Engineering Mechanics
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Module 5
Lecture No 42
Planar polar coordinates: solved examples

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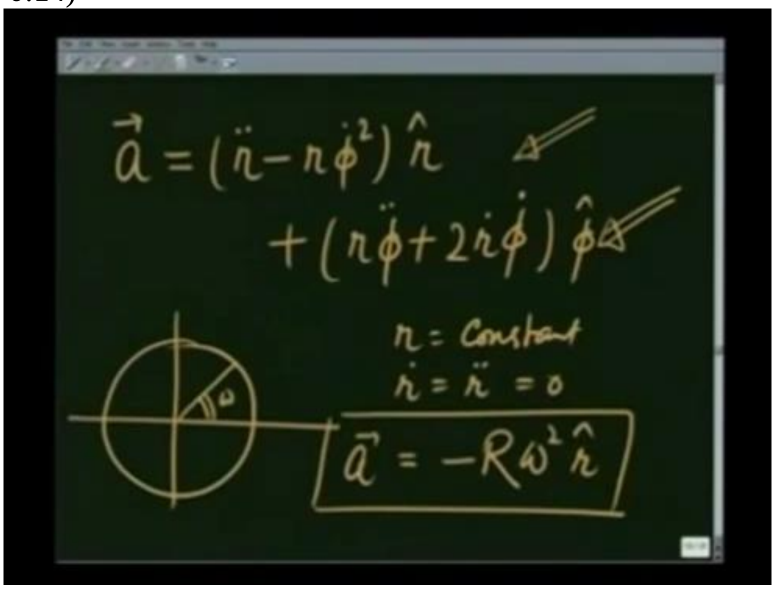

I will solve 1 or 2 examples using planar polar coordinates for particles moving in a plane which will give you some idea as to how to use these coordinates.

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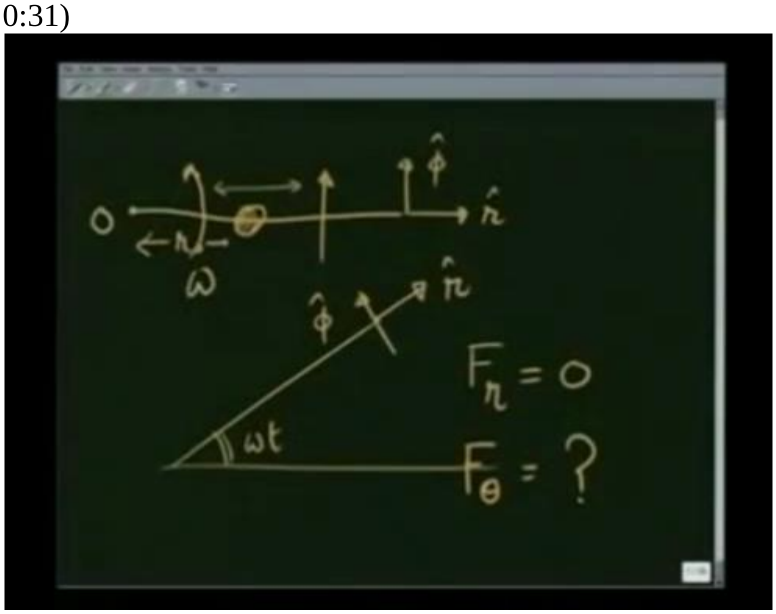
e 

As my 1st example of application of planar polar coordinates, let me take a wire that is rotating with a constant angular speed omega in a plane and let me put a bead here at a distance initially R0 from the centre about which the wire is rotating and I want to know how the position of this wire changes with time, how its velocity changes with time, what is the force that the wire applies on the bead?

Obviously, if it is frictionless, there is no force applied in this direction. There is force applied in this direction because the wire forces the bead to move in this way. And I am going to describe the motion using planar polar coordinates. So right away I see my R direction is going to be this. This is going to be my phi direction.

In general, if wire is making some angle omega T from its initial position, this is going to be the R direction and this is going to be the phi direction. The force in the radial direction is 0. Therefore the component of force in the radial direction I write as 0 and the component of force in theta direction I do not know but it is nonzero and I wish to calculate this.

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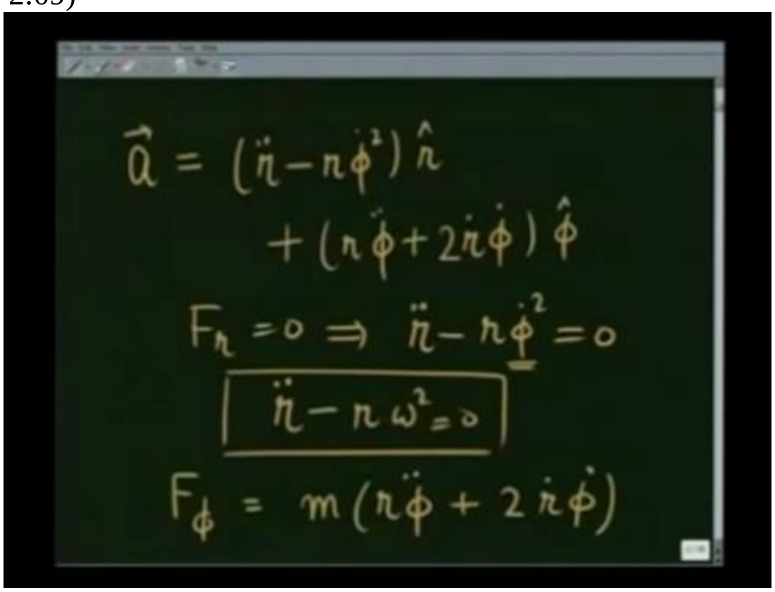

If I write the acceleration of the particle in planar polar coordinates, it is going to be R double dot - R phi dot square in R direction + R phi double dot + 2R dot phi dot in phi direction. And as I have just told you, FR is 0 and this implies R double dot - R phi dot square is 0. And since I know that the particle is moving with a constant angular speed, so phi dot is a constant which is equal to oh omega. Therefore the equation becomes R double dot - R omega square is equal to 0.

Similarly in the theta direction, F theta is going to be equal to MR phi double dot + 2R dot phi dot. Since the wire is moving with a constant angular speed, this term is 0.

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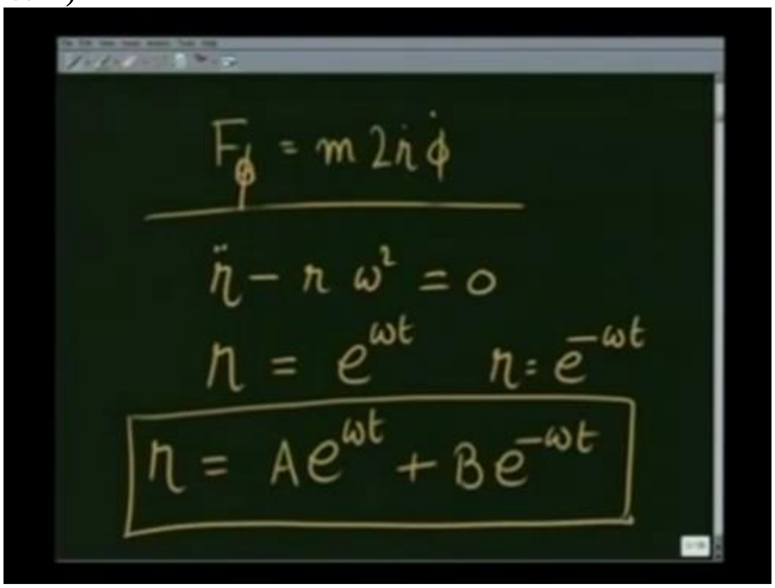

And therefore I have F phi that is the force that the wire applies in the phi direction to be equal to M2 R dot phi dot. To start solving the equation, let me go back to the earlier equation R double dot - R omega square is equal to 0. You can see by section that this has 2 solutions, E raised to omega T or R is equal to E raised to - omega T.

And therefore the general solution R is going to be A where A is a constant E raised to omega T + B where B is another constant - omega T. This is how R is going to change as a function of time. The constants A and B are fixed by the initial condition.

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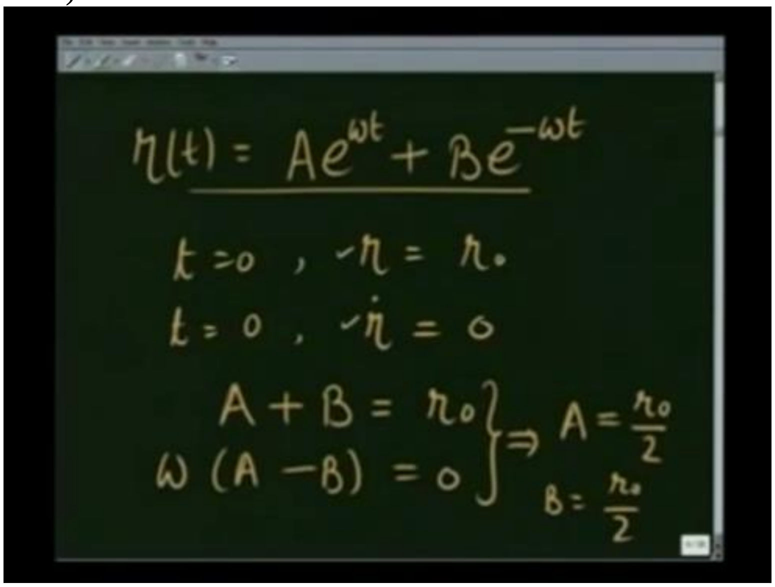

So the solution R as a function of time is AE raised to omega T + BE raised to - omega T. And if I am given that at T equal to 0, the particle is at some initial position, R0 and at T equal to 0 suppose it was not moving in R direction so that R dot was equal to 0, then I have, writing these quantities from this equation at T equal to 0, A + B is equal to R0 and if I take the derivative omega A - B is equal to 0 then these 2 equations give me A is equal to R0 over 2 and B is equal to R0 over 2.

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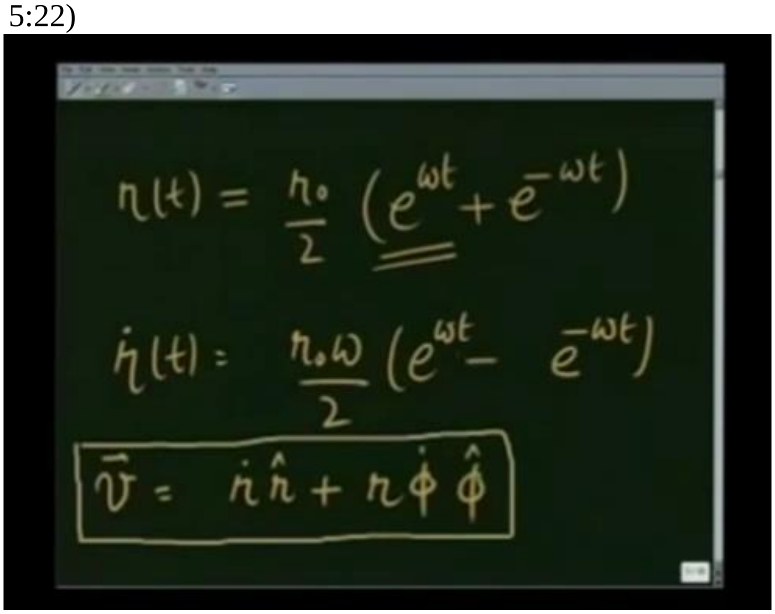

And therefore the general solution RT for a particle on a wire that is rotating at a constant angular speed and the particle started from a distance R is equal to R0 with no initial radial speed is R0 divided by 2 E raised to omega T + E raised to - omega T. You can see R increases exponentially. R increases very fast with time.

How about the velocity in radial direction? It is going to be equal to R not omega divided by 2 E raised to omega T - E raised to - omega T. General velocity V is going to be R dot R + R in phi direction and you can substitute the values and find the general velocity. How about the acceleration of the particle?

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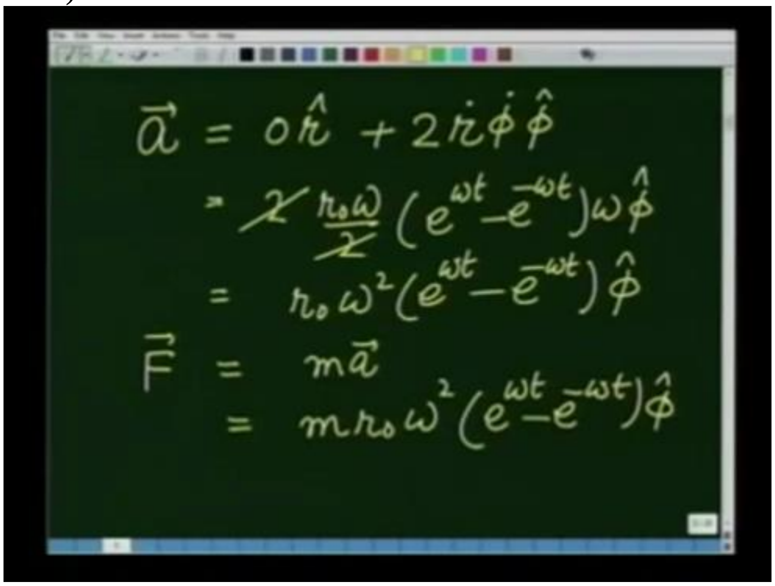

The acceleration as you have already calculated is 0 in R direction and 2 R dot phi dot in the phi direction. Using the expression for the R derived earlier, this comes out to be 2R0 omega by 2 E raised to omega T - E raised to - omega T times omega which is the value of phi dot in phi direction. This 2 cancels and the acceleration therefore is R not omega squareE raised to omega T - E raised to - omega T in phi direction.

This is the general expression for their celebration for this bead moving on the wire. How about the force? The force applied by the wire is given by MA and is therefore equal to MR not omega square E raised to omega T - E raised to - omega T in phi direction. So you see that we have solved this problem in planar polar coordinates in a simple manner. I would urge you that you try solving the same problem using X and Y coordinates.

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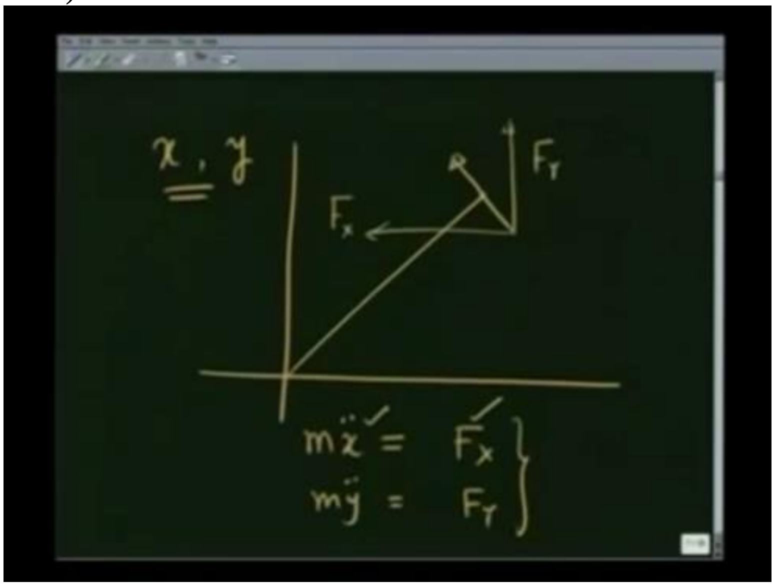

Just to give you an idea, when the wire is moving, wire is applying a force like this which is going to have a component in X direction, which is going to have a component in Y direction and therefore the equations you will end up writing would be MX double dot is equal to FX which is going to be the component of the force in this direction, MY double dot is going to be equal to FY which is this component and solve these with respect to time.

F changes with time, X changes with time. This solution is going to be slightly more complicated. So in such situations, use of planar polar coordinates help us.

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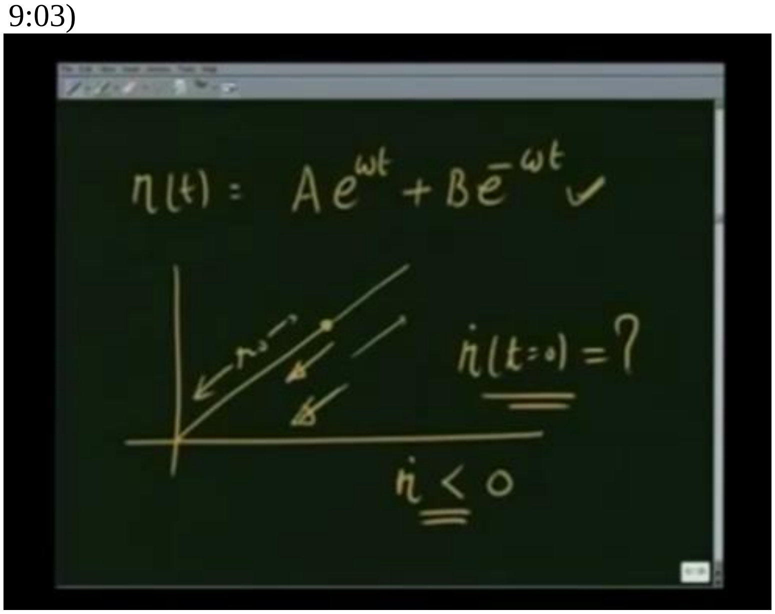

Also once I get the general solution, an interesting situation is when I ask you recall this isE raised to I omega T + B E raised to - omega T. I am going to ask, can I start the bead from a distance R0 and give it a velocity in this direction so that the bead never moves out, keeps on moving in. Okay, this is just an interesting application of the solution.

So what I am asking is, what should R not at T is equal to 0 be? So that R dot is always less than 0. That is R keeps on decreasing. Let us see that.

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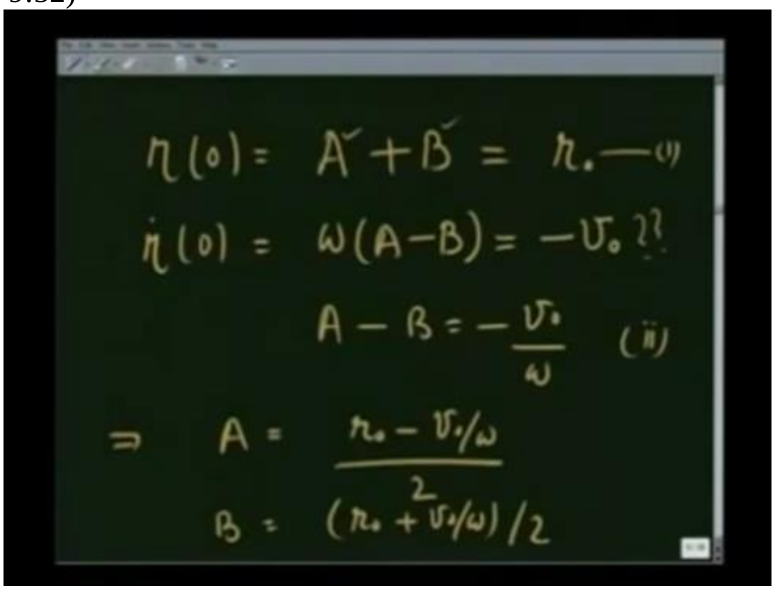

So I given that R at 0 which is going to be A + B is equal to R0. I have already calculated R dot and at 0 this is going to be omega A - B and I want to have velocity going in so - V0. I want to find that V0 with which I push the particle in so that it keeps on moving in. I want to find that velocity. Right? So what I want is that such A and B so that the particle keeps moving in.

This is my equation number 1. Equation number 2 becomes A - B is equal to - V0 over omega. This is my equation number 2 and this gives me A is equal to R0 - V0 over omega divided by 2. And Bis equal to R0 + V0 over omega divided by 2.

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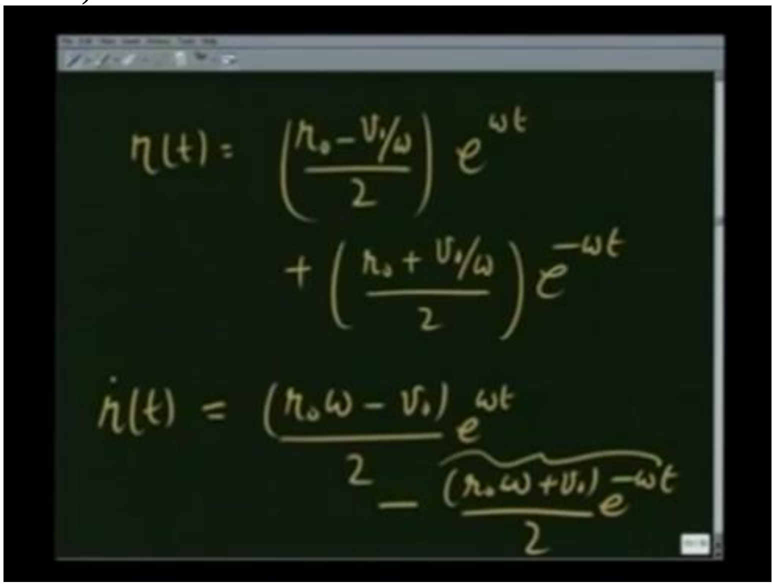

So my solution is going to be R0 - V0 over omega divided by 2 E raised to omega T + R0 + V0 over omega divided by 2 E raised to - omega T. And therefore R dot as a function of time is going to be R0 omega - V divided by 2 E raised to omega T - R0 omega + V0 divided by 2 E raised to - omega T. This term is already negative.

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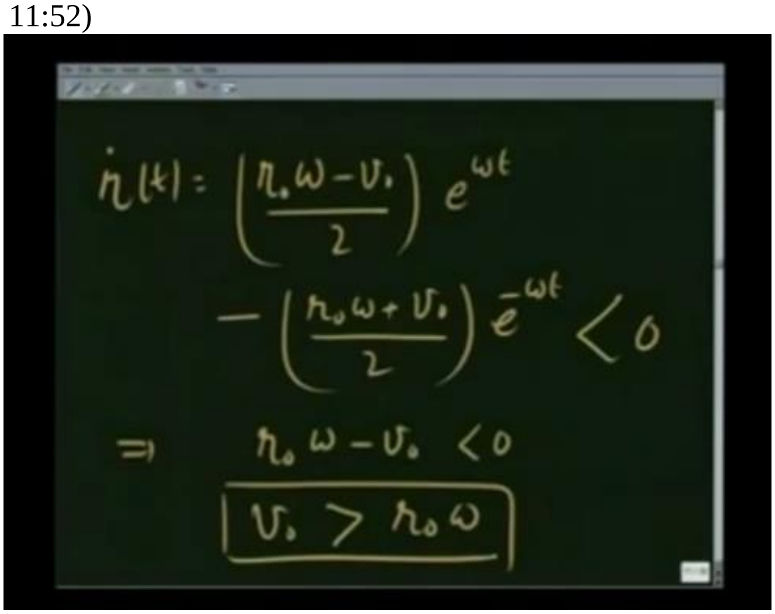

And if I want the velocity to remain negative, that means I want R dot which is equal to R0 omega - V0 over 2E raised to omega T - R0 omega + V0 divided by 2 E raised to - omega T less than 0. This implies R not omega - V0 should always remain less than 0 or V0 should remain R0 omega. So as another example of this, what we have shown is that depending on the initial condition, if I choose not velocity of pushing the particle in so that it is greater than R0 omega, the particle would keep on moving in.

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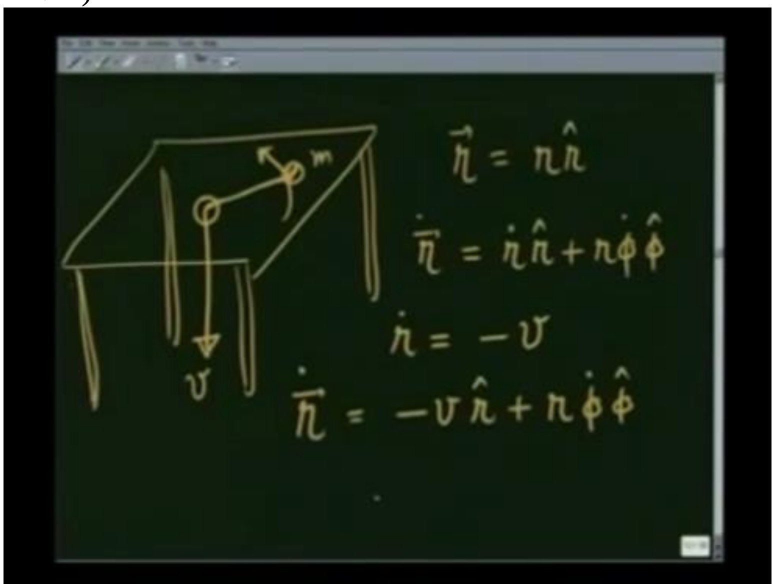

As another example of the application of planar polar coordinates, let me take a particle which is moving secularly on a table. Here is the particle of mass M. It is tied with a thread and thread is passing through a hole in the middle of the table. And I am pulling this thread or string in with a constant speed V. Again, since the motion is circular, it helps to use planar polar coordinates.

So the position of the particle at any time is given as R is equal to R unit vector R. Its velocity R dot is equal to R dot R + R phi dot phi. R dot is given to be constant and R is decreasing. So its sum - V and therefore the velocity R dot is equal to - VR + R phi dot phi. I want to find the subsequent motion.

How phi dot changes with time? How R changes with time? What is the force F needed to pull this thread down? Obviously the particle is moving in this circle and therefore there is and the table is frictionless, there is no force in phi direction.

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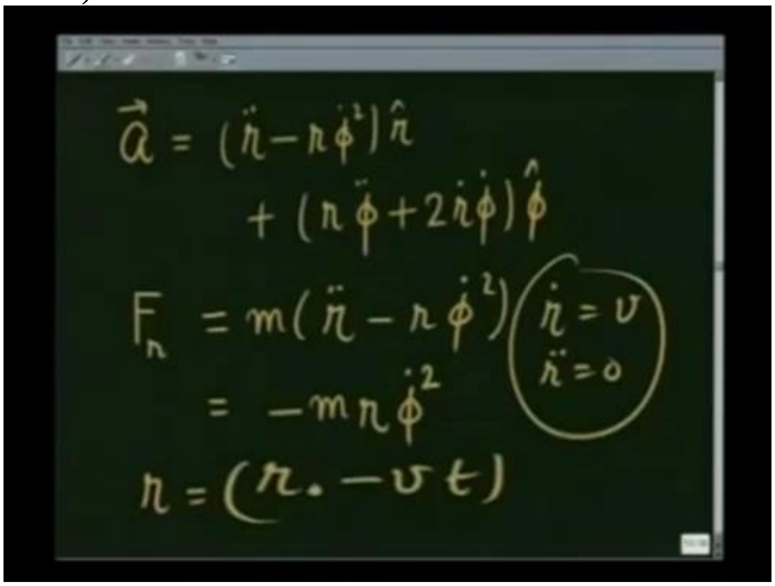

So let us go back to what the acceleration of the particle is. It is R double dot - R phi dot square in R direction + R phi double dot + 2 R dot phi dot in phi direction. The force that I am pulling it in with is in radial direction. Therefore that is equal to M R double dot - R phi dot square.

And since it is being pulled in with constant speed, that is R dot is equal to V and R double dot therefore is equal to 0. I have FR is equal - MR phi dot square. Also, R is going to be its initial R0 - VT because R just keeps on decreasing at a constant rate. On the other hand since there is no frictional force, there is no force in phi direction.

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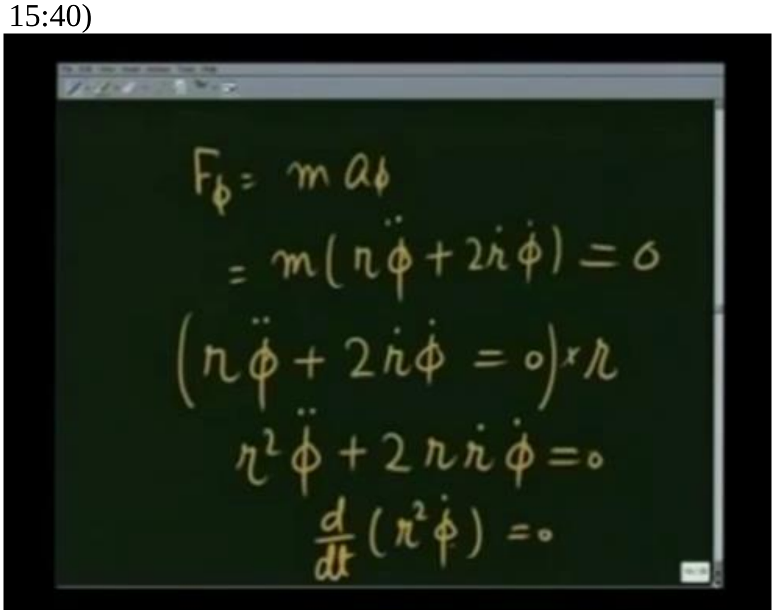

And therefore I am going to have F phi which is equal to MA phi which is equal to MR phi double dot + 2 R dot phi dot is equal to 0. So R phi dot double dot + 2 R dot phi dot is equal to 0. I can easily integrate this equation if I multiply this whole thing by R so that I get R square phi dot + 2R R dot phi dot is equal to 0. And you can easily see, this is nothing but derivative of this is phi double dot, R square phi dot.

First term gives you R square phi double dot. Second term gives you 2 R R dot and this is equal to 0. And therefore, R square phi dot is constant since the derivative of R square phi dot is 0.

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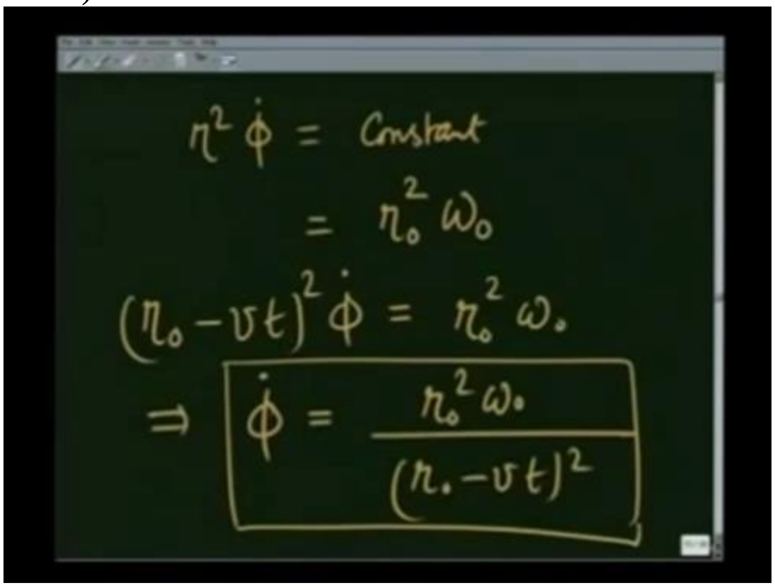

And therefore R square phi dot is a constant. So if the initial position was R0 and its initial angular speed was omega 0, this remains fixed. As I already told you, R changes according to R0 - VT square phi dot the food is going to be equal to R0 square omega 0. And therefore the angular speed if I am pulling the wire in with a constant is going to change as R0 square omega 0 over R0 - VT square. So we have obtained how the angular speed of the wire changes.

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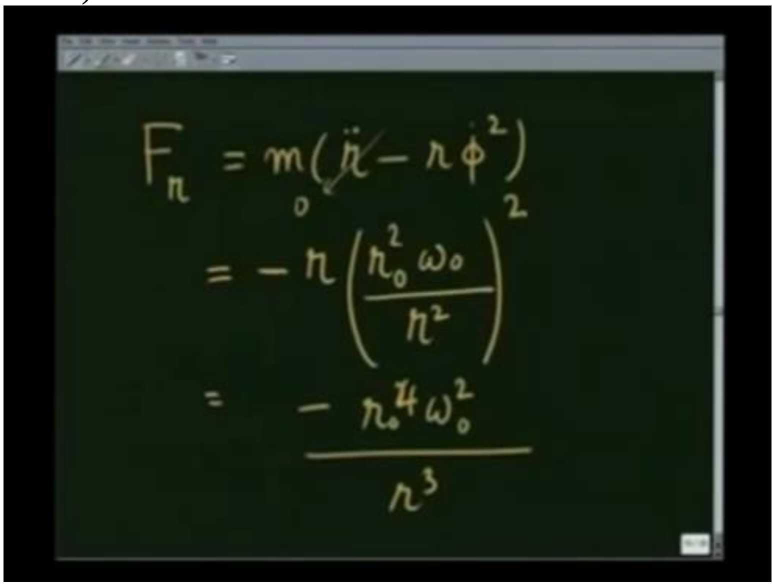

How about the force needed to pull the wire in? That as I told you earlier is FR which is equal to MR double dot - R phi dot square. R double dot is 0 because I am pulling the wire in with constant speed. And therefore this is going to be equal to R - R phi dot square is nothing but R0 square omega 0 square over R square and I square it which is going to be equal to - R0 raised to 4 omega 0 square over R cubed.

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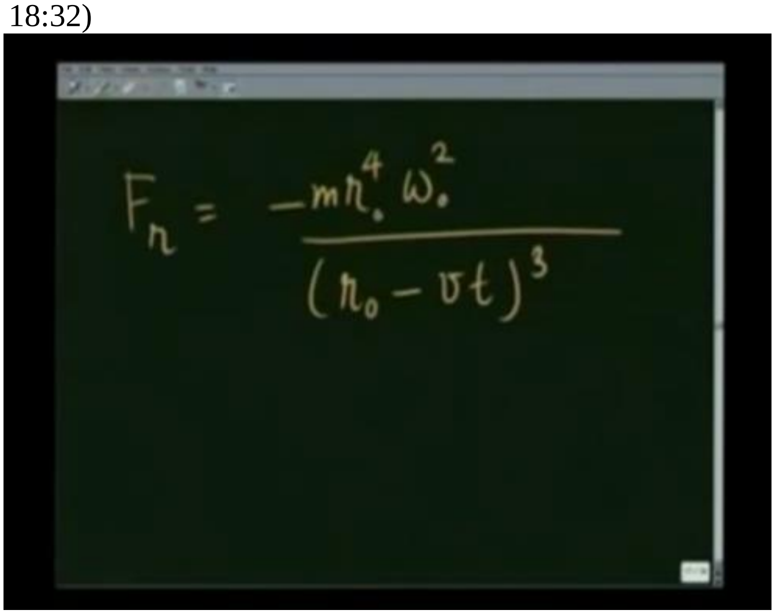

And therefore the force needed to pull it in is equal to M - R0 raised to 4 omega 0 square over R0 - VT cubed. So as time increases, this quantity becomes smaller and smaller and force that you need to pull the wire in with constant speed keeps on going up. So with these 2 examples, I have shown you how to use planar polar coordinates.

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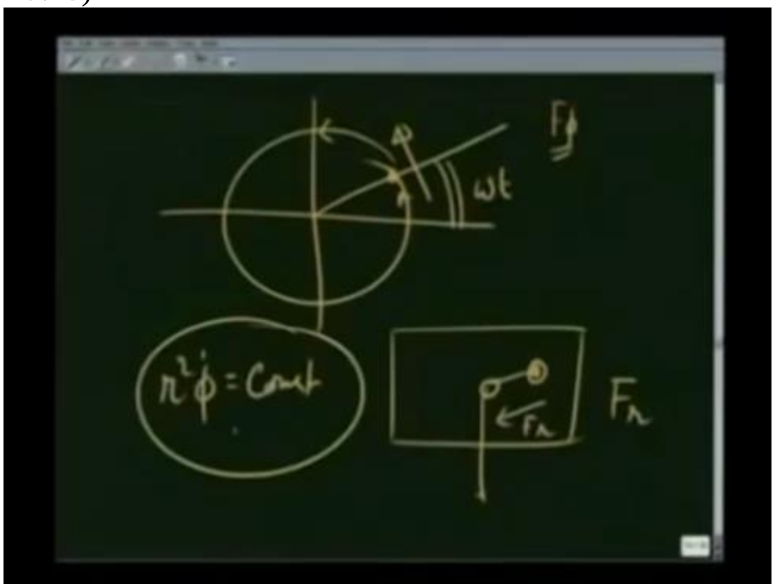

In one case where there was a bead on a wire and the wire was moving with some speed omega. So this was omega T, the force was in phi direction. In the other case where we are pulling a mass and the force is in radial direction. In this case, we can solve for F phi, in this case we solve for FR.

In this case I also got R square phi dot to be a constant and you would recall from your previous courses in your 12^{th} grade that this is nothing but statement of conservation of angular momentum. In this case, the force is radial. And therefore the conservation of angular momentum takes place.