Engineering Mechanics
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Module 5
Lecture No 43
Description of motion in cylindrical and spherical coordinate systems

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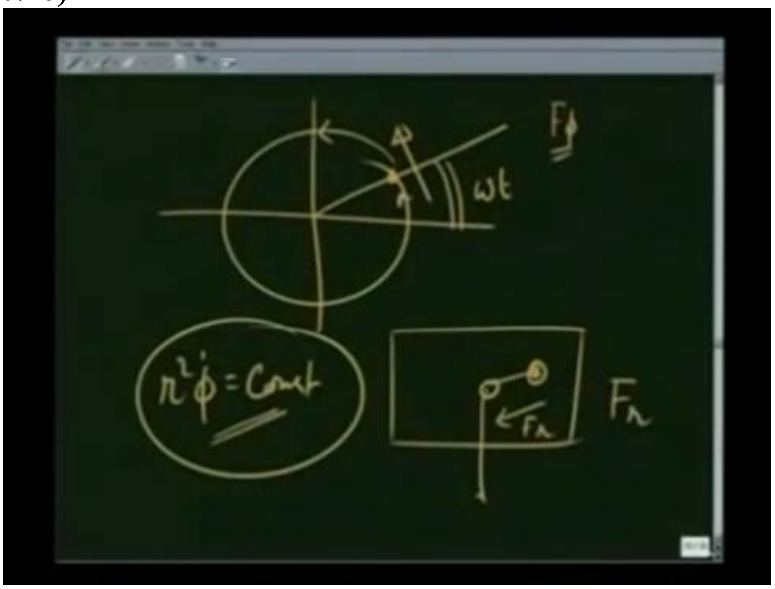

Having done the planar polar coordinates, the obvious question to ask is, what is the corresponding system of coordinates if I want to describe a three-dimensional motion?

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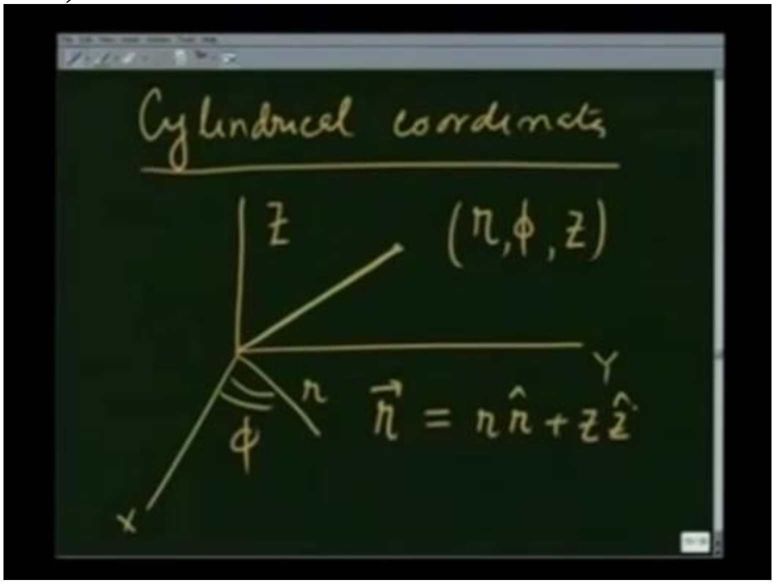

A very straightforward extension of this to 3 dimensions is the cylindrical coordinates in which I describe a motion in the XY plane in terms of this R and phi. Notice how I have changed the orientation of X and Y and requires some getting used to and the 3rd dimension is treated as Z itself so that the position of a particle is going to be given by R which is in the plane X and Y, phi and Z. Z giving you the 3rd dimension.

Obviously, the vector R which is in this direction is going to be given as R, unit vector R + Z unit Z. Let me show it more clearly in the next picture.

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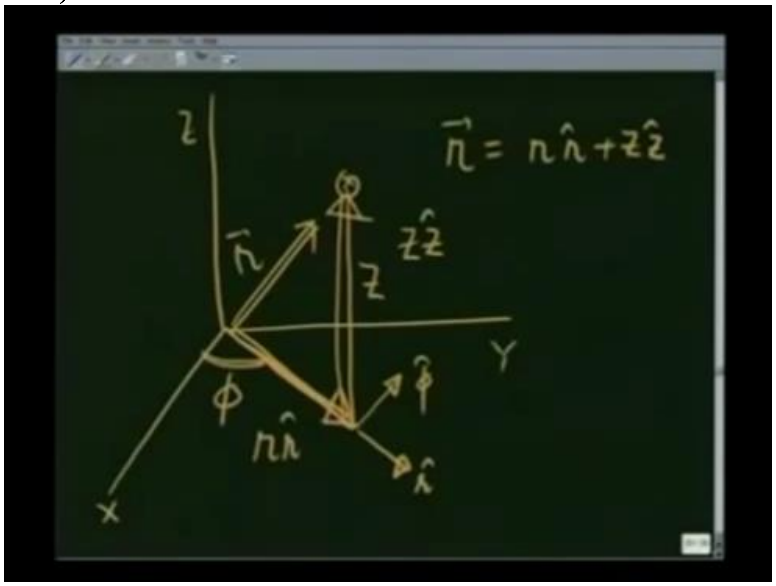
e 

So I have X, Y and Z. The position of the particle is here which is at a height Z from the XY plane. In the XY plane, the radius vector makes angle phi from the X axis. So this vector here is nothing but R R the unit vector R is in this direction in the XY plane and this vector is nothing but ZZ unit vector. And therefore the position vector R is given as RR + ZZ.

The phi unit vector is again in the XY plane, perpendicular to R and perpendicular to Z like this. How about the velocity of the particle? Again its very simple in this case because the motion in the XY plane is being given by the planar polar coordinates and the Z direction is fixed.

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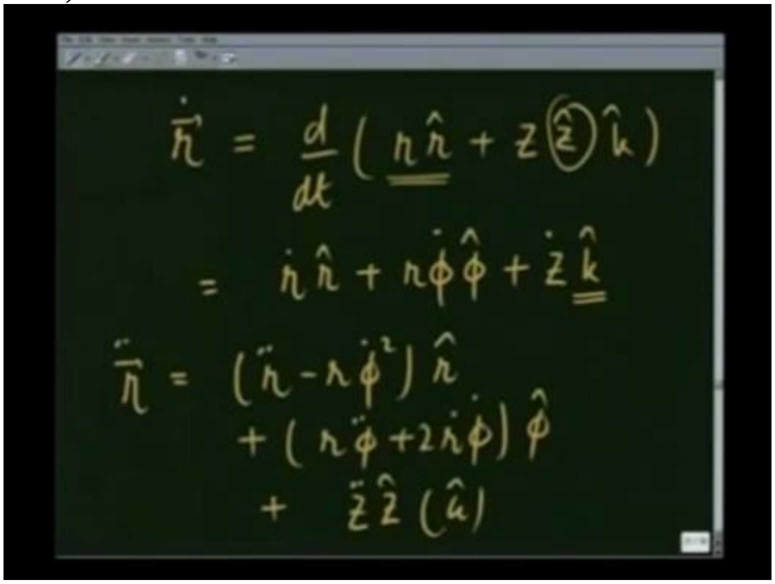

So therefore R dot velocity is going to be equal to D by DT RR + ZZ. You are more used to calling this K. So I am using them interchangeably. This is going to be equal to R dot R + I have already derived this. I am going to write the result right away without any derivation, R phi dot phi + Z dot K or Z dot Z. There is no derivative of this because Z unit vector is fixed.

Rest, derivation of the acceleration I will leave for you as an exercise. I will just give you the answer. It is going to be same as in the previous case for the plane in XY plane which is going to be R double dot - R phi dot square in R direction + R phi double dot + 2 R dot phi dot in phi direction + Z double dot Z or equivalently K direction.

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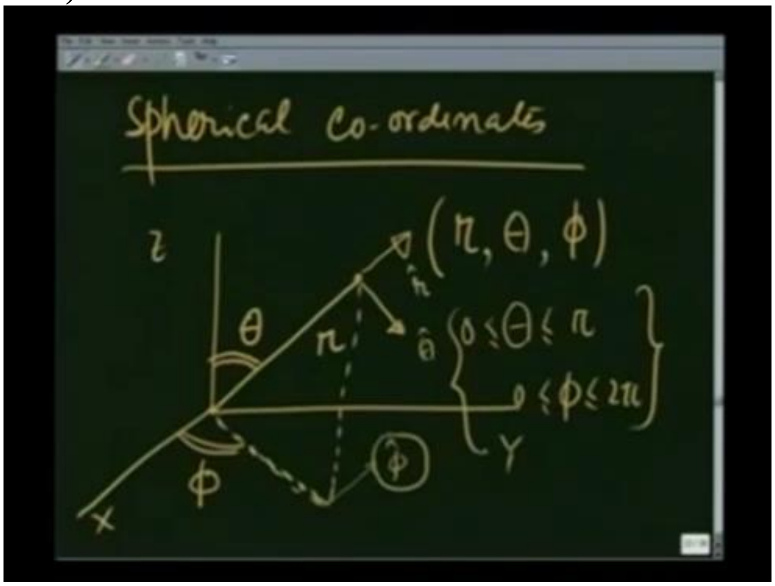

A more interesting extension is the spherical coordinates to describe three-dimensional motion. And in this case again if I make my X, Y and Z coordinate system, the position of a particle is given by its distance from the origin, the angle theta that this radius vector makes with the Z axis and angle phi that the projection of the radius vector, this is the projection I am showing again makes with the x-axis.

So now I am using R, the distance from the origin, the angle theta that is made from the Z axis and angle phi that the projection of R makes from backs access. The unit vector R is going to be in direction of the radius. Unit vector theta is going to be in the direction of increasing theta and unit vector phi is going to be in the direction of increasing phi.

You can see that the phi unit vector is confined to XY plane whereas R and theta are going to have component in all, X, Y and Z direction. And in the next few minutes, we are going to see how these unit vectors are represented. I just want to mention here that theta in this case varies from 0 to pi and phi varies from 0 to 2 pi and I will leave it for you to sort of work out that this covers the entire space. And R obviously varies from 0 to infinity.

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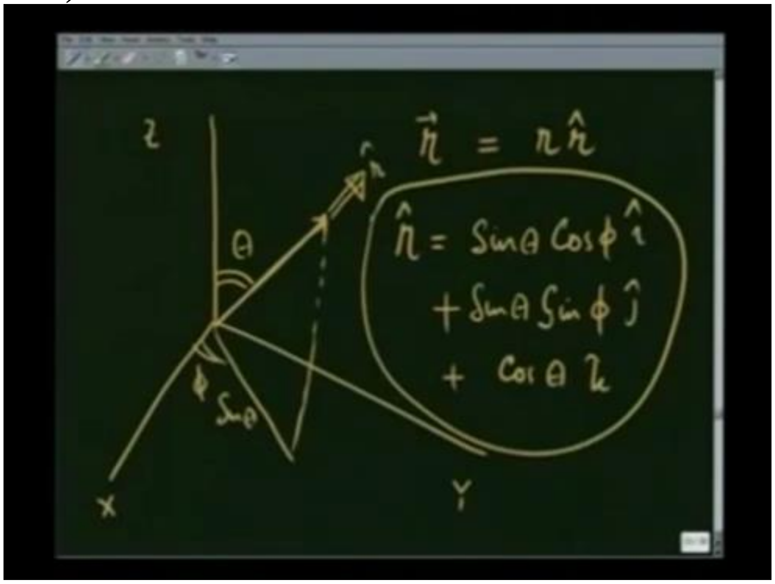

Let us then see that I am representing a particle by vector R. This vector R which is RR again where unit vector R is in this direction. The components of unit vector R are therefore going to be it is very easy to see that the Z component is going to be cosine theta. The projection on the XY plane of this is nothing but sine theta and therefore the X components is sine theta.

This is phi cosine of phi I + sine theta. And the Y component of this projection, sine of phi J + cosine of theta K. This is the unit vector in R direction. How about the unit vector in theta direction?

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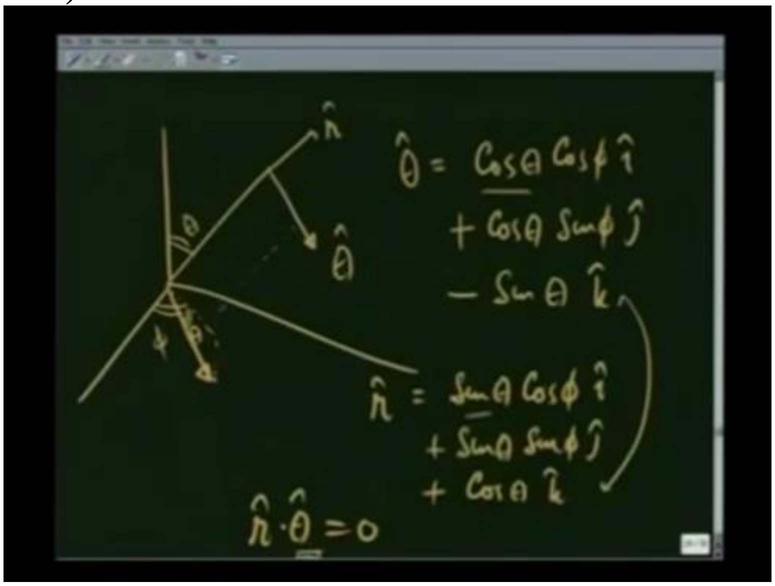

Unit vector in theta direction is going to be like this. This is R, this is theta. If I transport it down, just to make it look easy, this is pointing down from the plane of X and Y axis by an angle theta because this angle is theta. And therefore you can see, the components of theta in X and Y direction are going to be cosine of theta cosine of phi because this angle is phi I + cosine of theta sine of phi J and its Z component is in the opposite direction, so - sine of theta K.

Recall that R was sine of theta cosine phi R + sine theta sine of phi J + cosine of theta K. You can see right away that R dot theta is equal to 0. These 2 together give you cosine theta. cosine theta times sine theta, cosine square phi + cosine theta sine square sine square phi - these 2 give you - sine theta cosine theta giving you this.

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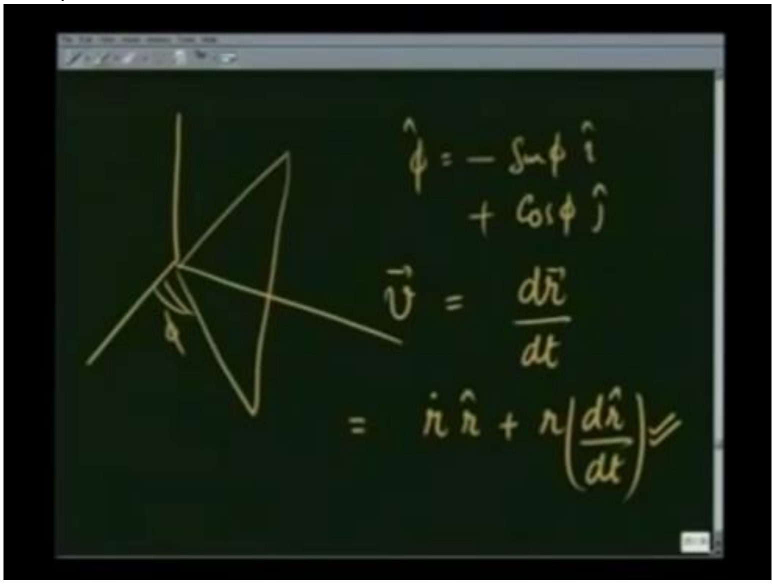

Similarly, if I were to write the components of phi vector, phi vector as I told you earlier, is confined to XY plane and therefore phi unit vector is just like in polar coordinates which is equal to - sine phi I + cosine phi J. So I have all 3 unit vectors. Now how about the velocity?

Velocity is equal to DRDT which is equal to R dot R + R DRDT. Expression for DRDT, rate of change of the unit vector in this case is going to the slightly more complicated because now it has all 3 components but we can geometrically see how this is going to be.

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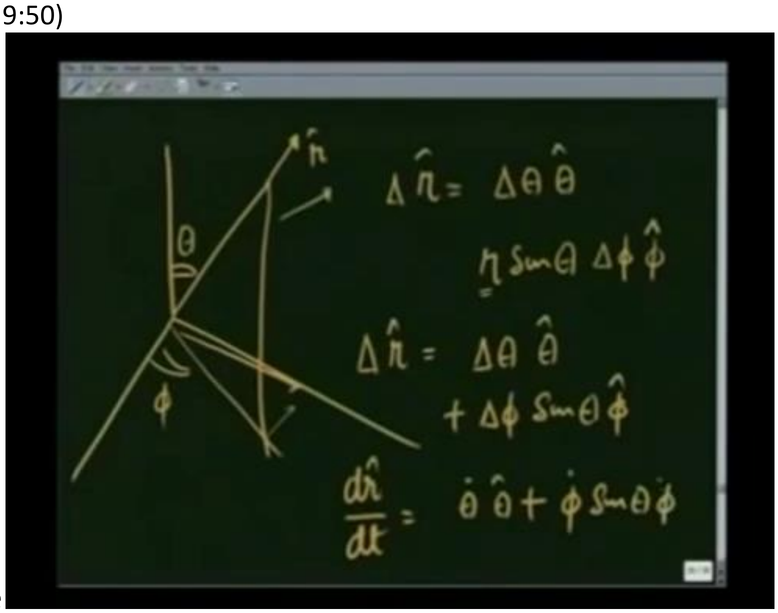
e 

This is vector R. If I were to change only theta and keep phi fixed, then you can see the R unit vector is going to change from here to like this and that change is going to be equal to Delta theta in theta direction. On the other hand, if I keep theta fixed and move phi, change phi, then the R unit vector this component is going to change and that component is nothing but R sine theta.

Since this is a unit vector, this R is 1 and it is going to change by delta phi in phi direction. And therefore the net change in delta R in R unit vector was going to be delta theta in theta direction + delta phi sine theta in phi direction. And therefore DR by DT is going to be theta dot theta + phi dot sine theta phi.

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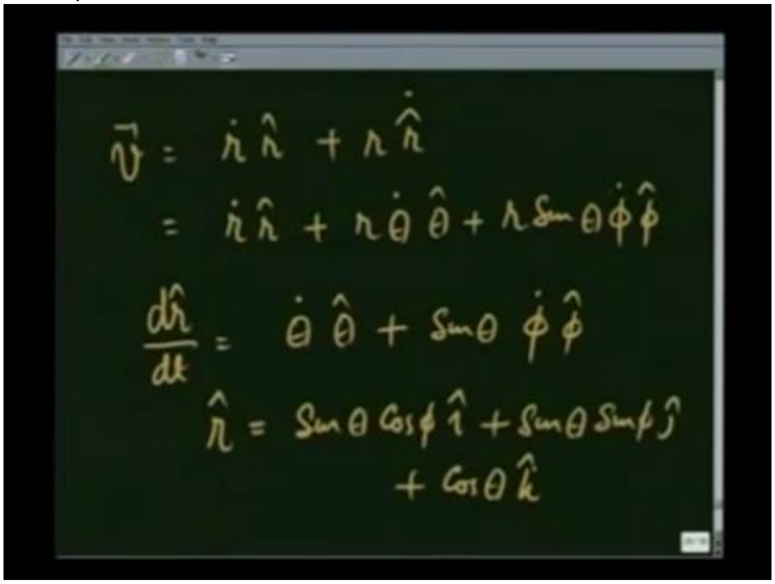

Therefore the velocity which is R dot R + RR unit vector rate of change is going to be equal to R dot R + R theta dot in theta direction + R sine theta in phi direction. Let us see whether this which we derived to be theta dot theta + sine theta phi dot phi. Then we derived directly from the definition R was equal to sine theta cosine phi I + sine theta sine phi J + cosine of theta K.

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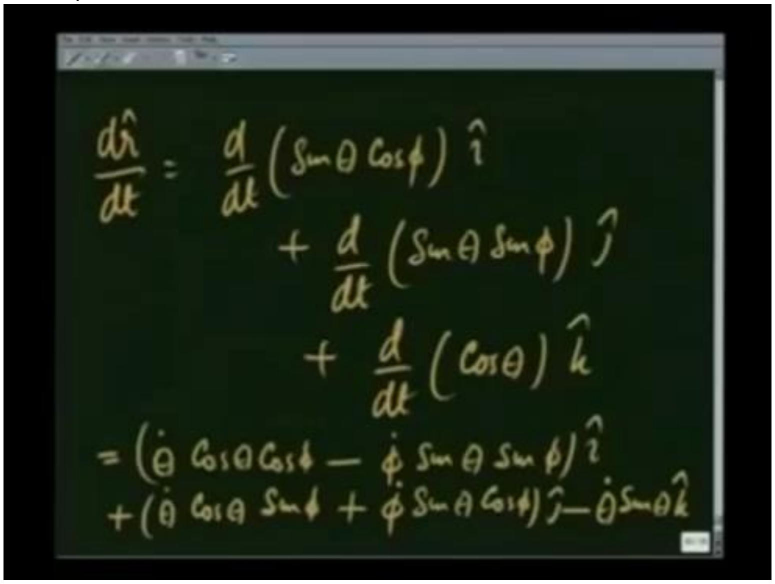

If I take the derivative, I have DR over DT which is equal to D by DT of sine theta cosine phi I + D over DT of sine theta sine phi J + D over DT of cosine theta K. This turns out to be equal to theta dot cosine theta cosine phi - phi dot sine theta sine phi I + theta dot cosine theta sine phi + phi dot sine theta cosine phi J - theta dot sine theta K.

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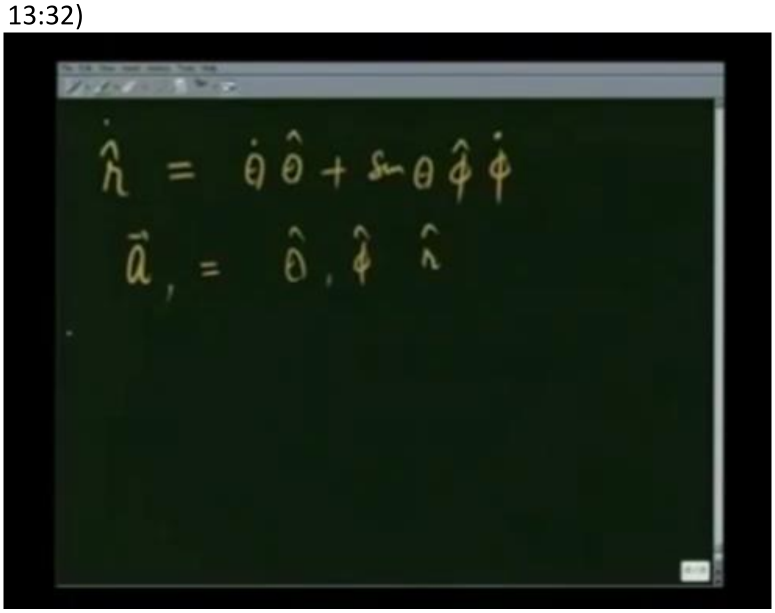

If you collect all the terms together, you can see easily that this comes out to be equal to theta dot theta + sine theta phi. sine theta phi phi dot. Rest of the exercises, that is calculating the acceleration and finding it in terms of theta, phi and are, I will leave as exercise for you.

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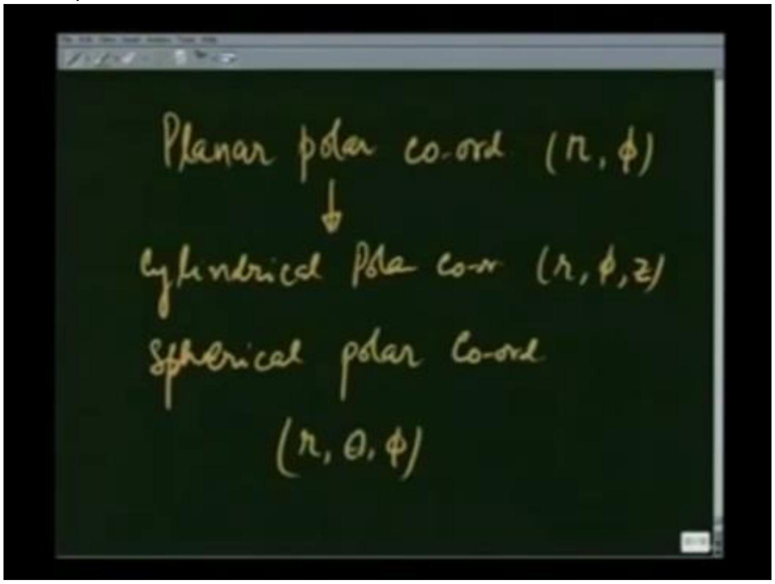

To sum the lecture I would like to value that we covered planar polar coordinates in which I describe the motion in a plane using radial distance and the angle from x-axis. We extended this to cylindrical polar coordinates in which I used R, phi and Z as my coordinate system and we also used spherical polar coordinates in which I used R, theta and phi as coordinates to describe distance or the displacement of a particle.

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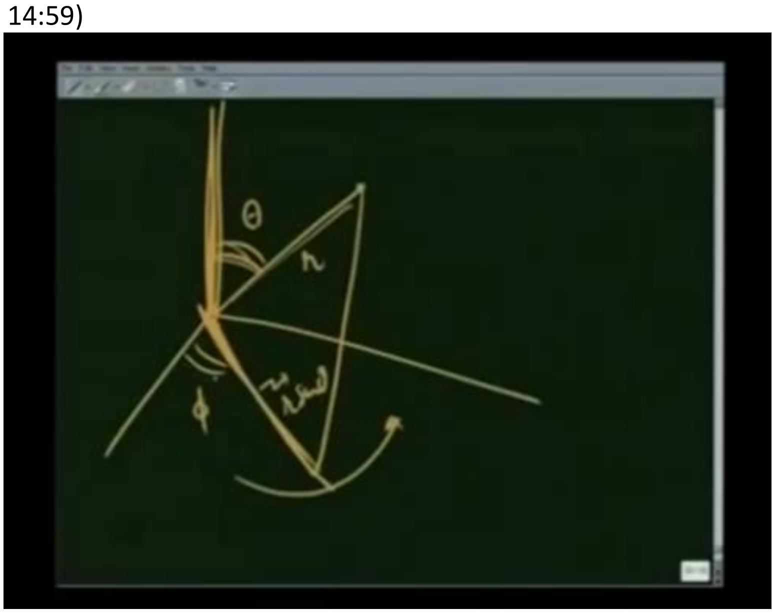

Before we just the end the lecture, I would just like to tell you how to think about spherical polar coordinates. You can think of these as 2 sets of planar polar coordinates. One set being R and theta, that is the particle moves in Z and this plane, Z and this plane of this line, Z axis and this line. With this angle being theta and this R, this describes one set of planar polar coordinate and the other set being the XY plane with these being R sine theta and this angle being phi.