Engineering Mechanics
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Module 5
Lecture No 47
Motion with dry friction-solved examples

In the previous lecture, we have seen how to solve problems using free body diagrams, isolating subsystems and considering forces on each subsystem. Also, how the subsystems affect the motion of one another was taken into account by looking at the constraints and equations for constraints.

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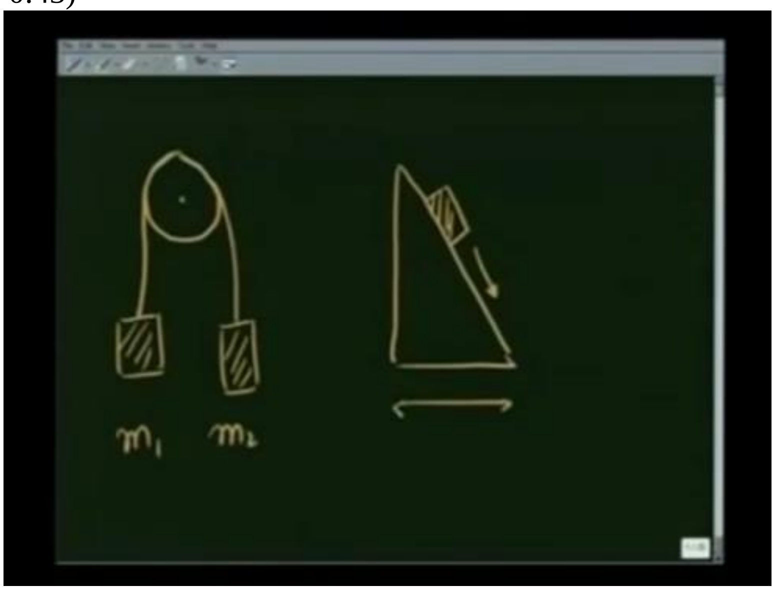

Two examples that we took were one, the Atwood's machine where I had 2 masses going around a pulley, M1 and M2. The other example that we took was a mass sliding down a wedge like this and the wedge would also move horizontally. In these examples, we ignore a ubiquitous force that is encountered everyday in our lives and that is the frictional force. This lecture is going to be about how do we take into account the frictional forces and how does it affect the motion of a particle.

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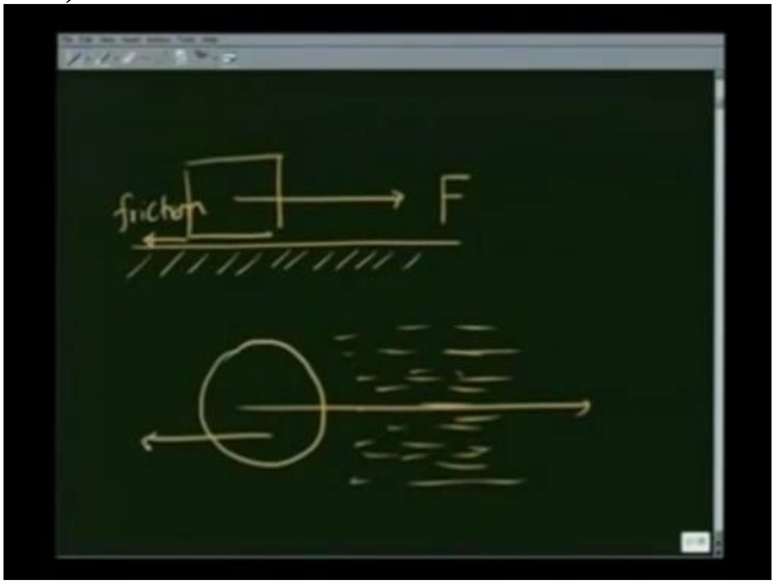

There are when a body moves, we see that its motion is usually resisted by the surface on which it is moving. So if I apply a force F this way, there is the surface applies a force called frictional force that opposes this motion. Even if the body is not moving, frictional force has a tendency to oppose the tendency to move.

Another example of frictional force is the drag force or the viscous force that we encounter when a body moves through a fluid. This force also opposes the motion. In this lecture, we would be looking at both kinds of forces. So let us start with a force of this kind which is when a solid moves on another solid.

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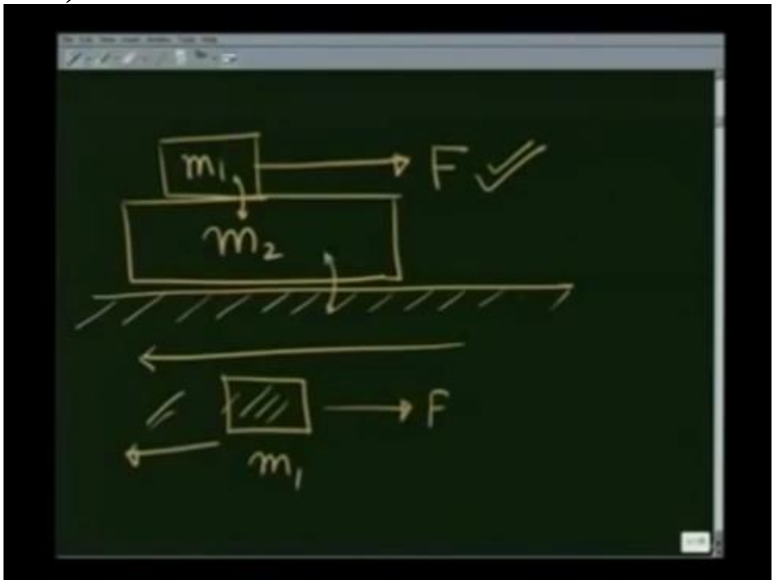

Let us take a mass M1 and another mass M2 and let me pull this mass with some force F. If you recall your everyday experience, you would see that the top mass or the bottom mass does not move until the force has reached certain value. And that is because their tendency to move is being opposed by the frictional force either between these 2 masses or between this mass and the surface on which this mass is resting.

So 1st question is, is the frictional force of constant amount or does it vary as I vary this force? Let us give an argument. If suppose the frictional force were of constant amount then that means, this mass M1 which is on the top, experiences a constant force in this direction. Then even if I did not apply any of this force F, because of this force, the mass would tend to move this way and I know that does not happen.

And that means, the frictional force adjusts itself as I apply the force. How does it adjust itself? It is just sufficient to oppose the motion or the tendency to move

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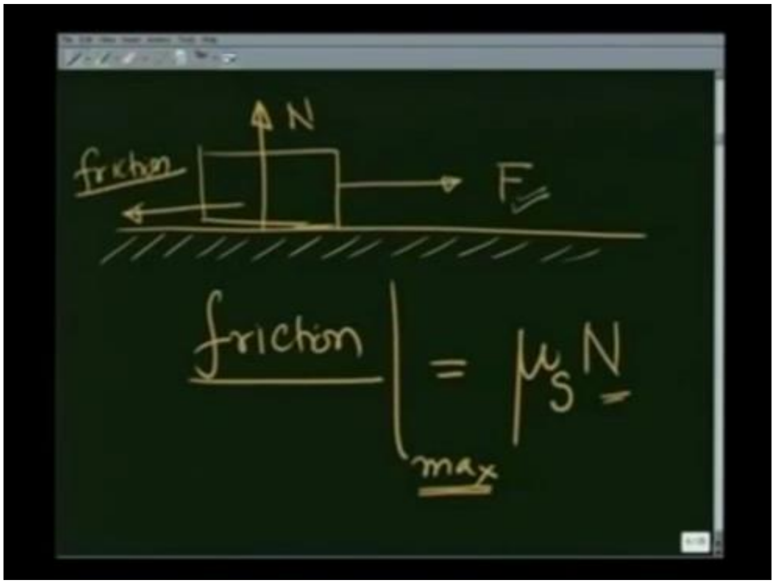

So for example if I take a mass sitting on a surface and I try to pull it by a force F, its tendency to move will be opposed by this frictional force. It is observed experimentally that the maximum frictional force that a surface can apply on a given mass is given by myu where myu is a constant times N and I will put a subscript S here to indicate that this is static friction.

Friction is slightly different with the body starts moving and we will be discussing that. Notice that I have written a maximum here and that is to indicate that that is the maximum possible frictional force that is available. Of course, the frictional force adjusts itself as I vary this force.

So the maximum possible is this. Otherwise it is always less than just sufficient so that this body does not move. What is N? N is the normal reaction of the surface on the body. So N is the force that is perpendicular to the surface on which the body is moving.

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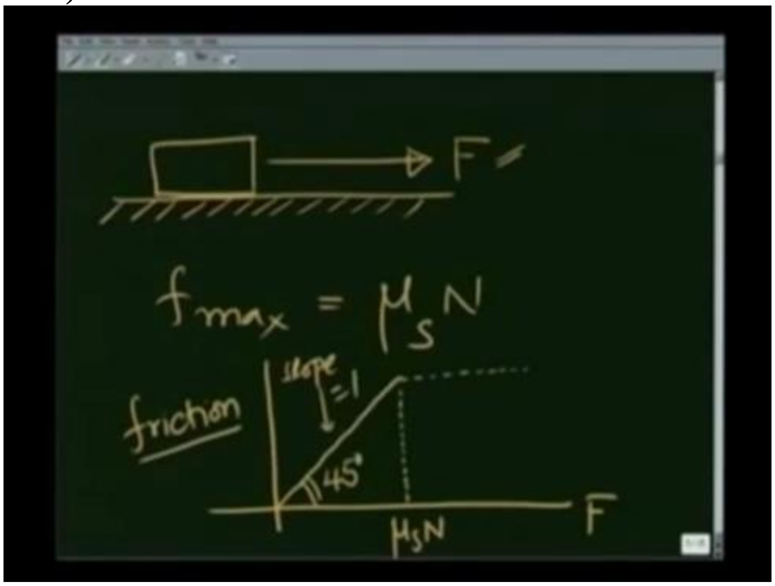

So the maximum frictional force on a block on a surface is given by F max which is myu S times N. If I apply a force here, the frictional force adjust itself so that the body does not move. If I plot the frictional force against the applied force F which is given here, you would see that frictional force goes exactly as F. This angle being 45 degrees so that the slope of this line is equal to 1.

And right when the applied force exceeds the maximum possible frictional force, the body would start to move and the frictional force may become a constant. I am drawing this line because this force drops slightly as I will explain in the coming few minutes.

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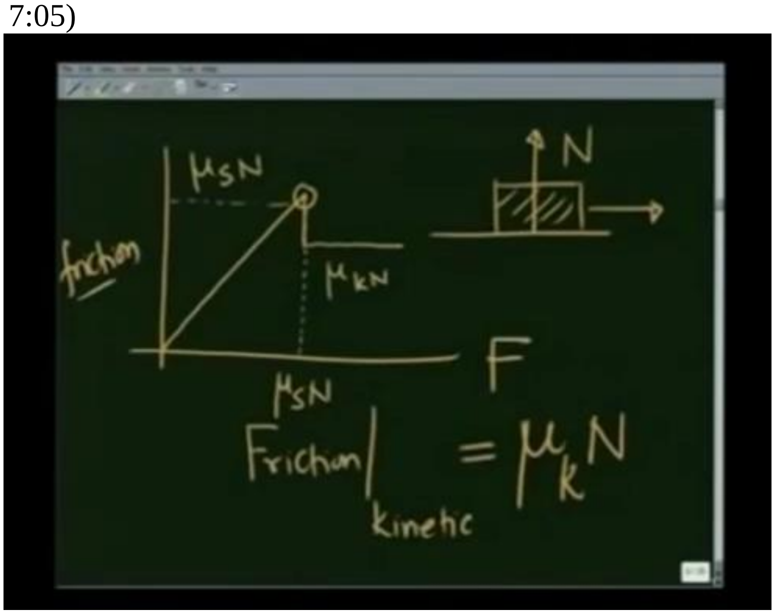

Of course once the body starts moving, the frictional force is slightly less. So that I write as friction kinetic and it is given by myu K the normal reaction so that if I again plot the applied force vs friction, it will go up at 45 degrees until the maximum value of the frictional force myu SN has been reached where N as I said earlier is the normal reaction of the surface.

And as the body starts to move, the force drops a bit but then is a constant. This is equal to myu KN. This point obviously is myu SN. Let me redraw it for clarity.

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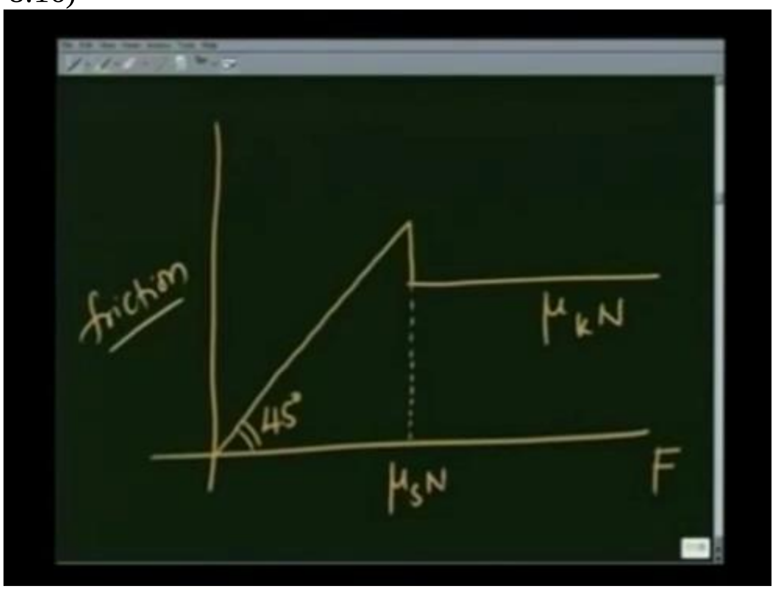

So as I take a body, apply a force, the frictional force increases in proportion to the applied force. This as I said earlier is 45 degrees. And then when it reaches the maximum, myu SN, the body will start moving and the frictional force drops slightly, this value being myu K. And to give you a feel for what myu S and myu K are, let me give their values.

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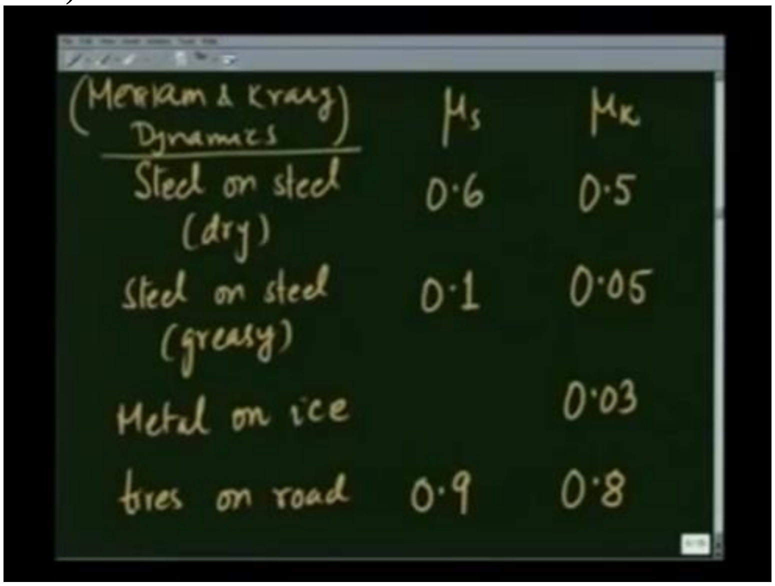

In the next slide for some materials, the value varies from material to material. So this is we will take steel on steel, dry surface. We will take steel on steel, greasy surface and we will also take metal on ice and tyres on road. Myu static for steel on steel is about 0.6 when they are both dry. Steel on steel greasy is 0.1. You can see why the grease steel, it reduces the frictional value quite a lot.

Metal on ice static is not known because it slips past quite a lot and tyre on road is 0.9. You can see, we really need a lot of friction for tyres on the road so that they do not slip. The moment body starts moving, steel on steel drops down to 0.5, steel on steel greasy drops down to 0.05, metal on ice is about 0.03, almost frictionless, tyres on road comes down to 0.8.

I must give you the source of this. This is Meriam and Kraig Dynamics. That has arrived taking she has values from. So what we have seen is, as the body moves on the surface, it can feel either the static friction and once it starts moving, then there is kinetic friction. Kinetic friction is slightly less than the static friction.

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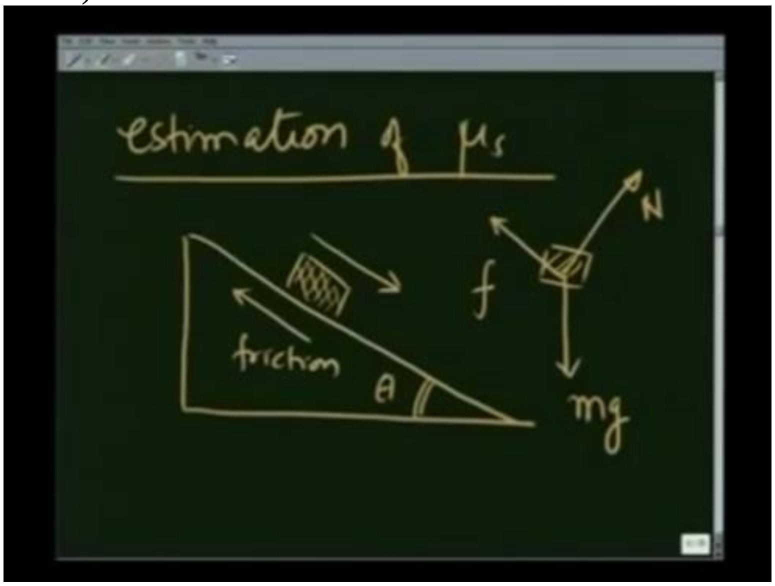

Let me now see how I can estimate the coefficient of static friction between 2 body. This can be easily done if I consider the motion of a block on a ramp or an inclined plane of angle theta. Since without friction, the body has a tendency to move this way, there is a frictional force trying to stop it this way.

If I make a free body diagram of this block, it has its own weight MG pulling it down, a normal reaction N due to the surface and a frictional force this way. Let us consider the body in equilibrium and see what happens.

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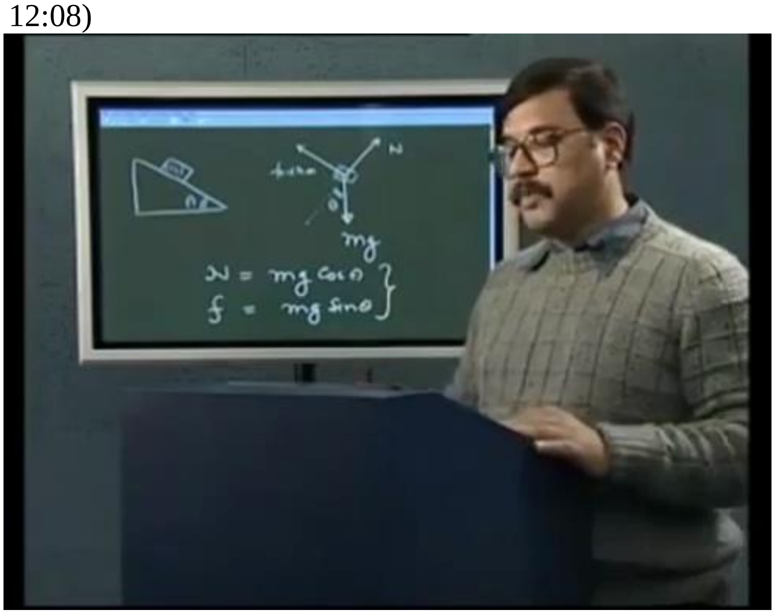

Value of the coefficient of static friction can be easily estimated if we consider a motion of a block on an inclined plane of angle theta and here is the free body diagram as I previously made. This is weight MG, normal reaction N and the frictional force this way.

This angle is also theta. If the body is not moving, then I should have all the forces balancing each other and therefore N equals MG cosine of theta. And the frictional force let me write it as F is equal to MG sine of theta. These are my equations.

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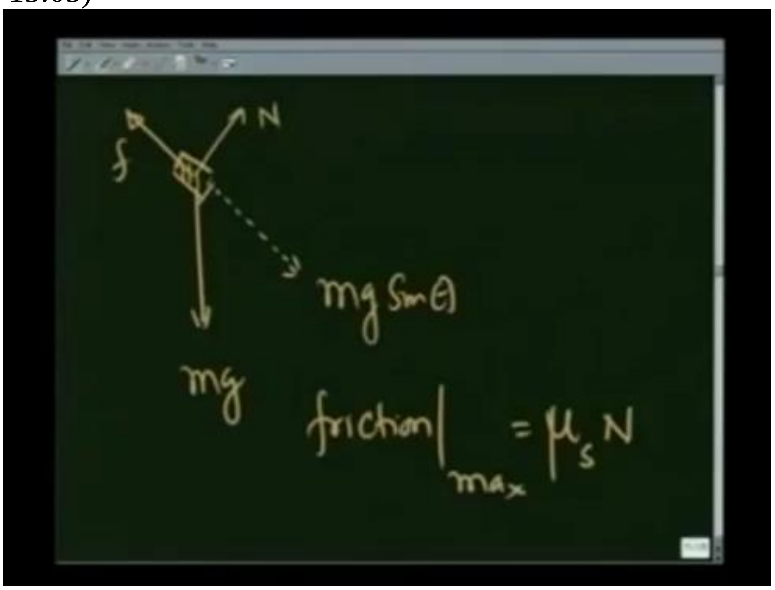

Let us see what happens when I start increasing theta. This body has weight MG and friction F and MG, this component of MG is pulling it down. As I increase theta, MG sine theta goes up. So the body is being pulled down by larger and larger force. When it surpasses the maximum possible frictional force, it will start sliding down. Let us see what is maximum possible frictional force. Friction maximum as I told you earlier, is equal to myu S times N.

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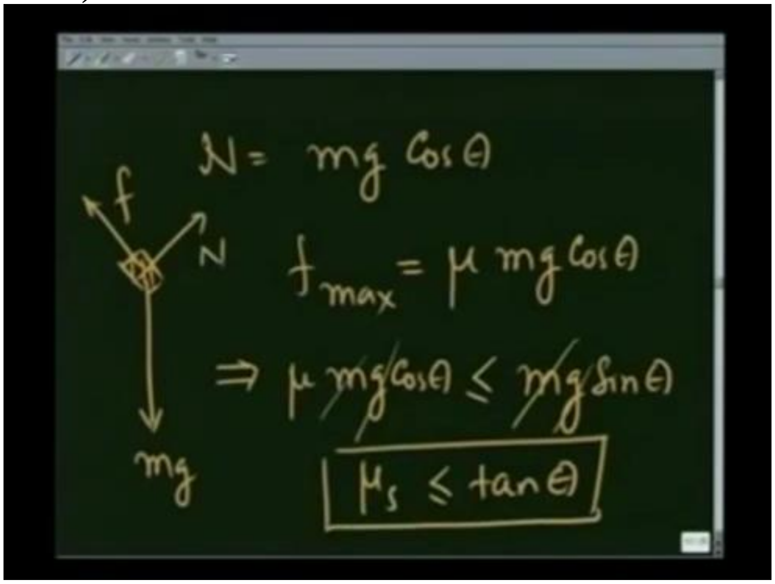

And as we saw in the earlier N is nothing but MG cosine of theta because there is no motion perpendicular to the inclined plane. And therefore, F maximum or the maximum value of friction is myu MG cosine of theta. And this implies that when myu MG cosine of theta, that is the frictional force becomes less than or equal to MG sine of theta, the body would start sliding down. Let me cancel M, let me cancel G.

And therefore when myu S is less than tangent of theta, the body will start sliding down. So you take an plane, keep tilting it until the body just starts sliding down and that angle, you take the tangent of would give you an estimate of the static coefficient of friction. That is the practical way of quickly estimating what the coefficient of static friction is.

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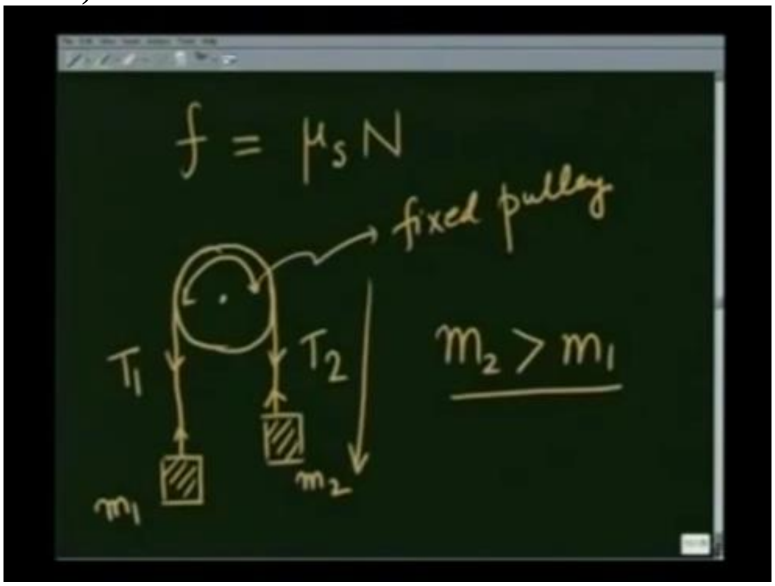

So we have seen that frictional force is given by myu depending on whether static or kinetic, myu S or myu K times the normal reaction of the surface on a body. Let me now take you back to Atwood's machine that we discussed in the previous lecture. A slight variation of that and see how frictional force changes the tensions here. Recall when I solved this problem in the previous lecture then I had assumed that all the surfaces are frictionless.

Now let me take the same problem where mass M1 and M2 R at two ends of a rope and it is passing over a pulley. The pulley is fixed. The pulley cannot move. Let me just write, fixed pulley. Only the rope slides over the pulley and there is friction between the family and the rope. Earlier when we solved this problem, we assumed that the tensions here were equal.

Now due to the frictional force, the tensions are going to be different. Let me call this T2, let me call this T1. Let the mass M2 be greater than M1 so that the rope as a tendency to move this way. Let us see how friction affects things here. I am pretty much repeating what I have done in the previous lecture on static friction. I do it here for completeness.

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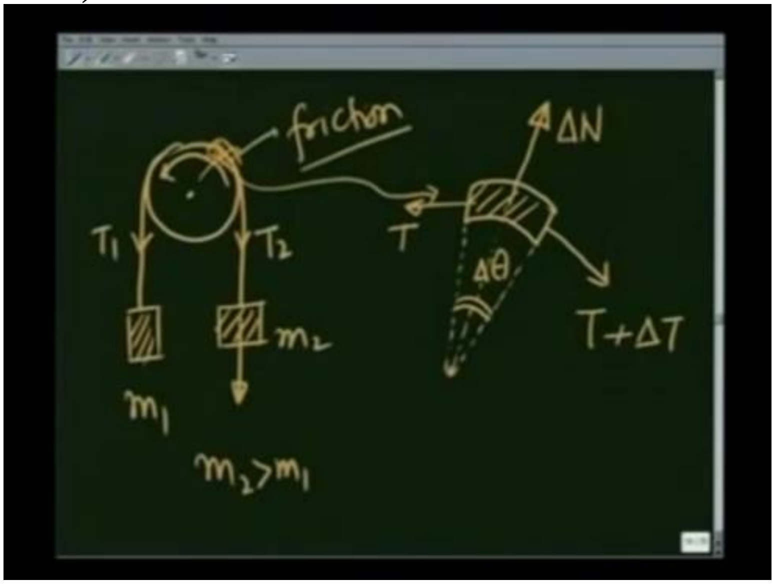

Since the rope has a tendency to move in this direction, the frictional force on the rope is going to act this way. This is the frictional force. This is mass M2 as I said earlier and this is mass M1. If I want to see how friction changes, let me for that consider a small piece of rope. This is somewhere here that I am making slightly bigger here and consider its free body diagram.

If I join from the centre of this to the centre of this pulley, there is normal reaction on this small piece of rope, let me call it delta N. Since the tension on this side, T2 is going to be greater than T1 because M2 is greater than M1, the tension increases this way. Let this tension be T + delta T. Let the tension on this site be T net. This small piece of rope make an angle delta theta at the centre of the fixed pulley.

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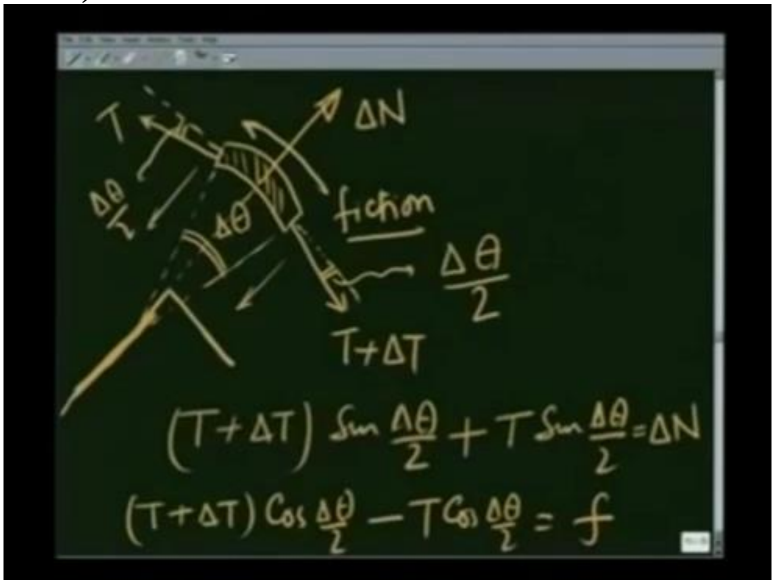

So let us make a neater picture of this part only. So here is this piece of rope. It makes an angle delta theta at the centre, tension in this way is T + delta T, tension this way is T, this is the normal reaction. And the rope is in equilibrium. Am I missing any force? Of course I am missing. Of course because the rope is not moving, there is friction that balances things. Balances T, the imbalance between T and T + delta T so that the rope does not move.

Let us now balance forces. The components of tension in this direction balance N. If I take the components parallel to this line and perpendicular to this line, this is the direction for N, this is direction perpendicular to it. The components of T in this direction balance N and therefore I am going to have T + delta T. This angle as you can see is going to be delta theta over 2. And this is a very small angle.

So sine delta theta over 2 can be estimated by delta theta over 2 itself + T sine delta theta over 2 is going to be equal to delta N. This angle again is delta theta over 2. And the 2^{nd} equation, T + delta T cosine of delta theta over 2 - T cosine of delta theta over 2 is going to be equal to the frictional force F. Let us then write these equations again.

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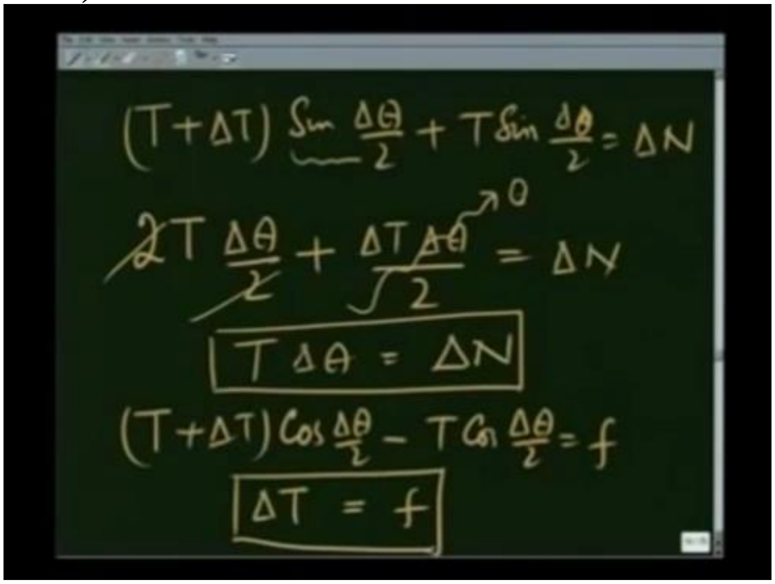

I have T + delta T sine delta theta over 2 + T sine delta theta over 2 equals delta N which can be approximated for a small delta theta as 2T delta theta over 2. I replace this whole thing by delta theta by 2 + delta T delta theta over 2 equals delta N. This is 2nd order term so this can be taken to be 0 in the limit that I take delta theta going to 0.

So I have, this cancels. T delta theta equals N or which I had earlier written as delta N because this is a very small normal reaction. And the other equation which was T + delta T cosine of delta theta over 2 - T cosine of delta theta over 2 equals F can in the limit delta theta going to 0 be written as delta T equals F. These are my 2 working equations that will determine for me how the tensions are related when friction is present.

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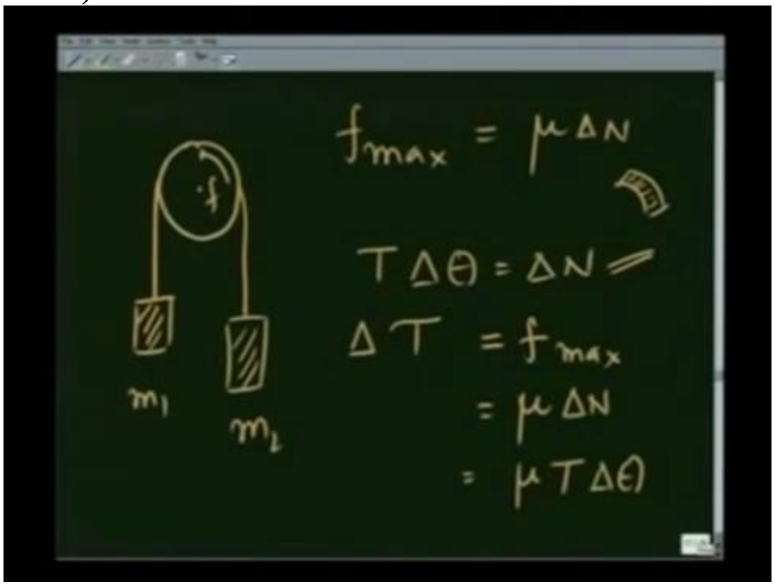

Let us now take the extreme case of when mass M2 just balances mass M1. This is the friction so that in that case, friction is going to be maximum and that is going to be equal to myu delta N on that small piece. So I have equations from earlier T delta theta equals delta N and delta T equals F max which is myu delta N which in turn from this equation is equal to myu T delta theta.

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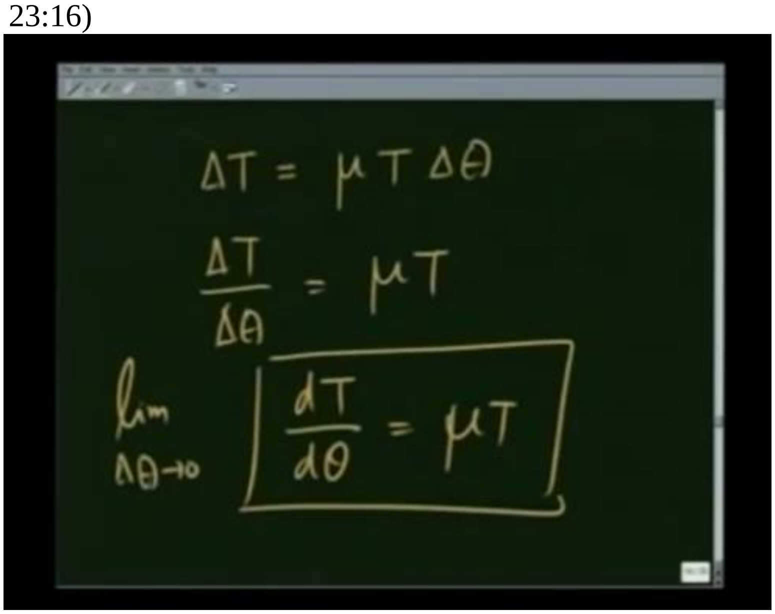

So I have a relationship between tension, change in it with respect to change in theta. That gives me delta T equals myu T delta theta or delta T over delta theta equals myu T. Limit delta theta going to 0, it gives me DT over D theta equals myu T. This equation is quite easy to solve. And you have been doing it in many different places.

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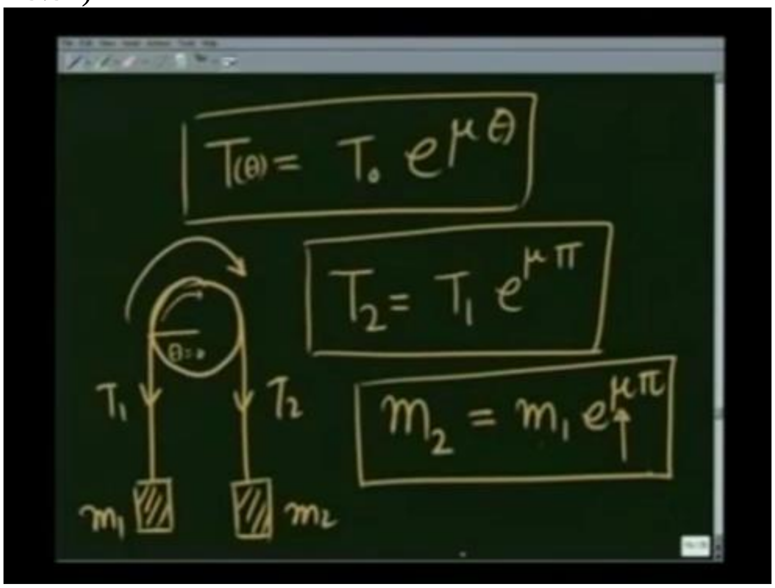

The solution for this comes out to be T equals some T0 E raised to myu theta where this T is being measured as a function of theta. T0 is at theta equals 0. So let me just again make a picture and show you that this was mass M1, this was mass M2, T1, T2, theta is increasing this way, T is increasing this way and if I take this to be theta equals 0, as theta increases, tension also increases so that you can see in this case, T2 is going to be equal to T1 E raised to myu times pi.

Since the tensions balance the masses, I also have M2 equals M1 E raised to myu pi. G cancels from both the sides. You can see, because of the friction myu, a very small mass M1 can balance a mass much larger than itself which is M1 times E raised to myu pi. This has practical uses. In fact, unconsciously if you recall you have been using it. Suppose you tie a rope or a clothesline in your backyard, you generally put it on the nail and wrap it around if you times.

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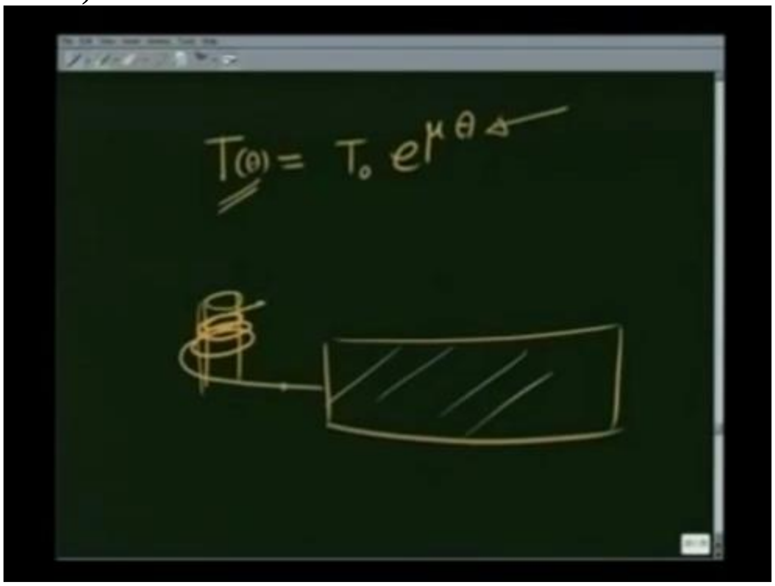

You are unconsciously using the fact that tension as angle goes up, goes as T0 E raised to myu theta so that with a very small force on one side, you can balance a large force on the other side. Another practical use is the capstans in the dockyards where a huge ship is stopped from moving by taking a rope and wrapping it around these capstans.

If you wrap it around many many times, theta goes up so that very small tension on one side, a huge tension builds up on the other side and that can stop a ship from moving. You can also do an experiment to check the validity of this expression at home. How do I do that?

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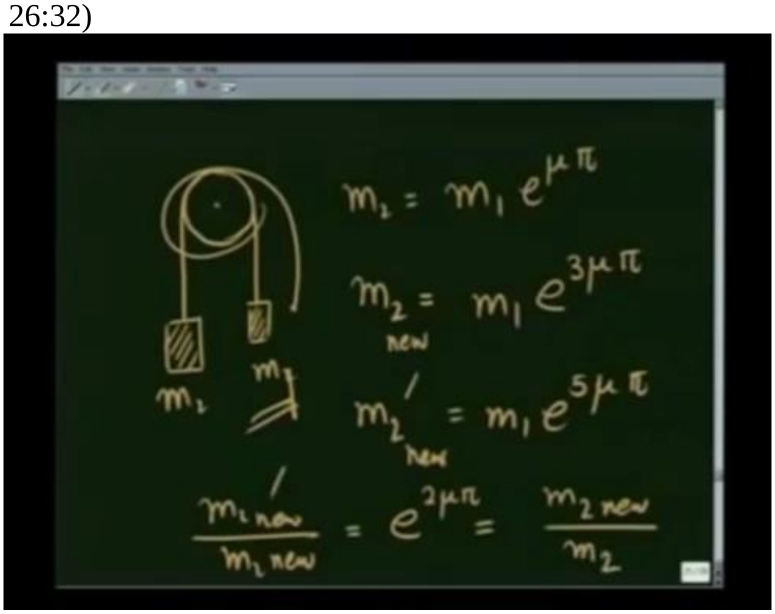

You take a pen and put a small object, maybe a pin or a key on one side and a large object on the other side of a string and try to balance it. As we said earlier, suppose this mass is heavier, M2 would be in this case be equal to M1 E raised to myu pi. So I will take this pen, wrap a string around it once and this relationship would be satisfied. Let me give it one more wrap.

If I do that, then I should be able to have M2, let me call it M2 new equals M1. Now it has gone around one more times. So I add another 2 pi to theta E raised to 3 myu pi. Let it go around one more time. M2 new prime is going to be M1 E raised to 5 myu pi. By seeing, how much mass can you balance with a given mass as you wrap the string around, you should be able to confirm the validity of this formula that we just derived.

You should see that M2 new prime divided by M2 new equal to E raised to 2 myu pi is also equal to M2 new over M2. This ratio would remain the same as you keep wrapping the string around more and more and more. Let us now solve a few problems involving friction. 3 of these problems are taken from the textbook of Meriam dynamics.

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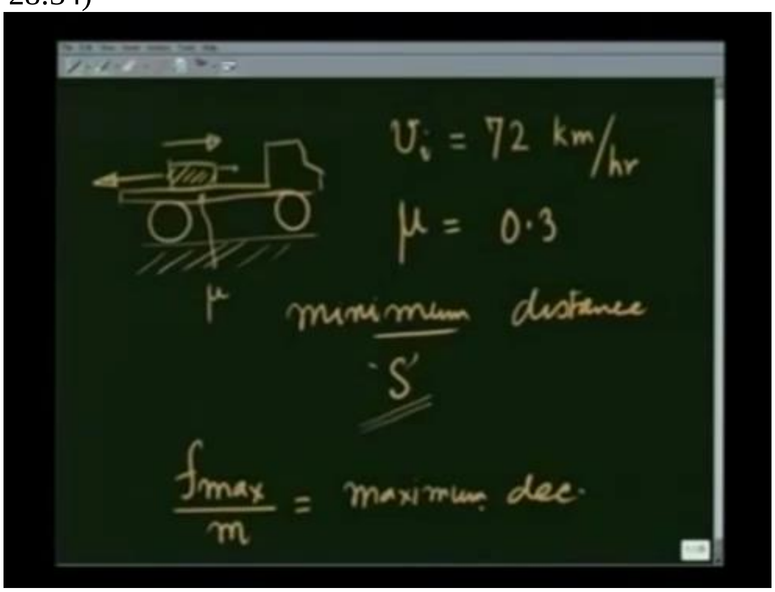

The 1st problem is if I have a truck moving at a certain speed and suppose there is a box here. The truck suddenly brakes. You know from experience, if the truck brakes, the crate or the box is going to move this way. If I am given the coefficient of friction here that is going to resist the motion of the box, so frictional force is going to oppose this movement.

What I want to know is, suppose initially the truck is moving at a speed of 72 km per hour and it brakes and suppose myu is given to be 0.3 between the box and the bed of the truck, what is the minimum distance S over which the track should stop if it uniformly decelerates so that the box does not move? To repeat, I am braking and I want the truck I want to find the minimum distance S so that the box does not move.

Obviously, when the truck slows down, I need the box also to slowdown. It should not happen that the truck slows down faster than the box can slowdown. If that happens, the box will start sliding. So the maximum deceleration on the box that I can have is F max, the maximum friction divided by the mass of the box is maximum deceleration or negative acceleration.

And it is this deceleration that is that is what the truck can also have. If it slows down faster than that, the box will start sliding. The friction will not be able to stop it.

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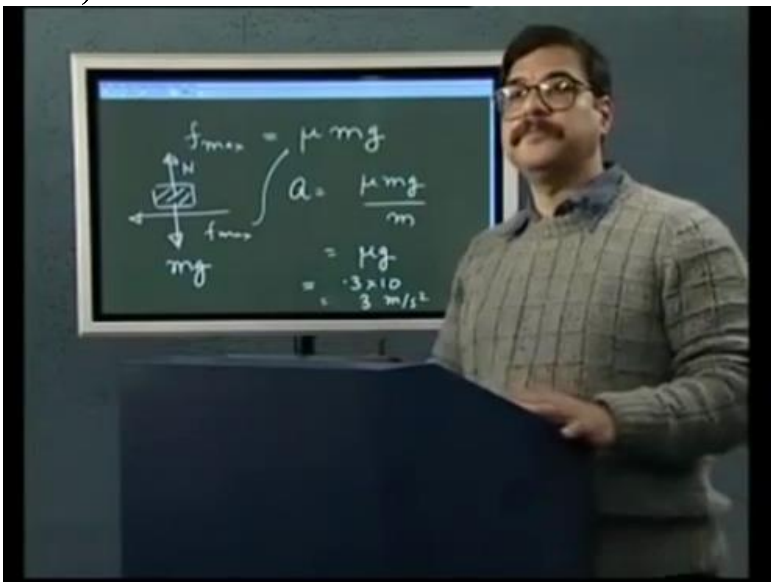

So let us see what is the maximum friction? F max is going to be myu mass of the box times G because for this box which is not moving in vertical direction N is same as MG. So F max is given by this formula. So maximum deceleration A is going to be myu MG divided by M which is equal to myu G which in our case if I take G to be approximately 10 metres per second square is going to be 0.3 times 10 which is 3 metres per square. And that is the maximum deceleration allowed for the truck also.

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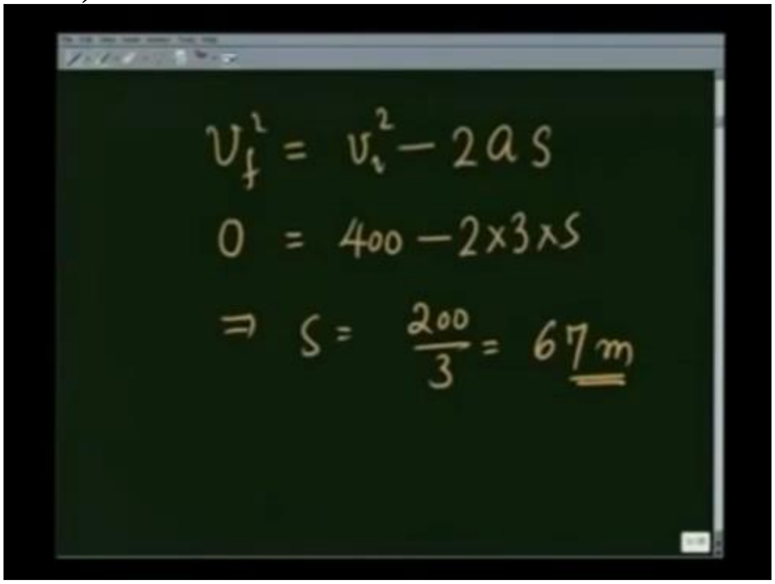

And so V final square for the truck which is 0 is equal to V initial square - 2 times the acceleration times S. This is going to be 72 km an hour which if you change to metres per second comes out to be 20 metres per second and 20 metres per second square is 400 - 2 I have already calculated A to be 3 times S. That is maximum A. So S minimum that much and that gives you S equals 200 over 3 which is 67 metres.

Of course if the truck slows down and stops at a distance larger than this, I can do with smaller F. That is no problem. But the maximum A allowed is 3. And therefore S minimum has to be 67 metres. That is one example of solving problems using frictional forces.

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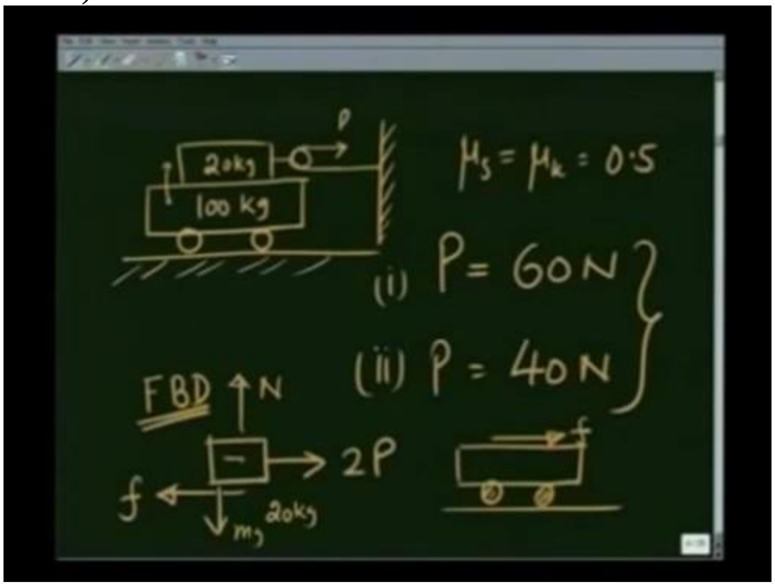

As the 2nd example, I take a crate or a trolley which can move on its wheel without friction. This is taken to be 100 kilogram. On top of it, I put a box of 20 kilogram and pull it with a pulley here pull it by a force P, other side of the string attached to the wall. The coefficient of friction whether static or dynamic let us take it to be the same, between these 2 masses is given to be myu S almost the same as myu K is equal to 0.5.

And we would like to know what are the accelerations of 2 masses in case P is 60 newtons, case 1 and case 2, when P is 40 newtons. Let us see what happens in the 2 cases. Obviously, when this P is pulling it, since this is a rope going around the pulley, the net force on the upper mass is going to be equal to 2P that I need not go over again and since this mass has a tendency to move this way, it will be opposed by a friction F.

That is the free body diagram of mass 20 kilogram. Similarly the free body diagram for the trolley is going to be, there is nothing here. The only force that by Newton's 3rd law is applied on this is the frictional force in the opposite direction, F.

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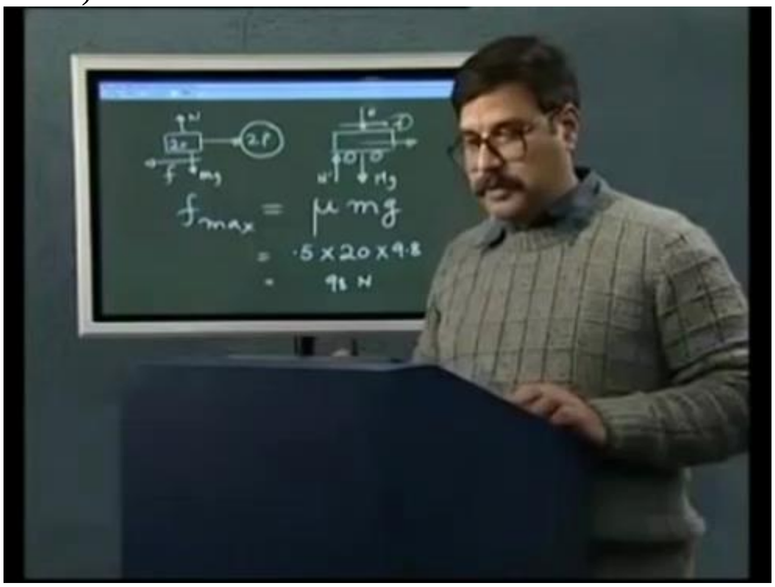

If I draw it again showing only the relevant forces for horizontal motion the free body diagram for 20 kilogram mass is 2P this way, friction this way and for the trolley is friction this way. So the trolley would move in this direction or accelerate in this direction due to the frictional force. The 20 kilogram mass will also move in this direction because of 2P and the force.

Let us see what P I should apply so that they do not slide. That would happen when F is maximum possible. Let us calculate what is F maximum possible that is myu MG. Myu is given to be 0.5, M is 20 for the upper mass, N, MG and free body diagram I should also show these forces here.

MG, N prime and there is an N here. All right? Times 9.8 and that comes out to be 98 newtons. And that comes out to be 98 newtons and therefore if 2P happens to be greater than 98 newtons, this fellow would accelerate in this direction.

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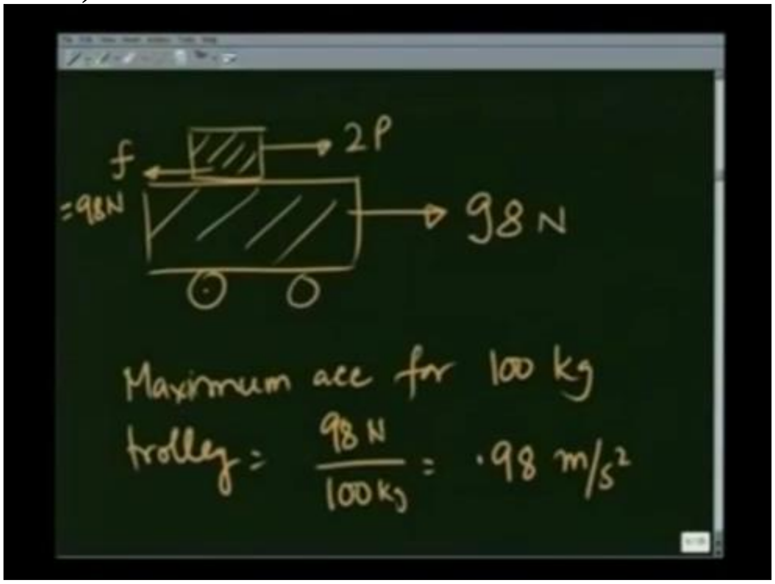

This fellow is moving this way. Maximum force it can have in this direction is 98 newtons. This has a force 2P and this has a force F maximum, 98 newtons. So the maximum acceleration for 100 kilogram trolley is equal to 98 newtons over 100 kilo or which is 0.98 metres per second square. If the acceleration of the top mass, 20 kilogram happens to be larger than this, then there will be sliding. If it is less, then there will be no sliding.

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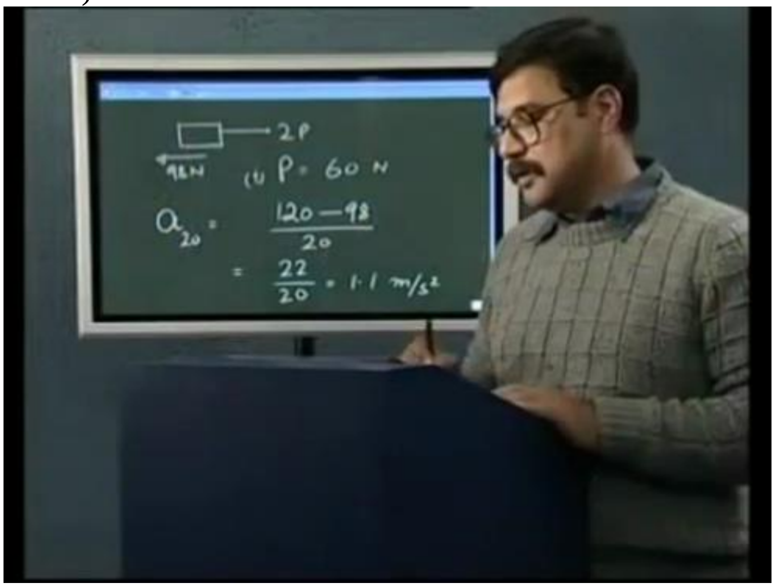

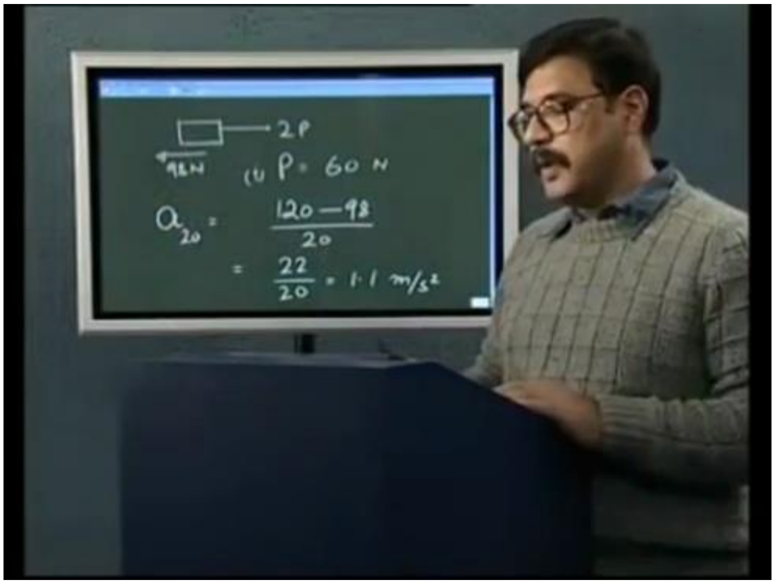

So let us see what happens when I apply a force of 2P here and this is 98 newtons. In case of P is equal to 60 newtons which was my 1st case, the acceleration of the upper mass, 20 is going to be 120 - 98 over 20 which is 22 over 20 which is 1.1 metres per second square.

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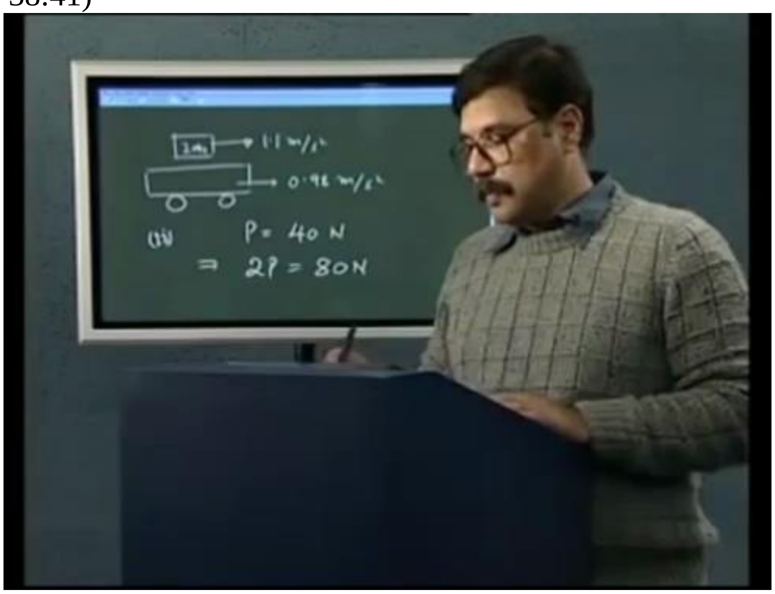

And therefore in this case, the 2 masses, the 20 kilogram mass and the trolley are going to have set different accelerations. This is going to move with 1.1 metres per second square and this is going to move with 0.98 metres per second square and they are going to slip on each other. Let us see the other case. When P is equal to 40 newtons which implies that 2P is going to be 80 newtons and in this case 2P happens to be less than the maximum force.

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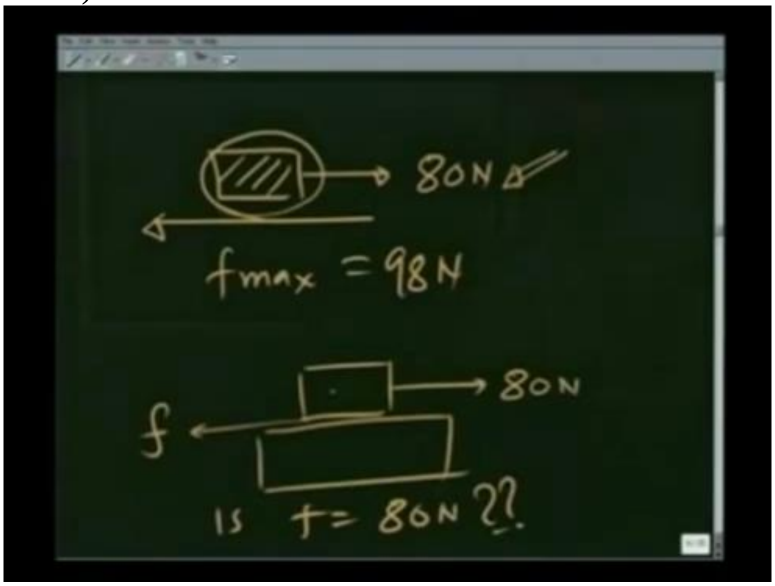

Maximum frictional force. This is 80 newtons and F max is 98. So the friction will adjust itself just to 80 newtons so that this fellow, this mass does not slip and therefore we can include that in this case, this is going to be 80 newtons and that is going to be just sufficient force F so that this does not slip. Is F equal to 80 newtons? The answer is obviously no. You can see it in 2 ways.

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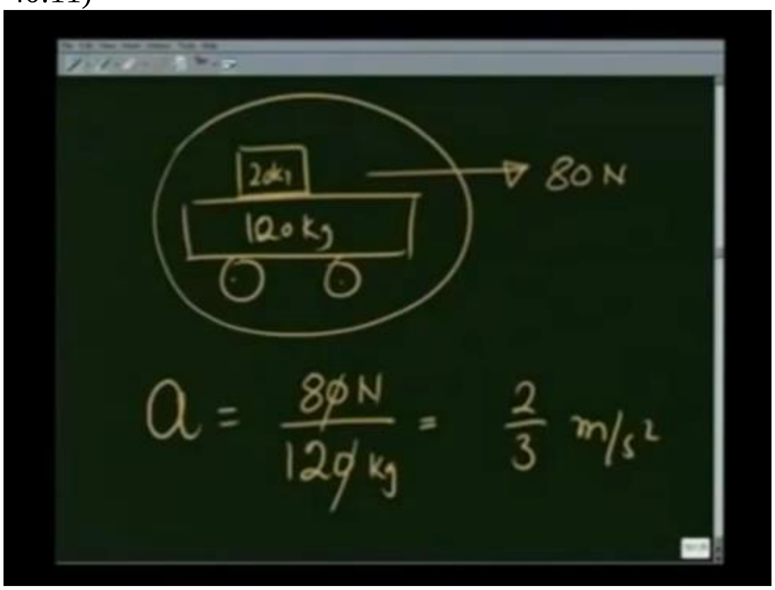

First if I consider the trolley and the mass, sorry 100 kilogram together, then on this entire system there is a net force 80 newtons. There is no relative isolation. They are moving together and therefore their acceleration together is going to be 80 divided by 120. This is newtons, kgs which is going to be two thirds metres per second square. But what about the frictional force in between? The question we started with is, is it 80 newtons? The answer is no.

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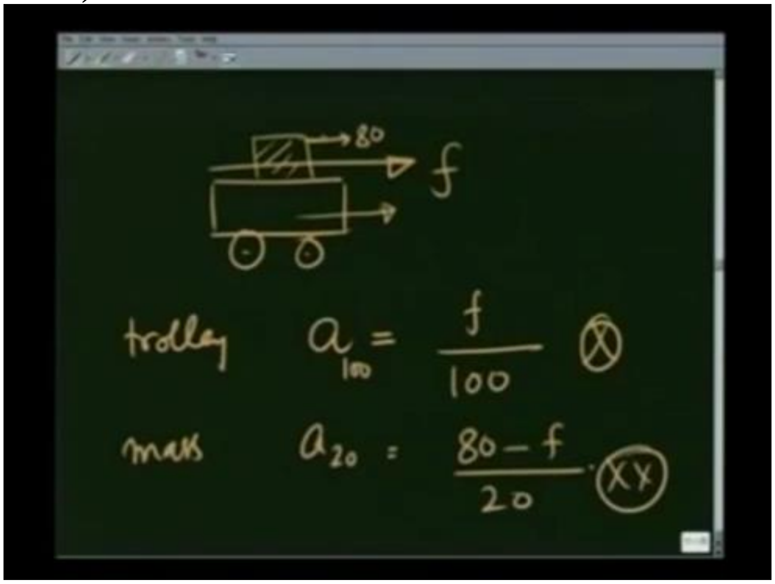

Because you see, the trolley has only one force acting on it, that is the frictional force. There is nothing to stop it. So the moment the restriction, this trolley is anyway going to move. If it moves and there is no slip in between the upper mass and the lower mass then this mass is also going to move and both move with the same acceleration. Let us look at it from that point of view.

In that case, the trolley has an acceleration which is going to be equal to the force, frictional force which I do not know over 100. Similarly, the mass let me write it trolley so 100 kgs. Mass 20 kgs is going to have an acceleration which is going to be 80 with the force with which I am pulling it - F over 20. And since there is no slipping. Because 80 newtons happens to be less than the maximum frictional force to accelerations, this and this one must be equal.

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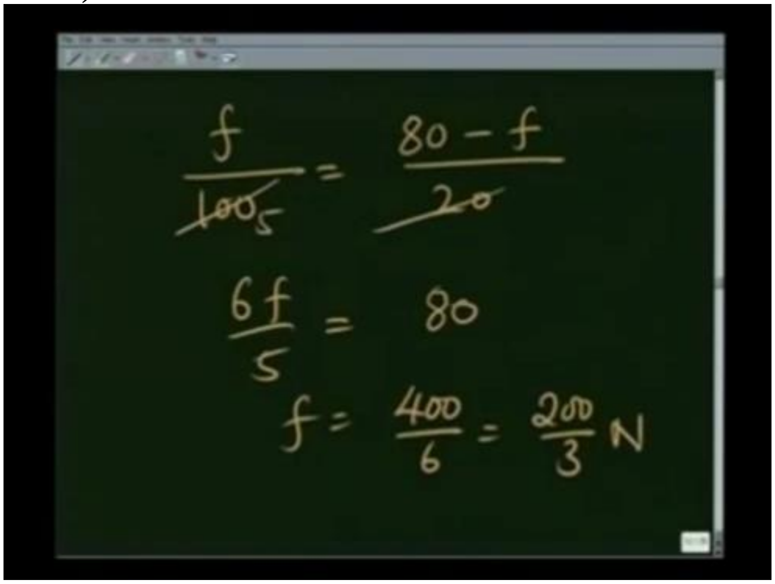

And therefore I should have F over 100 equals 80 - F over 20. That is fine. So I have 6F over 5 equal to 80 or F equals 400 over 6 or 200 over 3 newtons.

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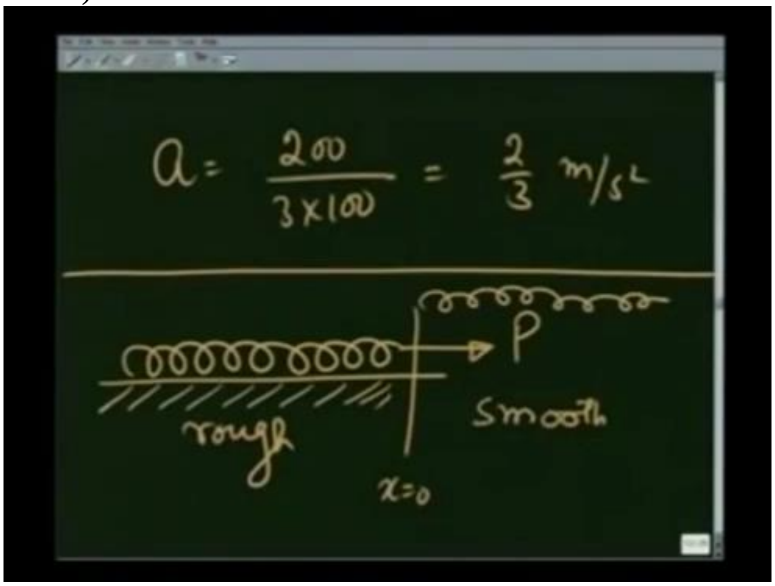

And that gives me the acceleration A, 200 over 3 times 100 equals two thirds metres per second square. So you see how friction changes the acceleration and how it affects the motion between 2 bodies that can apply frictional force on each other. As the 3rd example, let me take a chain which is on a rough surface. So this is a rough surface. And let us take this to be X equal to 0 and beyond this is smooth surface.

This is a rough surface. And I start pulling this chain with a constant force P large enough so that it comes into motion. So we are not making that any complicated. It starts moving and I want to know when this chain is fully outside the rough, it has come fully out, what is its speed?

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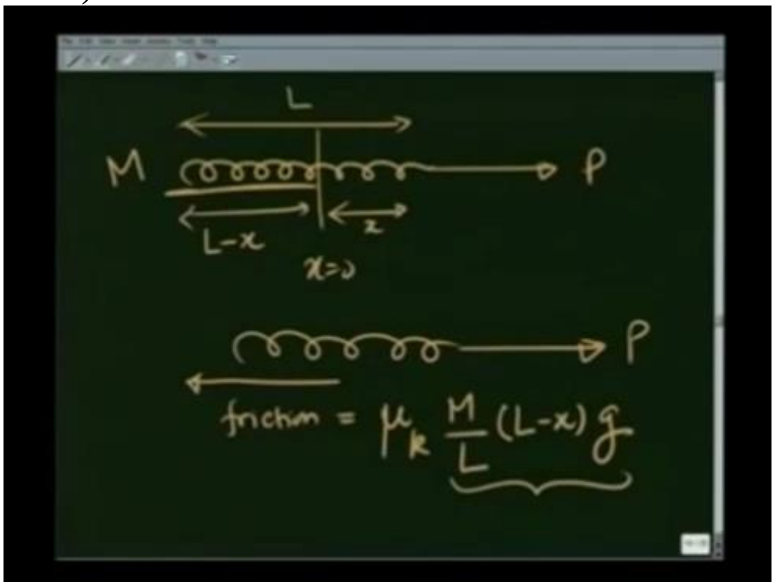

Okay. So let us now make a picture in between when it has come out partially from X equal to 0. The chain is like this. It is being pulled by force P. Let this distance be X. Let the total length of the change be L. This then is going to be L - X. Let the mass of the chain be M and I want to know when the chain has come out fully from the rough, what is its speed?

Let us see what are the forces acting on the chain. There is obviously this force P pulling it this way. There is frictional force but the frictional force acts only on this part. And so friction and since it is moving friction, its maximum is going to be myu kinetic M over L L - X, that is the mass of the part in the rough times G. This whole thing is the normal reaction on the part of the rough.

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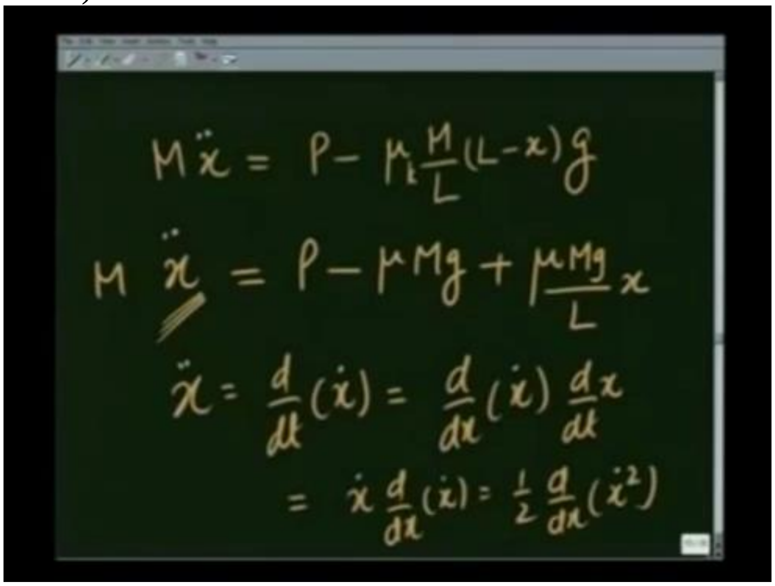

And therefore if I write the equation of motion the total mass M moves with acceleration X double door because if the chain has come out by distance X, it is acceleration is going to be X double dot and this is going to be equal to P - myu M over L L - X times G. For simplicity, I have dropped the subscript K here. It is understood, myu is kinetic friction of course.

So X double dot therefore is given as P - myu MG + myu MG over LX. Since I want to know what is the speed when the chain has been fully pulled out, I am not really interested on the variation of X with respect to T. So I use an old trick with are used in the previous lecture. I write X double dot which is really D by DT of X dot in terms of X and X dot. So this I can write using the chain rule as D over DX X dot times D over DT of X which is nothing but X dot D over DX X dot which is one half D over DX of X dot square.

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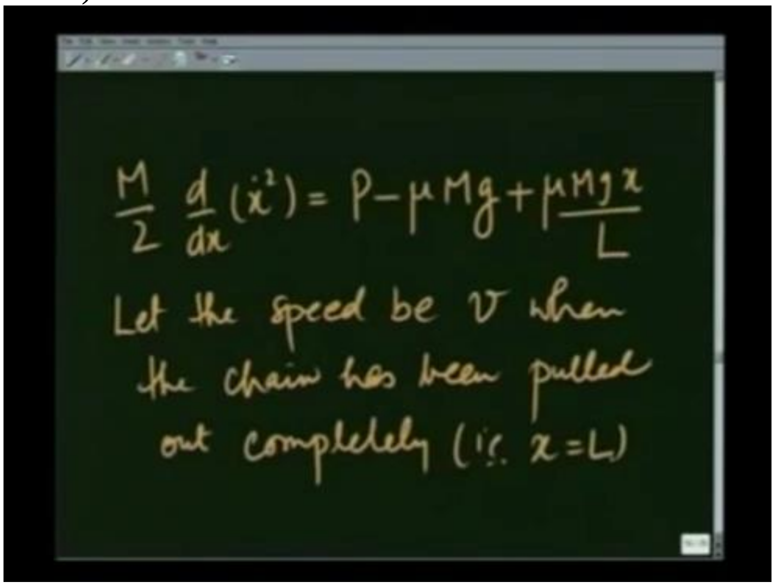

And therefore my equation becomes M over 2 D over DX of X0 square is equal to P - myu MG + myu MG X over L. Now I can easily integrate it from X equals 0 to L. So if I do that and let the speed be V when the chain has been pulled out completely. That is X equals L.

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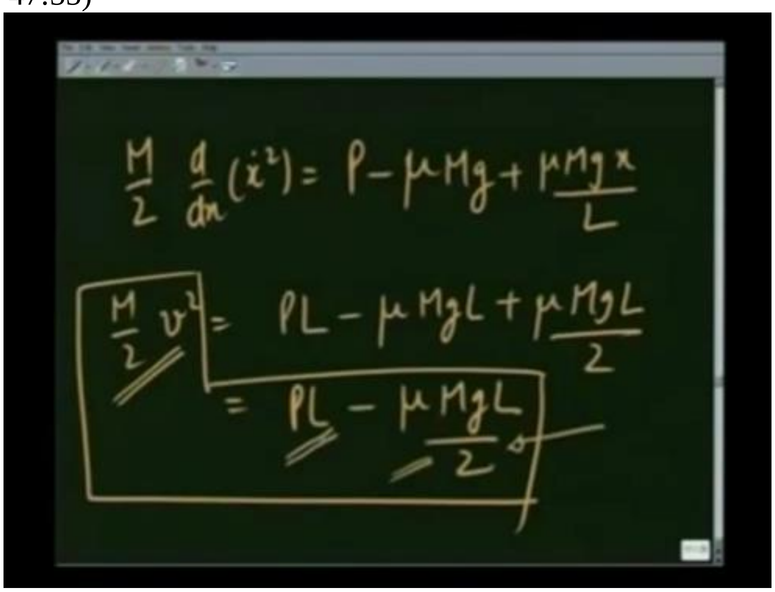

So by integration I get, I am integrating the equation M by 2 D over DX X dot square equals P - myu MG + myu MG X over L. By integration I get M over 2 V square equals PL - myu MGL + myu MGL over 2 which is nothing but PL - myu MGL over 2. And that gives me the speed of the chain after it has been pulled out fully.

You will see in a later lecture when we talk about work energy theorem that this is a gain in kinetic energy which is equal to the work done by the external force which is PL when the force moves the chain by distance L - the average frictional force which at the highest is myu MG, and the lowest is 0. So average is myu MG by 2 times L. This is the loss due to friction. So energy gain by work being done - loss due to friction is equal to the kinetic energy and this is how you can calculate the kinetic energy.

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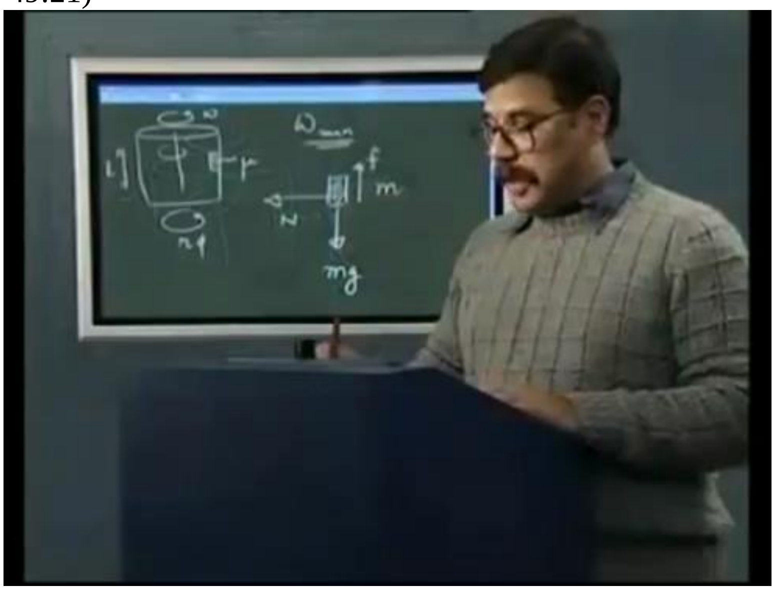

As the last example, let me take a cylinder which is moving around its axis with an angular frequency omega and let me take a mass here which is free to slide but because of this rotation, is stuck on the wall. Coefficient of friction between the wall of the cylinder and mass M is myu. I want to know what is omega minimum when this mass is stuck on the wall. If I may take the free body diagram of this mass M, it is being pulled down by its own weight and the friction opposes this and there is a normal reaction, N.

These are the 3 forces that are acting on this mass. Again, since this is a rotational motion, I am going to go to cylindrical polar coordinates. Because it is rotating this way, I will use R and phi in this direction and for this direction, I will use Z.

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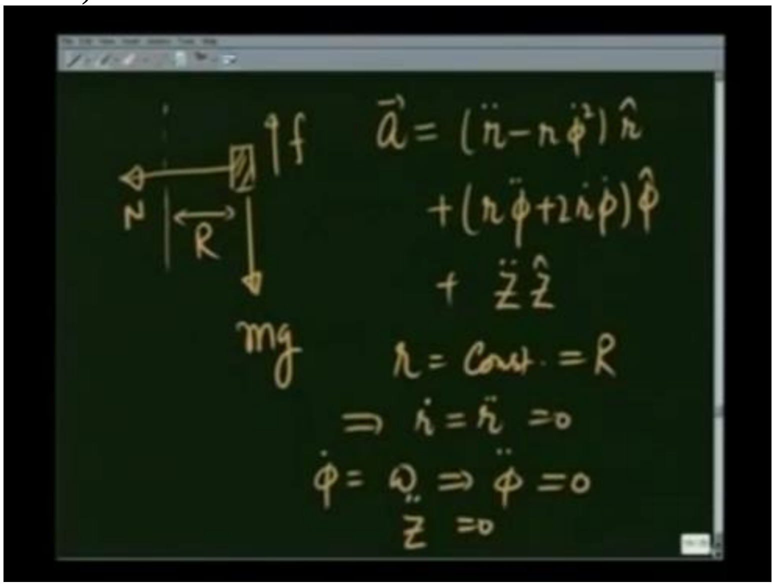

So let us see what happens to this mass. This is at a distance equal to radius of the cylinder from the centre. It is being pulled down by weight MG, that is force N on it and there is frictional force F on it. Its acceleration in cylindrical coordinates is going to be R double dot - R phi dot square R + R phi double dot + 2 R dot phi dot in phi direction + Z double dot Z in Z direction.

However in equilibrium when it is moving stuck on the wall, R is constant and this is equal to R. And therefore R dot equals R double dot equals 0. Similarly phi dot is fixed to be omega and this implies phi double dot is also equal to 0 and it is not moving up and down and therefore Z double dot is also equal to 0.

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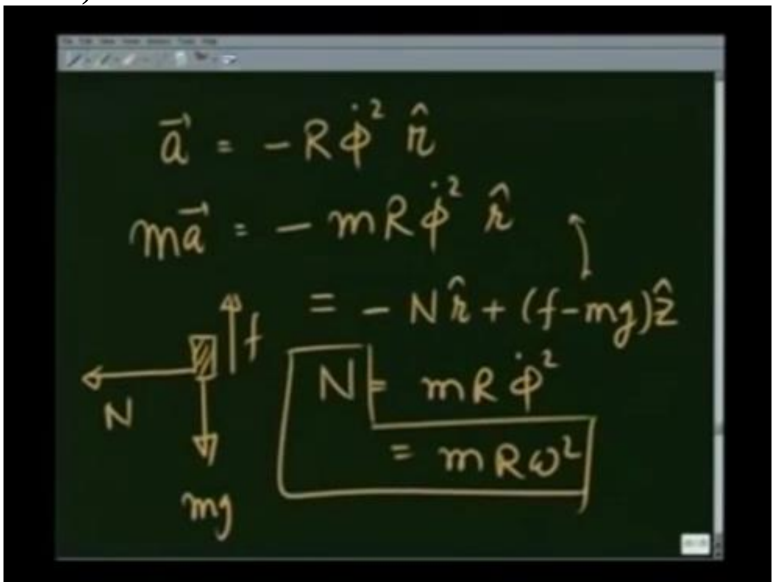

Therefore the acceleration of the mass is given as A is equal to - R phi dot square R. MA therefore is - MR phi dot square R. How about the forces on the mass? There is a force in the negative radial direction, N. There is MG and there is friction. So this should be equal to N in negative R direction + F - MG in that direction. Equating the 2 sides, we get N is equal to MR phi dot square which is nothing but MR omega square.

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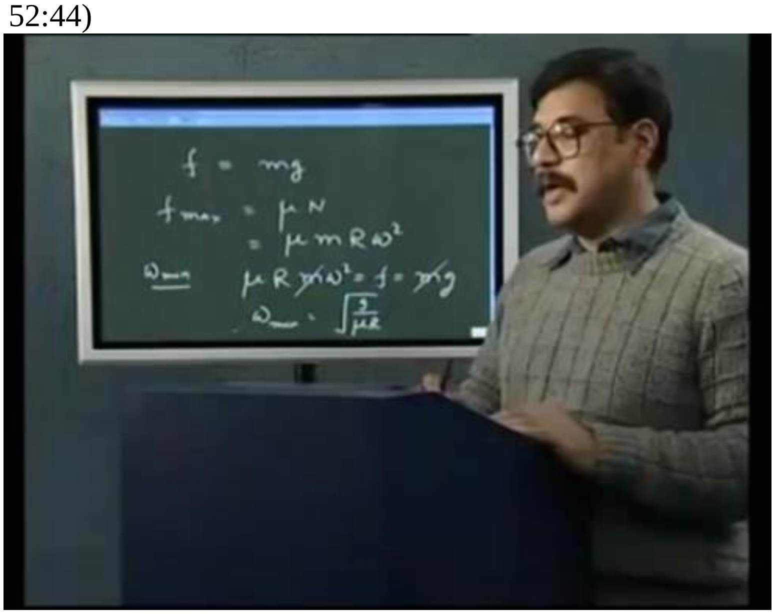

And the other equation gives me F equals MG. Remember F max is myu N. So that is going to be myu MR omega square. For the minimum omega I should be applying the maximum possible frictional force for that omega. Therefore for omega min my condition becomes that myu RM omega square is equal to F equals MG. This M cancels and I get omega minimum to be square root of G over myu R and that is your answer.

So what we have seen in this lecture is how frictional force acts on 2 bodies when they slide or tend to slide over each other and how we use this in solving problems. In the next lecture, we are going to look at another form of frictional force which arises when bodies move through fluids and that is the viscous or drag force.