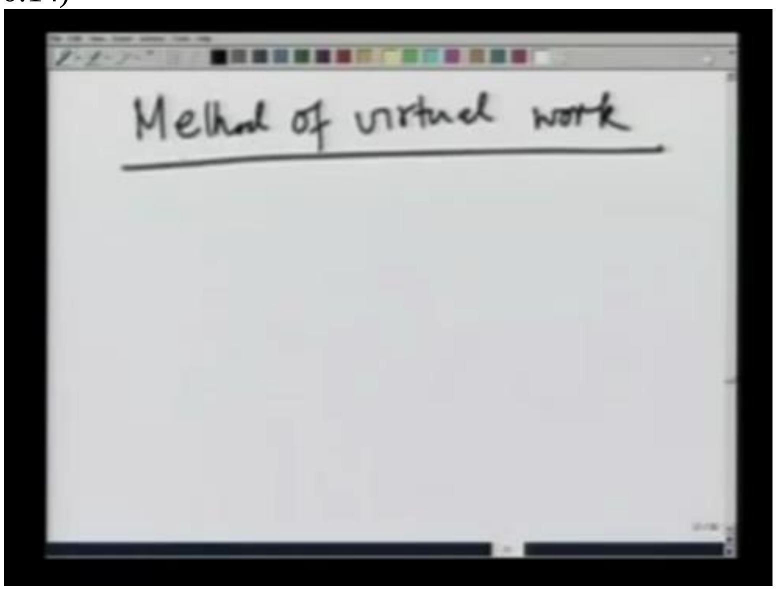
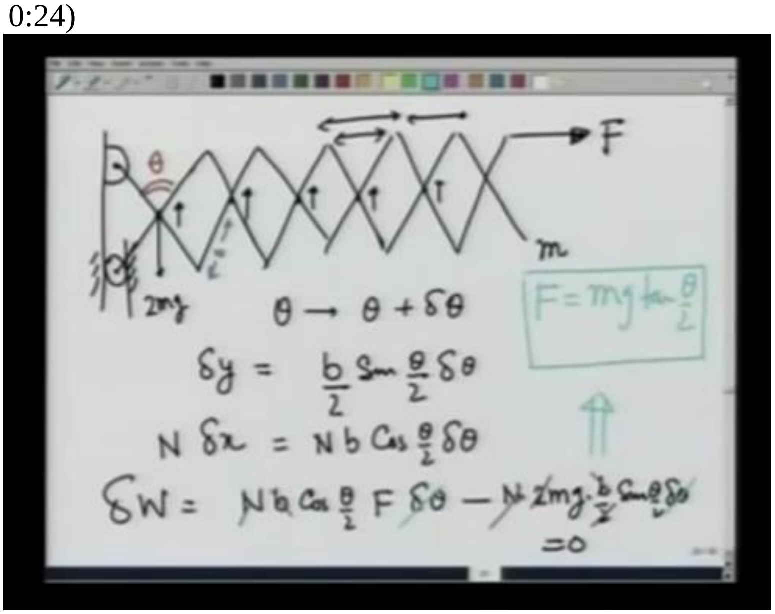
Engineering Mechanics
Professor Manoj K Harbola
Department of Physics
Indian Institute of Technology Kanpur
Module 4
Lecture No 40
Properties of plane surfaces – VIII: second moment and product of an area, solved examples

(Refer Slide Time: 0:14)

e 

How does the method of virtual work simplify or make life easy when the system becomes big?

(Refer Slide Time: 0:24)



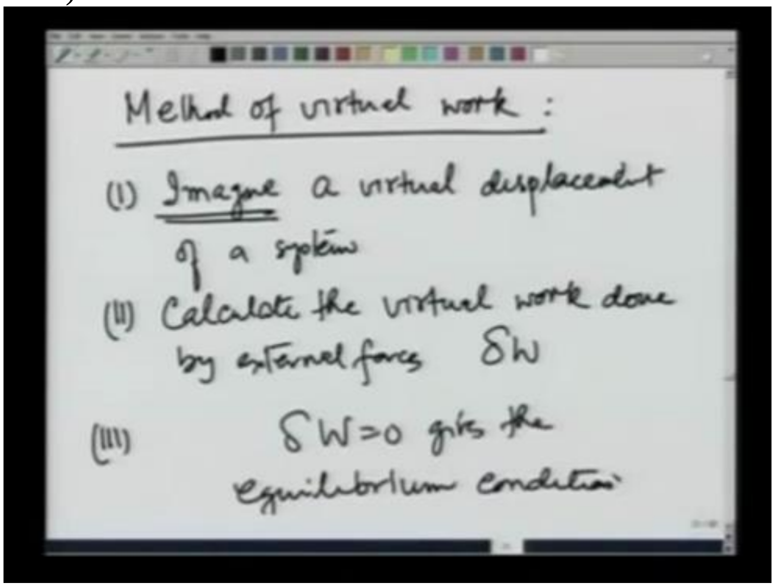
Let us extend the previous example itself. In this mechanism, if I take many many links, suppose N such links and so on. And I apply a force F here. Each bar is mass M and I want to calculate how much force F is required to keep this entire system in equilibrium. Again notice that this system is has only one degree of freedom with this data describing everything. Let the length here be B. Imagine again a virtual displacement of this system so that theta goes to theta + delta theta.

If that happens, all these points are going to move up. And all these gaps here are going to widen up. We have already calculated that delta Y for each point is equal to B sine of theta by 2 delta theta. B divided by 2. And delta X for each of this gap opens up by B cosine of theta by 2 delta theta. So if there are N such links, delta X for N is going to be N times as much, N times B. Delta Y for each joint is going to remain the same.

So work done, virtual work done is going to be NB cosine of theta by 2 times F delta theta - this delta Y at each link is doing work against 2 MG. And there are N points. So therefore I am going to have N times 2 MG times B over 2 sine theta by 2 delta theta and this should be 0. This N drops out, 2 cancels, so does B and so does delta theta. And therefore you again get and let me write it here to keep on the same slide.

F equals MG tangent of theta by 2. Same answer as earlier. So we figure out that no matter how long a link do we make, it does not matter, the force required to keep the system in equilibrium is still MG tangent theta by 2. But you see how easy it was. We did in 4 or 5 steps using method of virtual work.

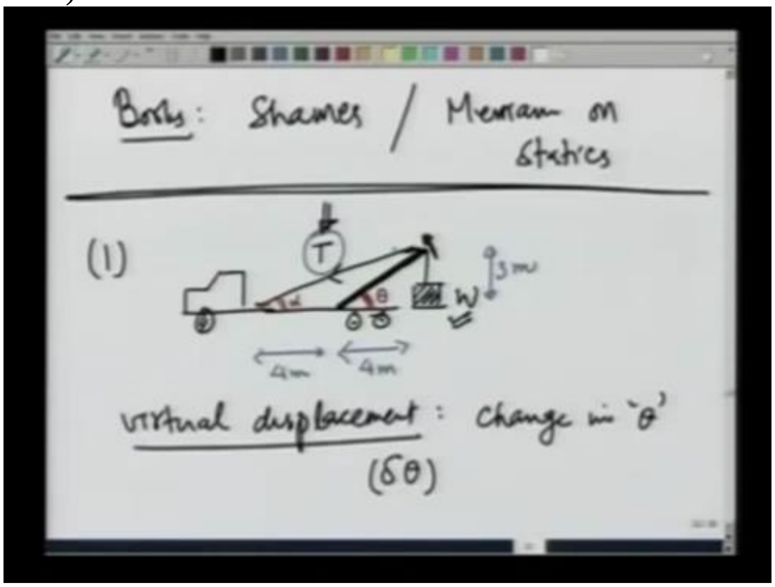
(Refer Slide Time: 3:30)



So let us recap what we did in method of virtual work. One, imagine and I keep using this word, imagine because this is the 1st to a displacement which we imagine. It does not really take place. Imagine a virtual displacement or virtual change of orientation and so on of a system. Two, calculate the virtual work done by external forces and three, call this delta W. Delta W is equal to 0 gives the equilibrium condition.

As in any other new concept, to master it, you have to keep doing many many problems. And I will again rest of the lecture I am going to do 3 or 4 more examples to make you more familiar with the method of virtual work.

(Refer Slide Time: 5:02)

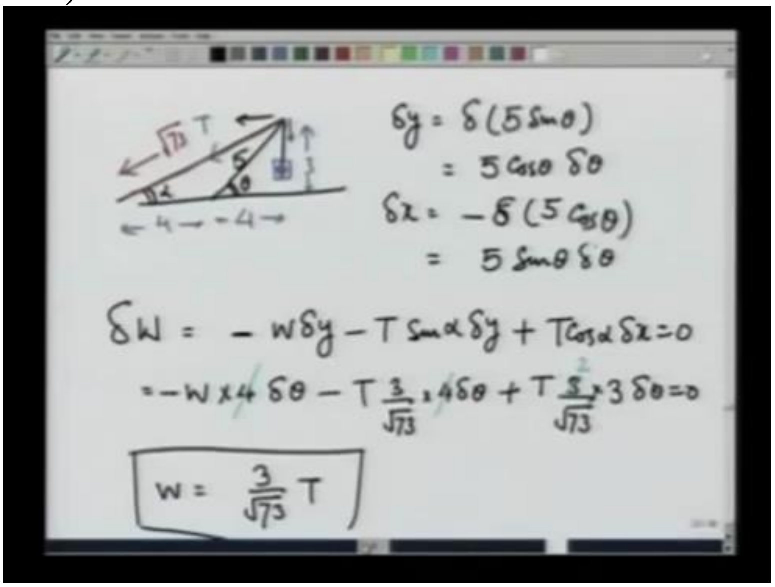


These examples are from the book by Shames on engineering mechanics and book by Meriam on statics. As the 1st example, let me take a crane on which there is a shaft here and by a string by applying tension T, one is holding a weight W. The distances given are 4 metre, 4 metre and 3 metre. We wish to calculate, given tension T, how much weight can be held here in equilibrium?

Let me call this angle theta and this angle alpha. If I want to use the method of virtual work then what we should do is imagine a virtual displacement of the system and here the virtual displacement is going to be described by the change in angle here. As you change, you pull this string in or pull the rope in, the system is going to move like this. So virtual displacement is going to be changed in theta and then calculate the work done by the external forces W and notice in this case the external force, tension is not a constraint force.

The constraint force is the force provided by the shaft because of its rigidity. So work done by that is going to be 0 because the displacement is always going to be perpendicular to this shaft and so we have to calculate work done by W as well as T, the tension and make it equal to 0. So let us see, at the given position where persons are given to be 3 metres, 4 metres and 4 metres what we have is virtual displacement is going to correspond to delta theta. And that will make this point move up, both in X direction and Y direction.

(Refer Slide Time: 7:47)



So what we have this situation schematically is theta, alpha, 4 metres, 4 metres, 3 metres, W here, tension T here. When I change theta to delta theta, if this length I this is going to be 5 metre, this is 3 and 4. So delta Y positive in the up direction is going to be delta of 5 sine theta which is going to be 5 cosine of theta delta theta. Similarly delta X is going to be in the negative direction. X is going to decrease.

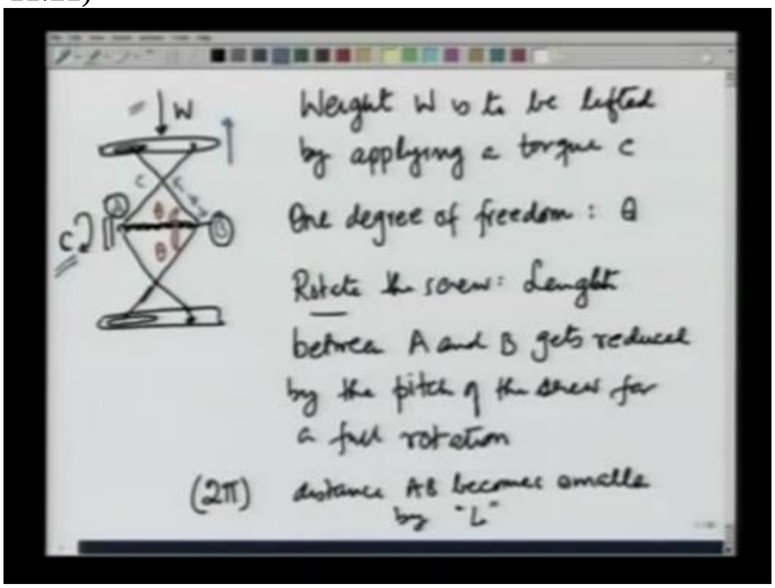
So I am going to write this as negative or just keep in mind it is to the left, is going to be 5 or delta of 5 cosine theta which is going to be 5 sine theta delta theta towards the left. We have calculated both X and Y components. And therefore total work done Delta W is going to be the weight and delta Y in opposite direction. So - W delta Y, the D has a vertical component with this point going down which is going to be T sine of alpha delta Y and T has a component going towards the left which is T cosine alpha so that is going to be T cosine of alpha delta X is equal to 0.

This gives me W. Delta Y, we have already calculated, 5 cosine theta delta theta. 5 cosine 4 theta from here is 4 times 4 delta theta - D sine alpha. Sine alpha is going to be 3 divided by this length. This length is 8 square + 3 square square root. So root 73. So T times 3 over root 73 times 4 delta theta + T cosine alpha is going to be T oh this is a - sign here. T cosine alpha is going to be T 8 over square root of 73.

Delta X is in the same direction, 5 sine theta is 3 delta theta is equal to 0. And that gives you again cancel 4, 4, 4 and this will give me 2 here. And that gives you, W is equal to 3 over root 73

T. That is the weight that can be held by this machine. You see again the calculation is very straightforward. As the final example of applying method of virtual work, I solve a problem that involves both a force and a torque.

(Refer Slide Time: 11:21)



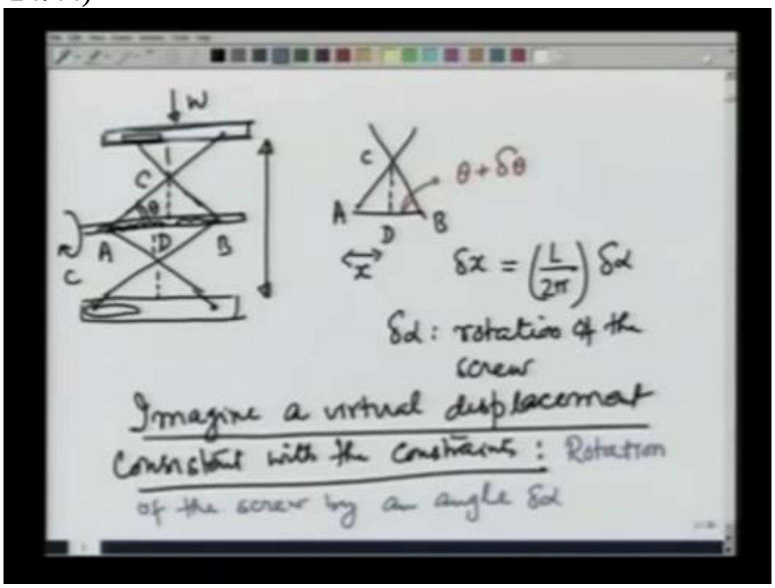
The example is that of a screw jack whereby the supports like this are fixed, the screw here and we apply a torque here, C. These go down and are fixed on a stand. One of these links is fixed and the other one can move in a slot and same thing on the top. One of them can move in a slot and the other one is fixed. The question that we ask is if there is a load W on this jack, what is the minimum torque C that is needed to lift this weight? So weight W to be lifted is to be lifted by applying a torque C.

You are given that this angle is theta. So is this and this length is B. Let us also call these points A, B, C. You can again see that there is only one degree of freedom and that is angle theta. The question is how do we calculate the force, the torque required so that this force W or the load W is balanced or can be lifted. To calculate this, let us see what happens when we rotate this screw.

On rotating this screw, the length between A and B, that is these 2 points, gets reduced by the pitch of the screw for a full rotation. So what is given? When I make a full rotation that is 2 pi rotation, the distance AB becomes smaller by L, length L which is the pitch of the screw. When A and B come closer, this platform gets lifted because this point gets lifted. So in rotating the screen to, I am doing some work and in lifting this, another work is being done.

The external forces and torques now are C, the external torque and W, the external load. The net virtual work done by combination of C and W should be 0. Let us calculate that and make that equal to 0.

(Refer Slide Time: 14:44)

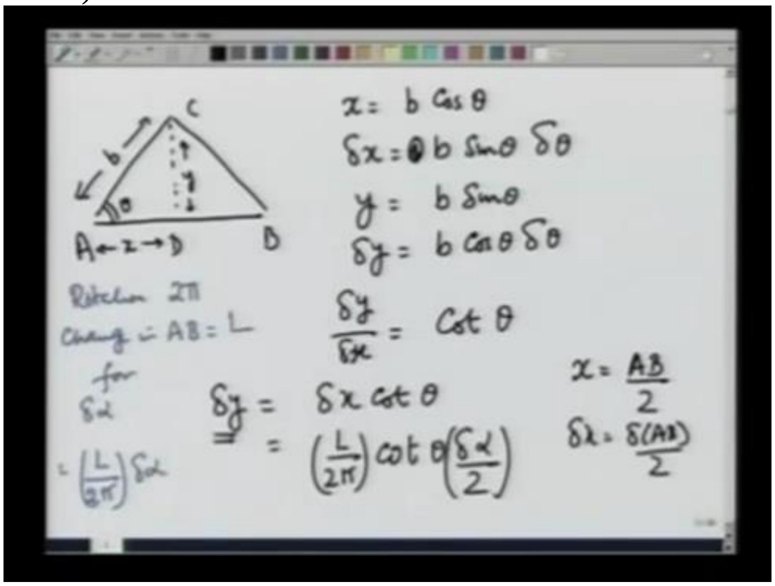


Let us call this angle theta which has already been given. So when the length reduces, this theta becomes slightly larger. Let us say this goes to theta + delta theta. And therefore this length between let us call this AB and the top of the triangle C and the perpendicular distance from C to ABCD. So this B, C, A and B. CD increases. The total length would increase by 4 times the increase in CD because there are 4 gaps like these.

So let us calculate that. Let us call this distance AD X so that delta X is the reduction in X and delta X is going to be L over 2 pi times delta alpha where delta alpha is the rotation of the screw. So now what we do is imagine a virtual displacement consistent with the constraints and that virtual displacement is going to be rotation of the screw by an angle delta X and that causes shortening of the distance between A and B and increase in the distance C and D.

From that I can calculate the virtual work done by applying the torque and the virtual work done against weight and making the sum total of that virtual work 0 would give me a relationship between the torque C and the weight W. Let us calculate that now

(Refer Slide Time: 17:09)

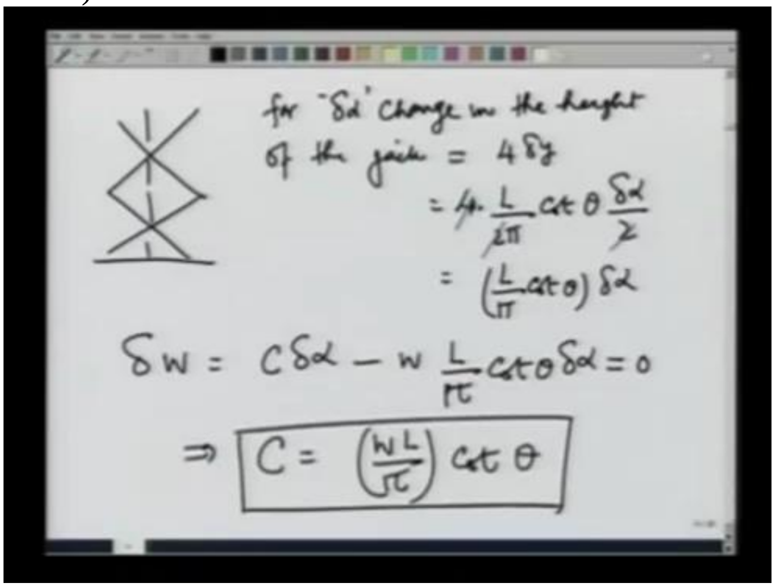


So in this triangle A, B, C and D, let us call this distance X and this distance Y. This angle is theta. Let us call this hypotenuse AC to be B. Then you can see that X equals B cosine of theta. Therefore delta X is going to be B sine theta delta theta. I am just wondering about the magnitude. So this sign - sign I am not worrying about. Y equals B sine theta. So therefore delta Y is going to be B cosine of theta delta theta.

And therefore delta Y over delta X is going to be equal to cotangent theta or delta Y is equal to delta X times cotangent theta. How much is delta X? That is equal to nothing but the change in the length AB . For rotation let me just write on the side, rotation 2 pi change in AB is equal to L. So for delta alpha, it is going to be L over 2 pi times delta alpha.

So therefore delta Y is L over 2 pi times cotangent theta delta alpha divided by 2. This divided by 2 comes because change in X, X equals AB divided by 2. So delta X is delta AB divided by 2. So we have calculated the change in delta Y related to a change, delta alpha.

(Refer Slide Time: 19:15)



Since in this screw jack, there are 4 lengths like this, 1, 2, 3, 4, so for delta alpha to change in the height of the jack, is going to be equal to 4 times delta Y which is 4 times L over 2 pi cotangent theta delta alpha by 2. This 2 cancels with 4 and this is therefore L over pi cotangent theta delta alpha. Let us now calculate the total virtual work done.

Total virtual work done is going to be C times delta alpha because of the torque and the length Y increases in the direction opposite to W. So this is going to be - WL over pi cotangent theta delta alpha. And by the principle of virtual work, this is 0 and that gives me an answer that the torque that is required to lift the weight minimum has to be WL over pi cotangent theta. And that is your answer.

So what I have done in this lecture is given you an introduction to the method of virtual work for you to appreciate the power of this method. It bypasses doing lengthy calculations and focuses directly on the external forces that we want to calculate. Of course we have not included friction here because friction is a constraint for that does virtual work. You will be learning about more advanced techniques using such methods in the future courses.