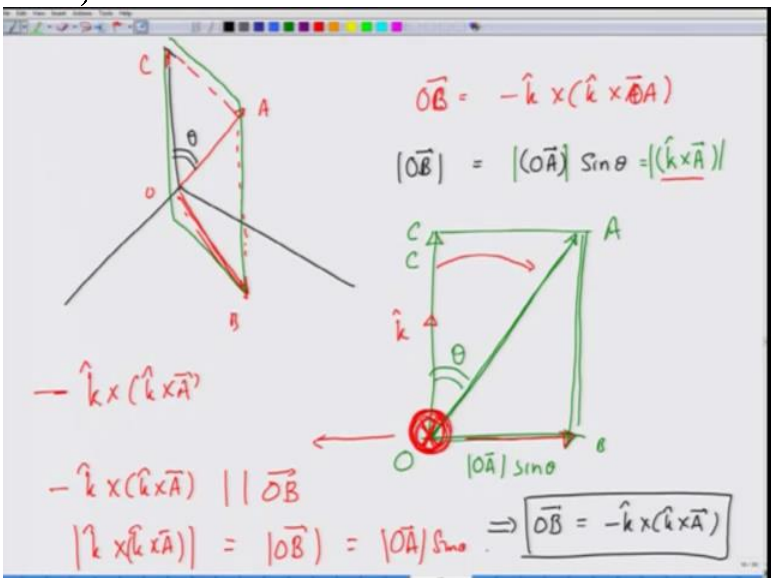

So now back to the problem. The problem that we have is this. I have this vector OA and its projection in the XY plane. That is why you see I am drawing kind of light going down is OB and its projection on the Z axis you see I am going parallel to this OB is OC. And I want to show that OB is equal to - K Cross K cross OA. Now see if you notice OB magnitude.

Right? If this angle from the Z axis is theta OB magnitude is going to be nothing but OA magnitude times sine of theta. Let me explain that a little better. Look at this plane. Look at the plane OABC. Let me draw this plane for you. This is my plane OAC and B. This is projection OB, this is projection OC. See what I am trying to do is to visualise things better, I am trying to draw a two-dimensional figure.

So I have drawn this figure OABC OABC and she is my vector OA. This angle is given to be theta. So you can immediately see that if I look at this triangle OAB, this is nothing but OA magnitude, this should be magnitude sine of theta. Okay? So this by definition is nothing but K cross A magnitude because this vector is K unit vector. K cross A is going to be nothing but A mod of K mod of OA sine theta.

Mod of K is 1. So this is nothing but modulus of K cross A. Notice, if I apply my right hand tool, K cross A Right? K cross K in this plane, I turn my fingers from K to A and this is going to be K cross A going into the plane of this paper. So this magnitude of K cross A is all right. It is OA sine theta but it is perpendicular to the plane OBAC. I want to bring it back to OB.

So if I take K cross K cross A. K cross A I have already shown right here, I am turning my pen around here. It is going into the page. If I turn my fingers from K to into the page, I get a vector in this direction. I want to switch it to the other side, I better put a - sign here to the left of the screen. So - K cross K cross A is parallel to OB and magnitude of K cross K cross K is nothing but magnitude of OB.

Because this is nothing but OA sine of theta. And therefore it is immediately clear that OB this implies that OB is in the direction of - K cross K cross A and has the right magnitude. And so this is proved.

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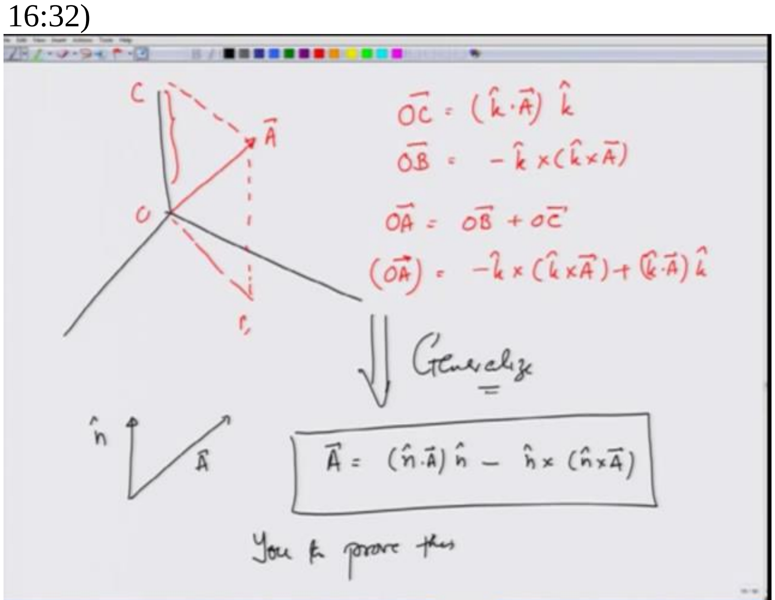

This also then tells you, again let me make this picture. This is my vector OA throwing light parallel to the Z axis. So this becomes OB. And throwing light parallel to this projection, this becomes OC. OC is nothing but K dot A. Right? This is nothing but it is OA sine theta in the direction K. And OB we have just shown, it is nothing but - K cross K cross A.

And OA is nothing but OB + OC. So a vector OA is nothing but - K cross K cross K + K dot A. A generalisation with this problem I can now generalise this. If you have a vector A and a unique vector N, then the vector A is equal to N dot AN . That is if I take the projection of A in the direction of N, multiplied by N - N cross N cross A. I leave this for you to prove. I will give you hints.

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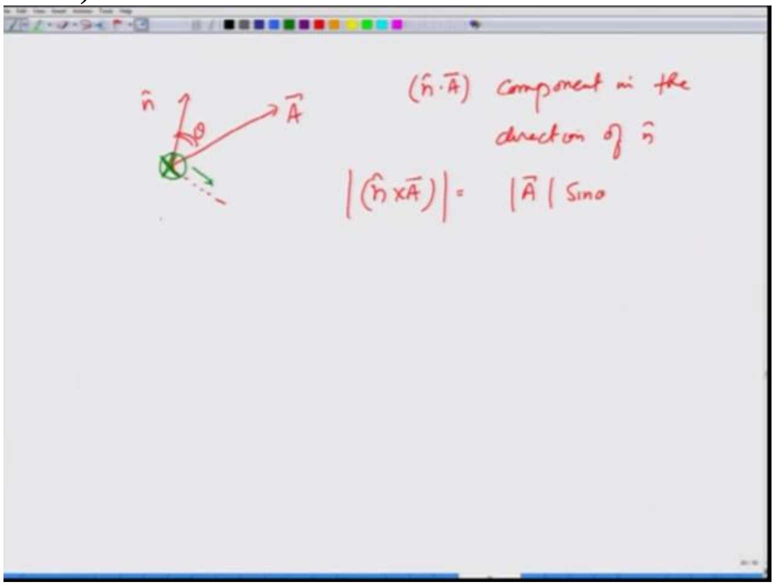

So again if I have a unit vector NA then N not A is component in the direction of N. N cross A is nothing but if this angle is theta, its magnitude is nothing but magnitude of A sine theta. Right? This is the projection perpendicular to N. And N cross A is going to be in the direction perpendicular to the plane formed by N and A. I have to bring it back in the direction of protection of A in the plane perpendicular to N.

And you use the same trick as we did earlier. This concludes our review of vectors. I will be giving an assignment on this topic soon.