Engineering Mechanics

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Module 1

Lecture No 08

Kronecker Delta and Levi-Civita symbols-II

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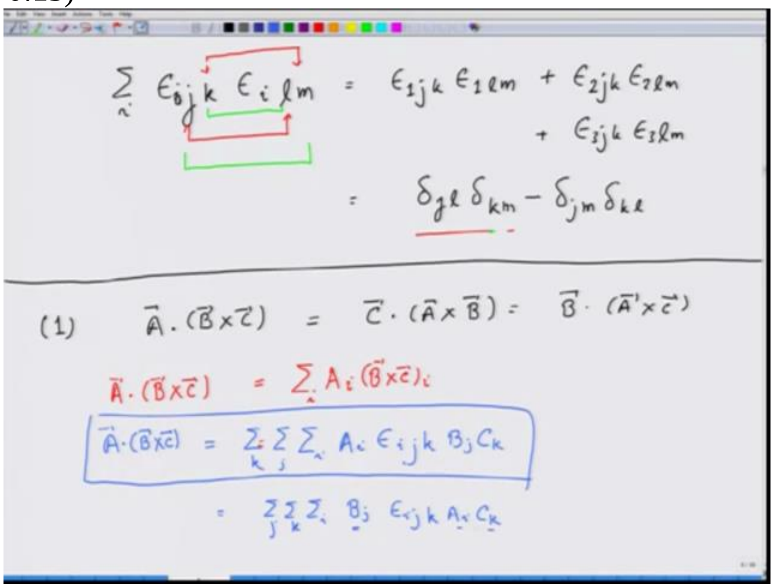

One problem I am going to do is prove that A dot B cross C is same as C dot A cross B is same as B dot A cross C. All right? So let us do that. Let us take A dot B cross C which is going to be nothing but AI B cross C I summation over I by definition of dot product which I can write as summation over IAI Epsilon IJKBJCK. All right?

And then I am summing over J, I am summing over K. So A dot B cross C is equal to this. Now let us see what do we do next? Next what I can do is I can change the order. Let me write this in the form of summation J summation K, I can sum in any order I like. This is commutative. IBJ Epsilon IJKAICK. All right? So now I will sum over BJ later and I will 1st sum over I and K.

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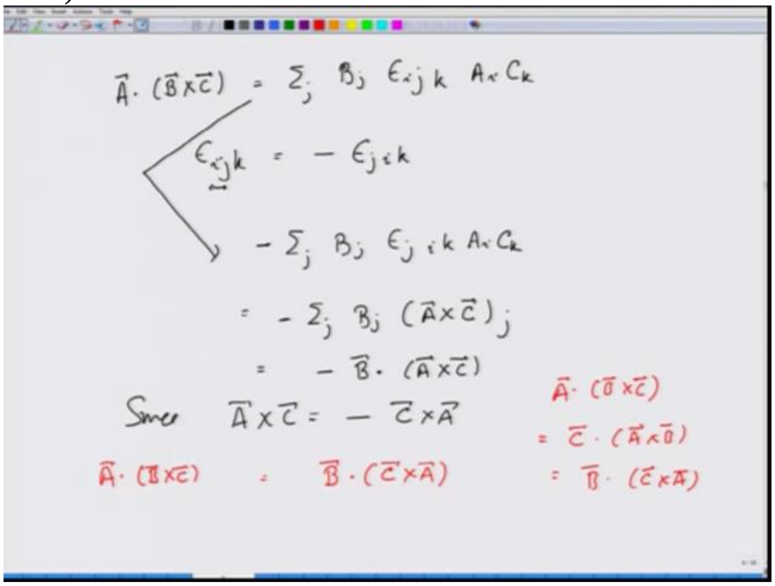

So what have we got? We have got A dot B cross C summation over JBJ Epsilon IJKAICK. I am now going to use the property that Epsilon IJK, this is another property which I did not say in the beginning, is always - Epsilon if I switch 2 of the indices, J I K. So I can write this as summation over J with a - sign BJ Epsilon JIKAICK which is nothing but equal to summation over JBJ times A cross CJ component with a - sign which is equal to B dot A cross C with a - sign.

Now since A cross C is equal to - C cross A I can equally well write this as equal to + B dot C cross A. So that is the answer, A dot B cross C is same as B dot C cross A. It goes in a cyclic manner. Right? So A dot B cross C the same as I bring C out dot A cross B is equal to B dot C cross A. What I wrote in the beginning in the previous page is not correct. So you can see how easily I could prove it.

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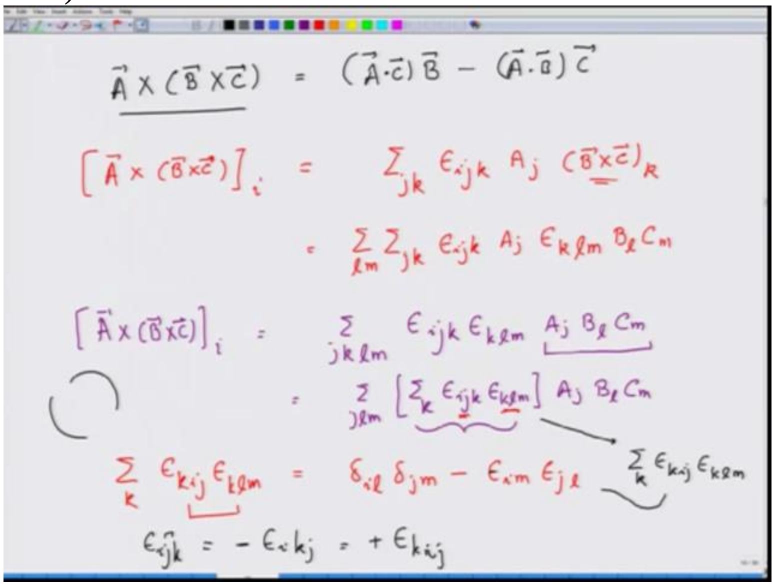

Let me use another problem where I will use these symbols. Let us see what A cross B cross C is. How it can be expressed easily? I am going to show that this is A dot C times vector B - A dot B times vector C. Let us how I can do that. So let us take the Ith component of A cross B cross C. This is going to be equal to summation over J and K Epsilon IJKAJ times B cross C which is another vector, it is Kth component which I can write as summation J, K, Epsilon IJKAJ.

How do I express the Kth component of B cross C is going to be Epsilon KLMBLCM and I am going to sum over L and M. So this is the whole expression. Now I am going to use some properties of Epsilon products. So let us 1st express this again. A cross B cross C Ith component that is X, Y or Z component is equal to summation JKLM Epsilon IJK Epsilon KLMAJBLCM. Notice that AJBLCM, there is no K in it.

So I can write this as Epsilon JLM summed over and then write summation K K sum only over this IJK Epsilon KLMAJBLCM. K sum is only here. Now I know already, I gave you a property that Epsilon KIJ Epsilon KLM summed over K is nothing but delta IL because the direct terms delta JM - Epsilon IM Epsilon JL. So I want to bring this Epsilon KLM is okay, Epsilon IJK into KLM form.

Let us do that. Epsilon IJK, I can switch J and K and write this as Epsilon IKJ and pick up a - sign because now I have changed the direction in that circle in the clock. And again I can switch I and K and write this as Epsilon KIJ with a + sign. So now I have the sum is nothing but summation over K Epsilon KIJ Epsilon KLM. That is nothing but this delta function. Sorry this is the delta function. I am sorry, this is the delta function. All right?

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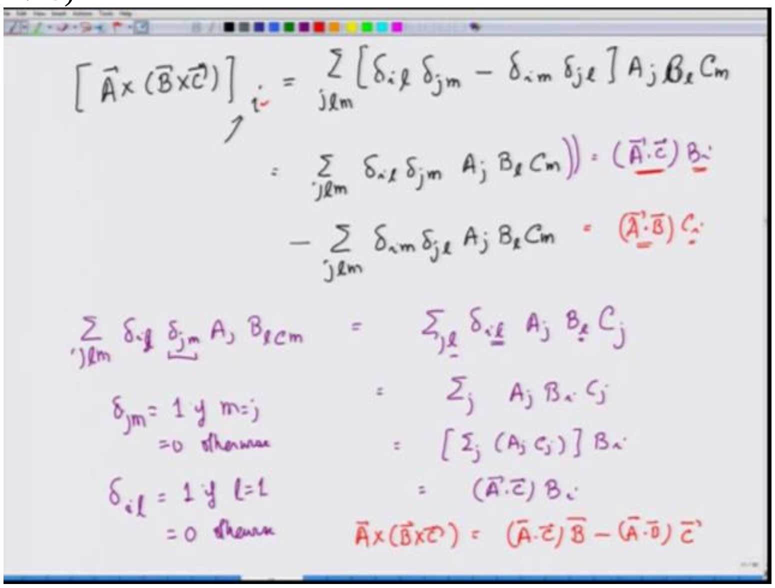

So I can now easily express A cross B cross C Ith component as delta IL delta JM - delta IM delta JL from the previous is AJLBLCM summation over JL and M. Notice I have to say over JL and M because on the left-hand side there is only Ith component. All J, L and M should go away. Let us see that what is it. This is summation JLM delta IL delta JM AJBLCM - summation JLM delta IM delta JL AJBLCM.

Let us do the sum. Let us take the 1st term, summation JLM delta IL delta JM AJBLCM. When I sum over J or M, let us sum over M 1st. M delta JM is equal to 1 if M equals J and 0 otherwise. So what this delta is doing is forcing M to be equal to J. So this is going to be equal to summation JL delta IL M when I sum over, it makes M equal to J.

So this is going to be AJBLCJ. If M is not equal to J, this term gives me 0. Similarly, now if I take delta IL is equal to 1 if L equals to I, 0 otherwise. So this is going to be I am going to sum over L I am going to get summation over J delta IL forces L to be equal to I. So I get AJBICJ which I can write as summation JAJCJBI.

Summation JAJCJ which is A1C1 + A2C2 + A3C3 as nothing but A dot C BI. So you can see that the 1st term is nothing but A dot CBI. Similarly, let leave it for you. You can show that this will come out to be A dot B dot CI. So you can see that the Ith component of A cross B cross C is nothing but this dot product times BI, this dot product times CI.

So 1st component is going to be A dot CB1 - A dot BC1. 2nd component, A dot CB2 - A dot BC2. And therefore, in general you can therefore write A cross B cross C is equal to A dot C B vector - A dot B C vector.

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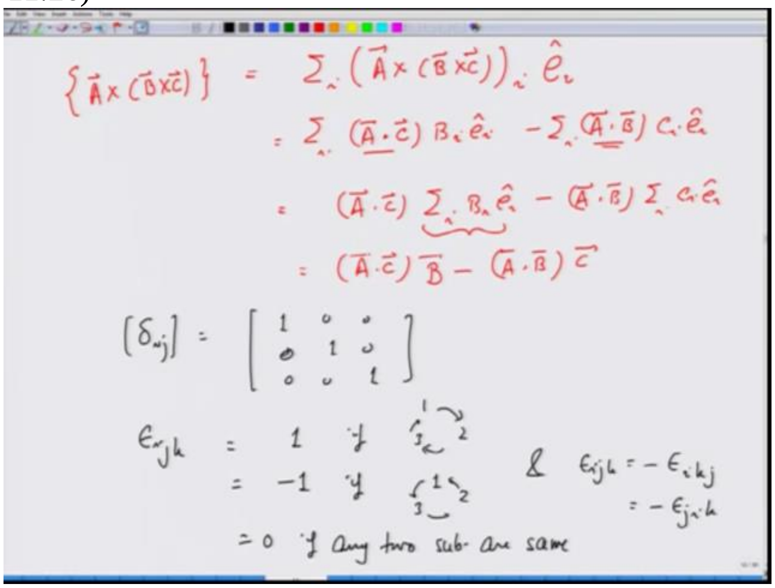

If you are very fussy about the notation, then you can also write that A cross B cross C is nothing but summation IA cross B cross C Ith component unit vector in the Ith direction. And this is nothing but A dot C BIEI summation over I - summation over I A dot BCIEI. Now A dot C and A dot B are scalar quantities. So they come out. A dot C summation I BIEI - A dot B summation I CIEI.

And these are nothing but A dot C. This is vector B - A dot B. That is your answer. So what I have tried to give you through these symbols and delta IJ which is in the matrix form an identity matrix 0, 0, 0, 0, 0 or symbolic Epsilon IJK which is 1 if let me make this symbol, 1, 2, 3 if you make a combination of 1, 2, 3 and going clockwise it is - 1.

If you make 1, 2, 3 combination counterclockwise or anticlockwise direction is 0 if any two subscripts are same. And Epsilon IJK is equal to - Epsilon IKJ or it is also equal to - Epsilon JIK. Using this, how we can make vector operations simpler to handle.