Engineering Mechanics
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Module 6
Lecture No 49
Equation of motion in terms of linear momentum and principle of conservation of linear momentum

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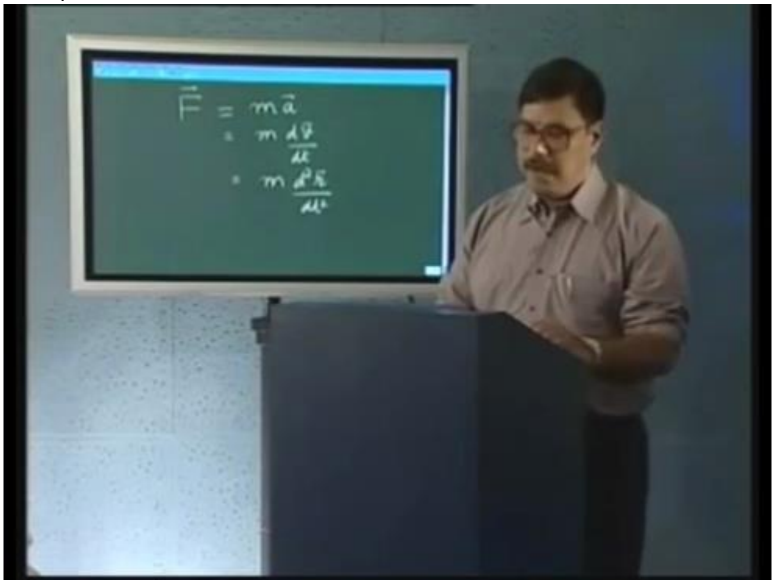

In the previous 2 lectures, we have been looking at the motion of a single particle and we saw that given a particle I essentially get its equation of motion and then solve it to get its velocity or its distance as a function of time. This is essentially what we do although we looked at constrained motion, motion with friction and things like that. Now we are going to make the problem slightly more difficult. We are going to ask a question, what happens when I have more than 1 particle?

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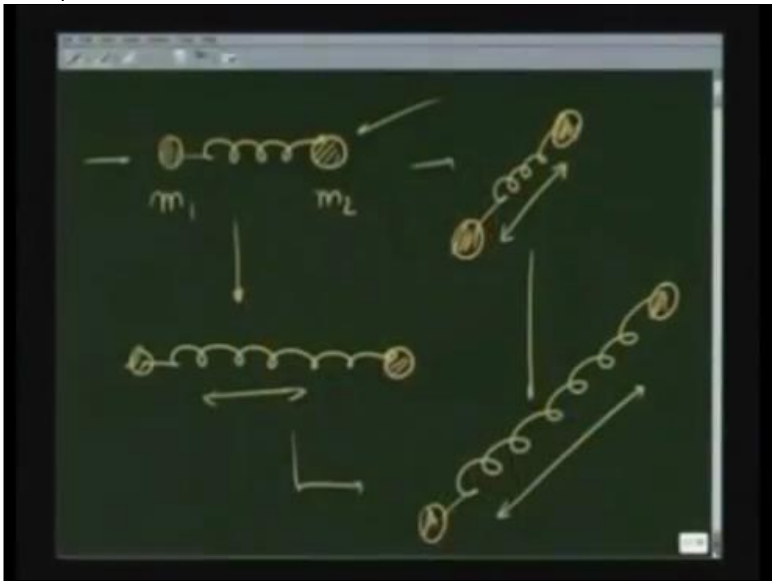

For example, let us take 2 particles say of mass M1 connected by a spring and mass M2 here. You know from experience, if I apply a force on it, it can do 2 or 3 different things. For example, it could either stretch so that particles go farther apart, it can change its orientation or can do both. And mind you, in all this process, there is a force that is acting on both the particles through the spring.

And I am also applying a force on this and a force on this. So how do we go about describing such a motion? And what happens when the number of particles increases? This is what we are going to look at and a quantity that becomes very useful in describing motions when many particles are involved is momentum. Let me motivate that by taking an example.

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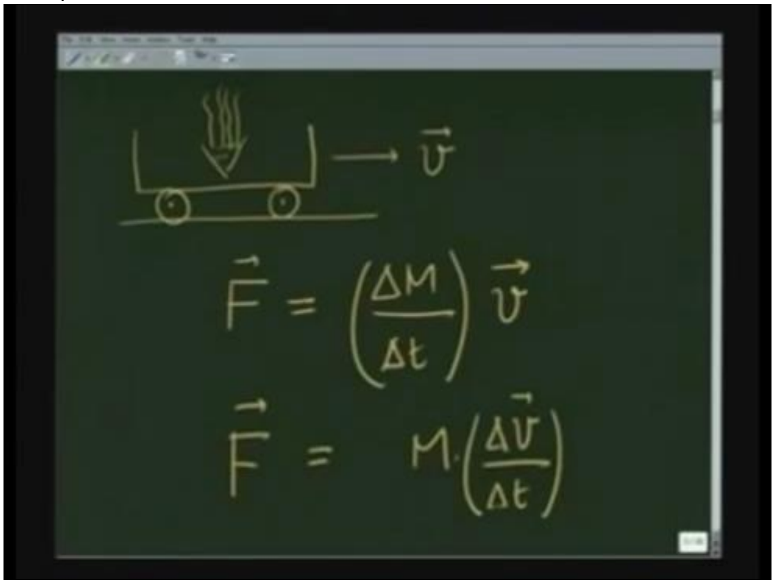

Suppose I have a cart which is moving on a horizontal track without friction. It is moving with velocity V and I start pouring some sand or some mass into it. Either I can put it vertically down or slowly put it in. You know from experience that the cart is going to slow down. In fact if you want to keep it moving with the same speed, you would have to apply a force and that force is going to be proportional to the rate of change of the mass of this cart times V.

On top of it, if the velocity changes, I have to apply more force. Compare this formula with the formula that we have been using so far which is a constant mass particle moving with an acceleration delta V delta T. So in general I have to apply a force if I want to move something with a constant velocity but its mass is changing or its mass is constant and its velocity is changing.

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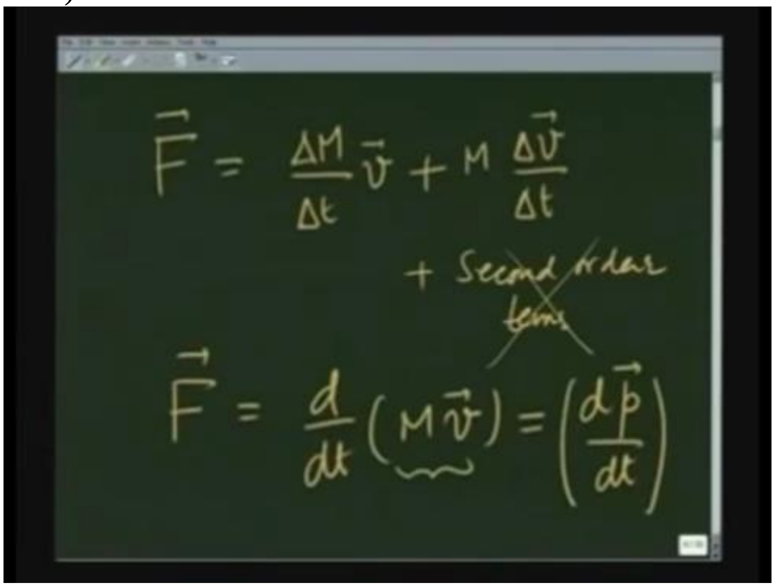

To combine the 2 things, the net force I have to apply when its mass is changing and its velocity is changing is going to be this. I have neglected second order term is M mass and velocity as they go to 0 when I take the limit Delta T going to 0. So in general I can write that the force is equal to D over DT MV.

This is the quantity which I define as the momentum of a mass moving with velocity V. So in general I am going to write F equals DP DT where P stands for momentum. Let us see how this concept helps me in solving problems in a slightly more convenient way.

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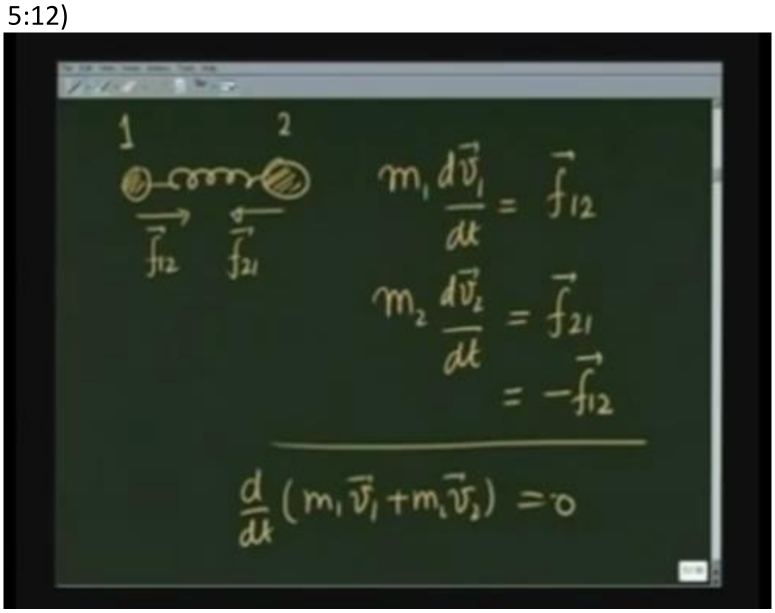

Let us go back to our example of 2 masses which are attached with a spring. Let us assume right now that we are not applying any force on the 2 particles. The only force that is acting between them is through the spring. Let the force en mass 1 be F12 in this direction and let the force en masse to F21. I am using this indices to indicate, F12 indicates force on 1 by 2 and F21 indicates force on 2 by 1.

If I write the equations, the Newton's 2nd law, equation for each mass, I am going to have M1 DV 1 over DT is equal to F12 and I am going to have M2DV2 over DT is equal to F21. But by Newton's 3rd law F21 is going to be opposite and equal to F12. So the magnitudes are the same, the direction is opposite. If I add the 2 equations, I get D over DT of M1V1 + M2V2 is equal to 0.

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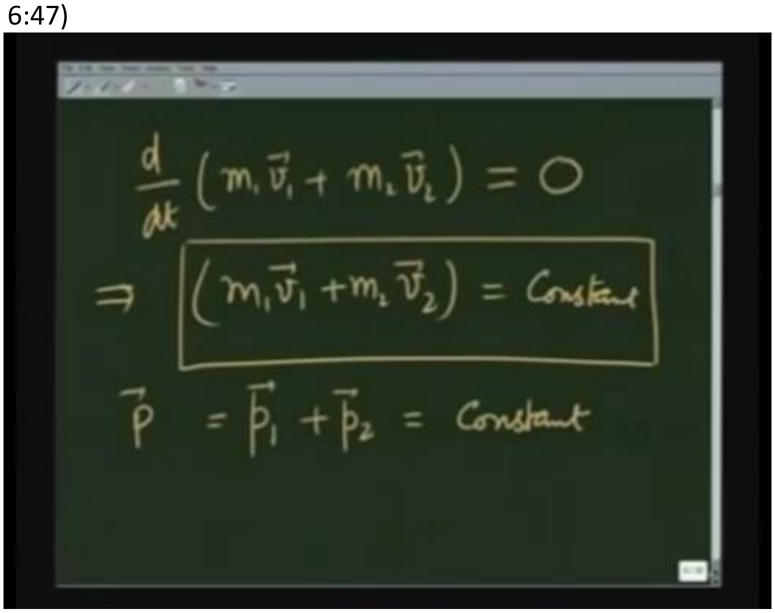

So we get D over DT of M1V1 + M2V2 is equal to 0. And what that implies is M1V1 + M2V2 is a constant. So what we learn is that no matter what the interaction between the 2 particles is, I have taken it to be most general, F12, F21. As long as Newton's 3rd law is satisfied, is operate, M1V1 + M2V2 or the momentum of the 1st particle + the momentum of the 2nd particle which I will call the total momentum of the system is going to remain a constant. This gives me an insight into the problem. The particles may be doing anything on their own.

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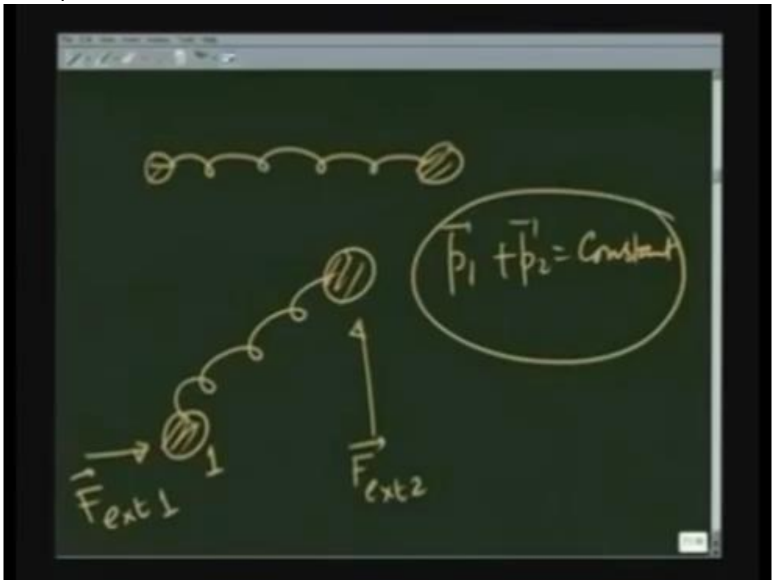

For example, as we said earlier, they could be stretching, they could be rotating. No matter what they do, this quantity is going to remain a constant. This is the statement of conservation of linear momentum in its simplest form. And when combined with other conservation laws like energy conservation, it gives me a great handle to fall mechanics problems. Let us see what happens if the forces were also applied on each of the particles.

For example, I could have on particle 1 that is a force and to distinguish it from the internal forces between the particles, I will call it F external on 1. Let me call force on this which is F external 2 and see what happens what the dynamics of the system is?

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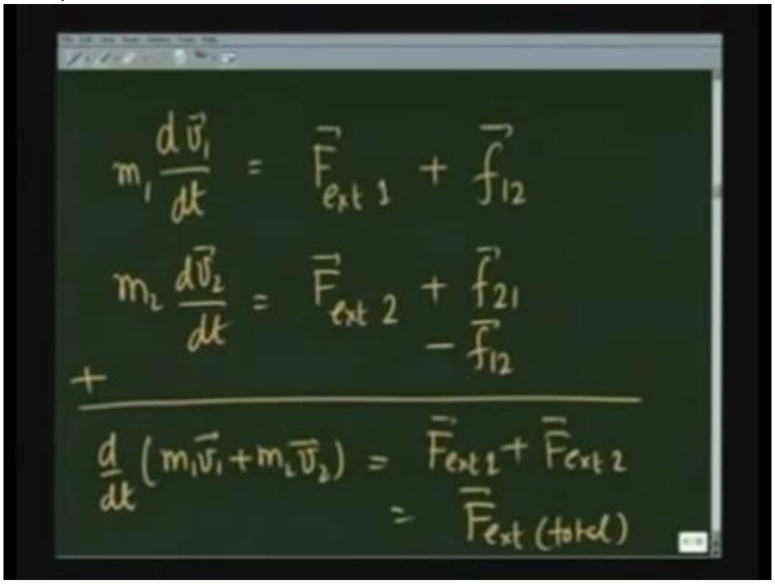

So now I am going to have DV1 over DT Times M1 is equal to F external 1 + F12. And M2 DV2 over DT is equal to F external on particle 2 + F21 which if you recall from the previous slide is - F12. And if I add the 2 equations, I again get that D over DT of M1V1 + M2V2 is equal to F external 1 + F external 2 which is the total force applied on the system.

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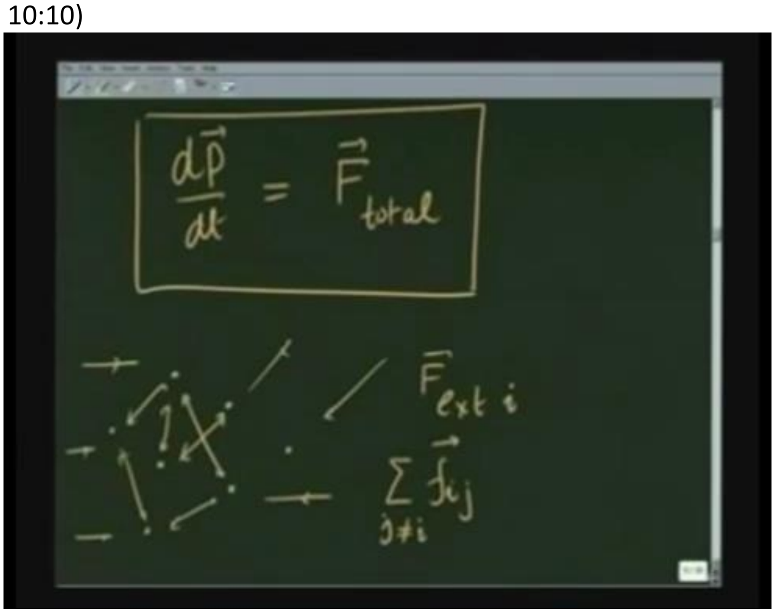

So as long as Newton's 3rd law is applicable, what I learn is that given a system of particles, so what we see is that the rate of change of total momentum is equal to the net or total force applied from outside no matter what is happening between the particles. I took an example of a two particle system. Is it true in general? Let us see.

So suppose I have a collection of particles, many of them and I apply force on each one of them, external force which I will call F external on Ith particle. In addition, they are also interacting with each other which I will call the forces on Ith due to J so that the net force on Ith is going to be sum over J that is force applied by all other particles but not I. It cannot apply a force on itself.

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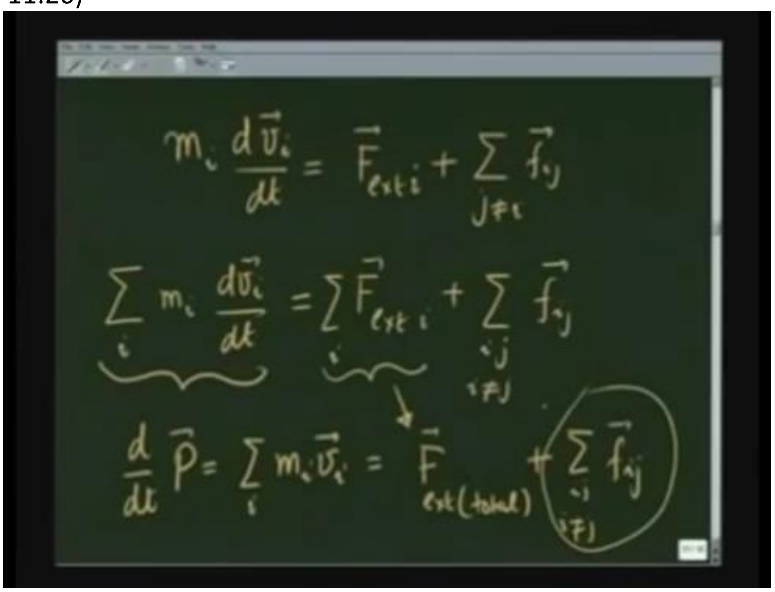

So that if I write the equation for Ith particle, it is going to be MI DVI DT is going to be equal to F external on the Ith particle + the forces due to all the other particles which are going to be equal to summation J J not equal to IFIJ. To see how the net mass of the all the particles move together, I sum over I so that I write this equation as summation over I MI DVI over DT is equal to F external I summation over I + summation IJ over both I not equal to J FIJ summed over.

This is the generalisation of the formula previously written for 2 particle system. This you recognise is the rate of change of total momentum P which I define as summation of individual momentum. This should be equal to this is the net external force. So F external total + this terms IJI not equal to J FIJ. Let us see what this term adds up to. You can already anticipate it should add up to 0.

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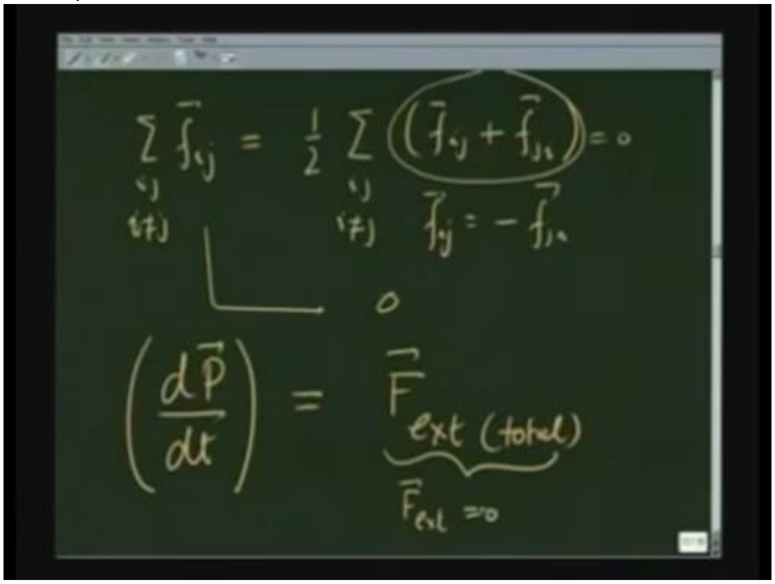

How does that happen? F IJ summed over I and J I not equal to J I can write as one half summation IJ I not equal to J F IJ and just interchange the indices JI. Because I am summing over I and J, it does not really matter. But by Newton's 3rd law, FIJ is equal to - FJI. So this term adds up to 0 and therefore this term is 0. And what we learn then is DP DT for a many many particle system is also equal to F external only where F external is the total force. It is the sum of individual forces applied on each particle.

This is a general statement and use them right away see that if F external is 0, then DP DT is going to be 0 and therefore net momentum is going to be conserved.

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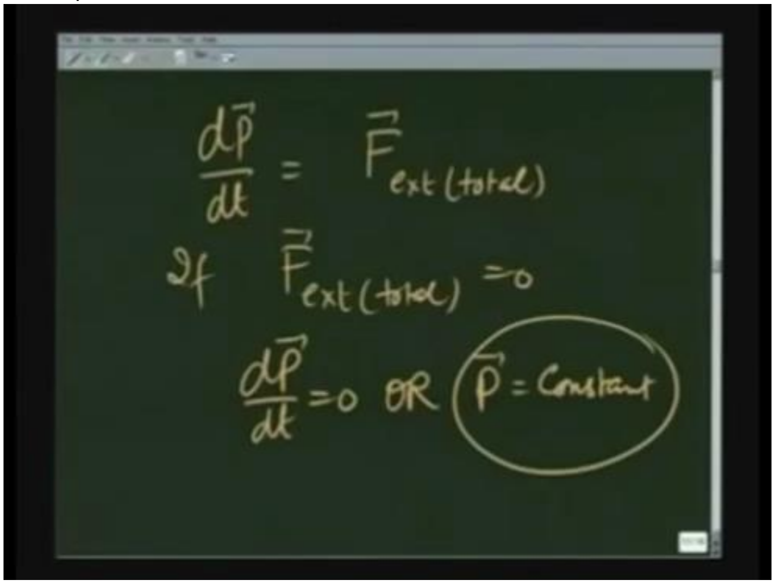

So let us see this again. DP DT is equal to F external total and if the total applied force from outside is 0, DP DT is 0 or equivalently P is a constant. So for a many particle system also, if there is no force applied from outside, the total linear momentum is conserved and that is the fundamental statement of physics. It is used in conjunction with other conservation laws and make solution of problems easier at times.