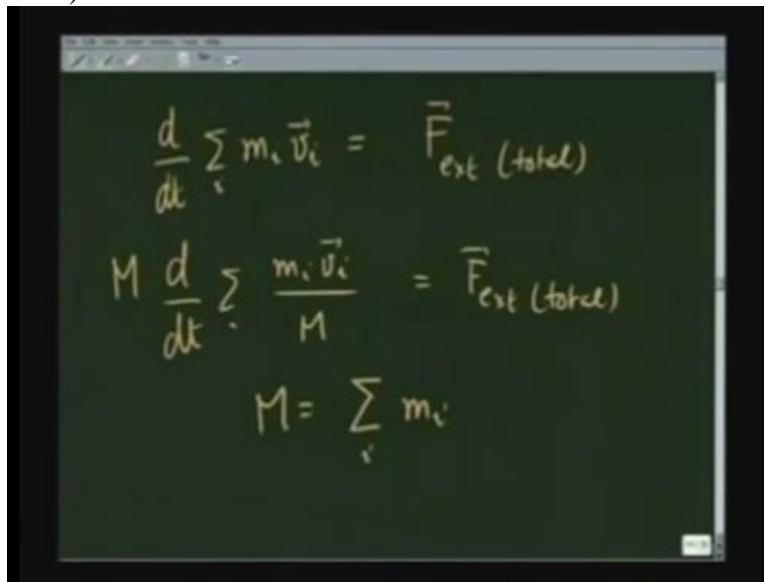


**Engineering Mechanics**  
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**Module 6**  
**Lecture No 50**  
**Linear momentum and centre of mass**

Let us then get a feel for how we can visualise the motion if I look at momentum.

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$$\frac{d}{dt} \sum_i m_i \vec{v}_i = \vec{F}_{\text{ext (total)}}$$
$$M \frac{d}{dt} \sum_i \frac{m_i \vec{v}_i}{M} = \vec{F}_{\text{ext (total)}}$$
$$M = \sum_i m_i$$

Let me again write this equation  $\frac{d}{dt} \sum_i m_i \vec{v}_i = \vec{F}_{\text{ext (total)}}$ . Since this is a collection of particles, mass is a constant. So let me multiply this by mass  $M$ , I will write in a minute what  $M$  is. And write  $\sum_i \frac{m_i \vec{v}_i}{M} = \vec{F}_{\text{ext (total)}}$  where  $M$  is the total mass of the system which does not change with time because I am considering all the particles. No particle is leaving the system or coming into the system.

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The image shows a chalkboard with two equations. The first equation is  $\vec{R}_{\text{centre of mass (CM)}} = \frac{\sum m_i \vec{r}_i}{M}$ . The second equation is  $\vec{V}_{\text{CM}} = \frac{\sum m_i d\vec{r}_i/dt}{M} = \frac{\sum m_i \vec{v}_i}{M}$ .

Then you see, if I define a quantity  $R$  centre of mass and from now on I am going to write it as  $RCM$  is equal to summation  $MIRI$  over  $M$ , then the velocity of the centre of mass  $V_{CM}$  is equal to summation  $MIDRI$  over  $M$  which is equal to summation  $MIVI$  over  $M$ .

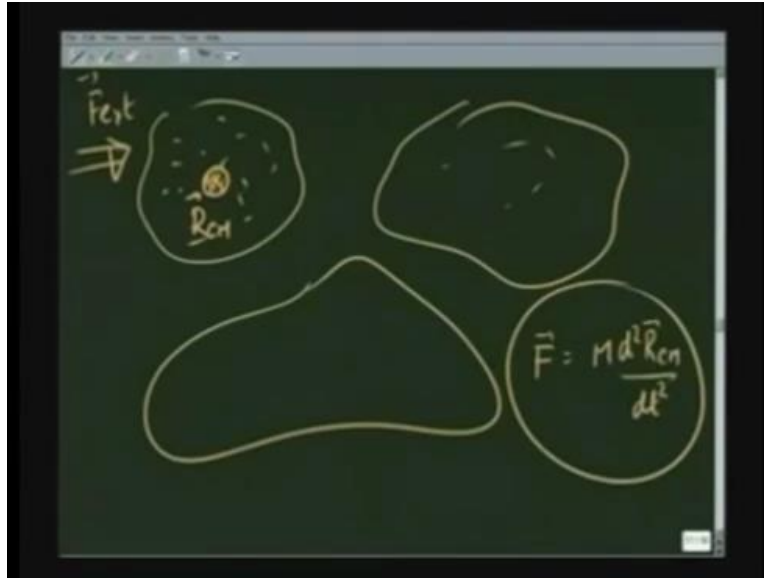
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The image shows a chalkboard with three equations. The first is  $M \frac{d}{dt} \sum \frac{m_i \vec{v}_i}{M} = \vec{F}_{\text{ext (total)}}$ . The second is  $M \frac{d\vec{V}_{\text{CM}}}{dt} = \vec{F}_{\text{total}}$ , which is enclosed in a box. Below the box is a diagram of a collection of particles and the label  $\vec{R}_{\text{CM}}$ .

And the equation  $M \sum \frac{m_i \vec{v}_i}{M} \frac{d}{dt}$  is equal to  $F$  external total can then be written as  $M \frac{dV_{\text{CM}}}{dt} = F$  total. I am dropping term external right now. What does this tell me? This tells me that I have a collection of particles. They may be interacting with each other, they may be doing many many things with each other.

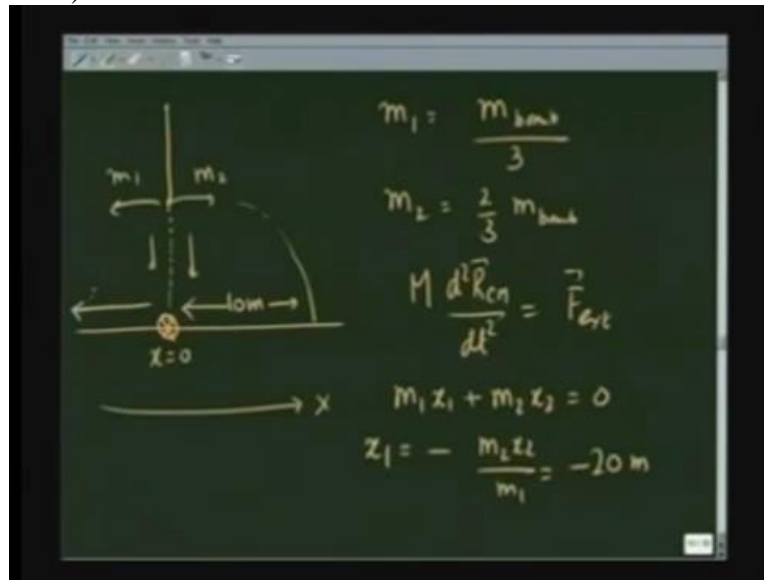
As long as the interaction force between the 2 particles is equal and opposite, there is going to be a point in the system denoted by RCM which is going to move as if it is a point particle with total mass M. This gives me a very nice feel about the system.

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So if I have suppose a body consisting of many particles and there is some net force on it,  $F$  external, this body may get deshaped, it may get different orientation but if I take a point which is same as the centre of mass, this would keep moving according to the equation  $F$  equals  $M \frac{d^2 R_{CM}}{dt^2}$ . And this gives me a very nice way of looking at the motion. I know one point how it would move no matter what the body does.

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To see how the concept of centre of mass helps in understanding or solving a problem for let us take an example where we drop a bomb vertically down so that it would have fallen at a place which I will call X equals 0. I am measuring X in this direction. Considering the bomb as a point particle, its centre of mass is sitting right here but before hitting the ground, it explodes in mid air and breaks into 2 pieces.

One of mass M1, one of mass M2 so that M1 is mass of the bomb divided by 3 and M2 is two thirds the mass of the bomb. So although the bomb explodes, no matter whether it explodes into 2 pieces, 3 pieces or 4 pieces, the centre of mass would still keep on moving as if nothing happened.  $M \frac{d^2 R_{\text{CM}}}{dt^2}$  is still  $F_{\text{external}}$ . And  $F_{\text{external}}$  is only the gravitational force.

So centre of mass would keep on moving this way and when the bomb pieces hit the ground, this would reach here. And that means, as far as the X coordinate is concerned, we are going to have  $M_1 X_1 + M_2 X_2$  is equal to 0. Suppose this piece fell 10 metres from where the bomb would have fallen 10  $X_2$  is 10 and therefore I have  $X_1$  is equal to  $- \frac{M_2 X_2}{M_1}$  and that comes out to be - 20 metres.

So the other piece is going to fall on this side at a distance of 20 metres. So what I am trying to show you through this example is that in many particle system, the concept of centre of mass gives me at least one point for which the motion still remains simple. And we are going to take

step by step by step how to make motion more complicated, that is how we take care of deformation, how we take care of orientational changes and so on.

But for the time being, we focus on the linear momentum, centre of mass motion and the simplest possible way I can describe the motion of the system. We just saw how the concept of centre of mass or the conservation of linear momentum helps in simplifying the solution of a problem. I would let you think what if the drag was also there, what the conservation of linear momentum as we applied just now in this bomb problem be applicable.