Engineering Mechanics
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Module 4
Lecture No 39

Method of virtual work-II: virtual displacement, virtual work and equilibrium condition in terms of virtual work

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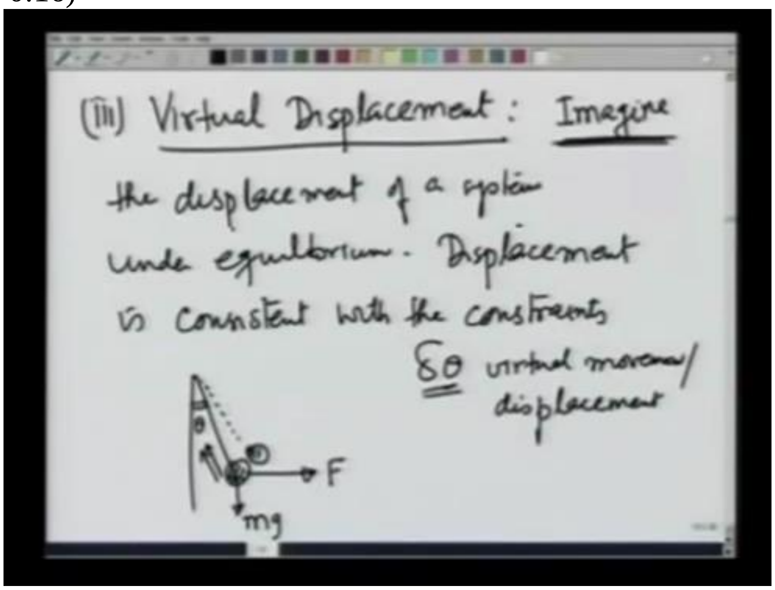

Next, I want to develop the concept of virtual displacement. Virtual displacement, I would like you to imagine and I want to emphasise this word, imagine the displacement of a system under equilibrium. And the displacement is such that the displacement is consistent with the constraints. So what you are imagining is suppose there is a body which is in equilibrium let us again take an example.

If I take the pendulum and suppose I apply a force this way, F, its own weight, MG and it is in equilibrium, imagine, just imagine that I displace it slightly. This is the only way I can displace the pendulum. No other way because this is the only degree of freedom allowed. That is I can change only theta.

So this displacement is consistent with the constraint. But it is actually not moving. I am imagining as if it has moved and this is known as virtual displacement. So and it is usually given by the symbol delta. So delta theta virtual movement or displacement. On the other hand, if I moved the bob along the string, the length of the string would change and that motion would not be consistent with the constraint condition. Therefore I cannot call that motion a virtual motion.

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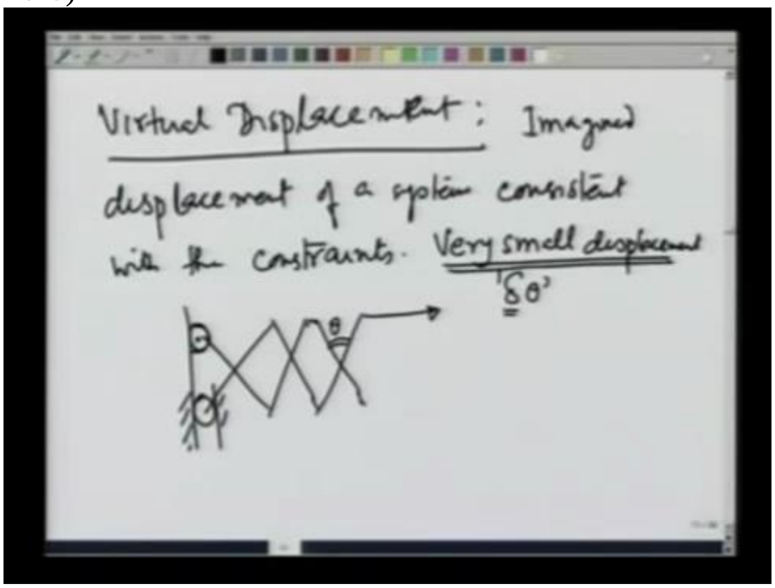

So virtual displacement is displacement again is an imagine displacement of systems, subsystems consistent imagine displacement of a system consistent with the constraints. I have already given you the example of the pendulum. This mechanism again if I go back to, the only way I can change its configuration is by changing this theta. So if it is in equilibrium under this force F here and its own weight, the virtual displacement would be delta theta.

One thing I should make it clear, I am writing it delta theta, so virtual displacement is also very small. Displacement. The change is very small. You imagine a very very small change. So it is about equilibrium, a very small change which we imagine and that is a virtual displacement.

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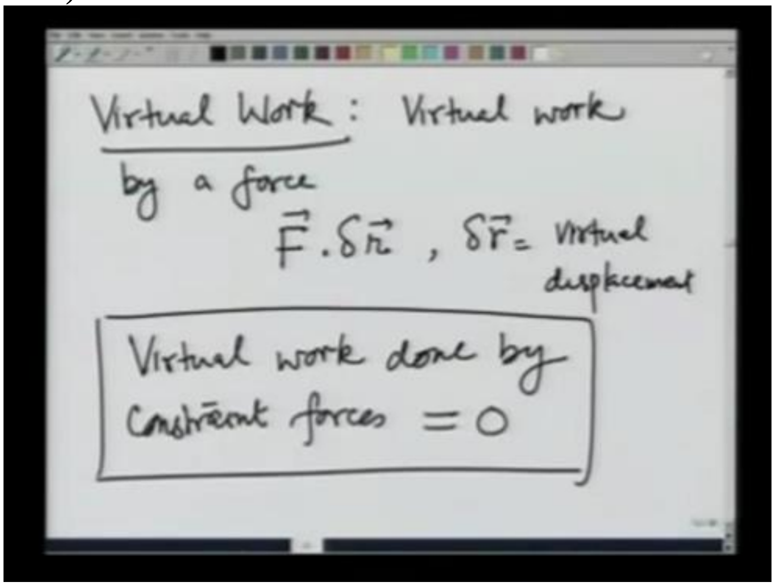

Next, associated with virtual displacement is something called the virtual work. So when you are making a virtual displacement, each particle in the system is changing its position. So virtual work by a force is force times delta R where delta R is the virtual displacement. Naturally, if I am doing a displacement consistent with the constraints and we have already seen that the constraint forces do not do any work, so virtual work done by constraint forces is equal to 0.

This is a very very important statement and that makes the development of method of virtual work possible. With these definitions, we are now ready to develop the method of virtual work.

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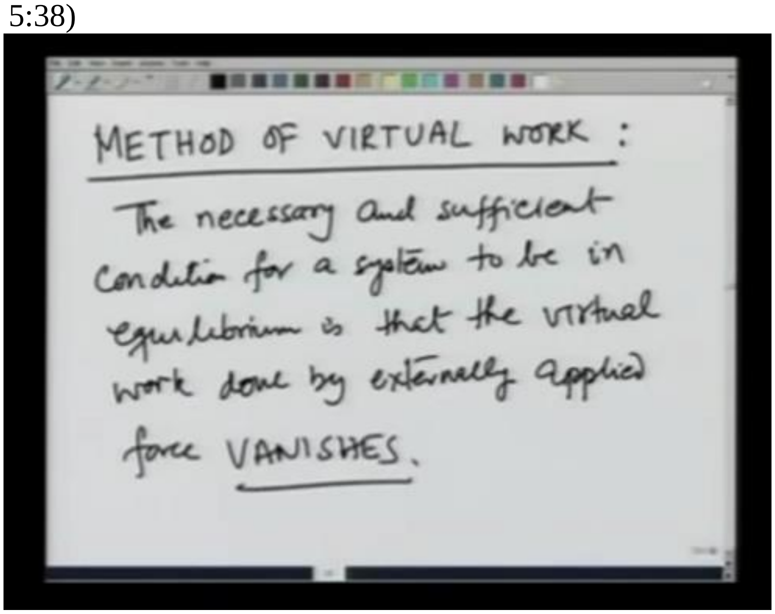

So in my 3rd of virtual work, we state that the necessary and sufficient both necessary and sufficient condition for a system to be in equilibrium is that the virtual work done by externally applied forces vanishes. Two comments are in order. One, that we are talking about system which has no friction. If friction is included, then the constraint forces also do, the frictional constraint force also does some work.

That we are excluding. Then because of the vanishing of work by constraint forces, it is only the work done by externally applied forces those which are not constraint forces but the forces that we apply from outside to achieve equilibrium. Then the virtual work done by externally applied forces vanishes.

Notice, we have avoided taking constraint forces into account and therefore when we apply this method, we can bypass calculating the constraint forces. Let us see how this method works.

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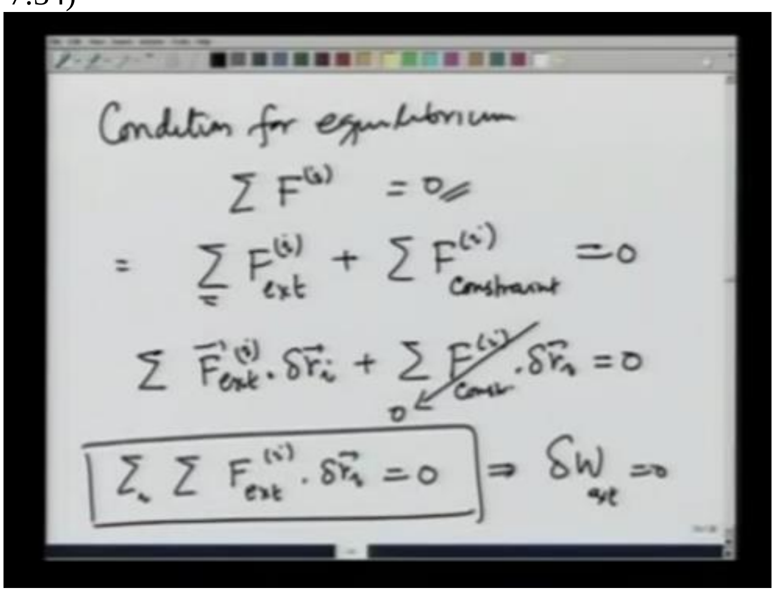

Condition for equilibrium of any system is that summation of all forces on any particle, ith particle, summation of all forces be 0. This may include the externally applied forces on the ith particle + F on ith which are constraint forces. Their sum is 0. If each particle is in equilibrium, the system is in equilibrium. Now let us calculate the virtual work done.

Virtual work done is going to be F external I dot delta RI. This summation refers to all the sources that are applied + summation FI. I refers to the ith particle constraint dot delta RI is equal to 0. It is 0 because right-hand side is 0. However we have already said that constraints do no work.

Therefore the right, this 2nd term vanishes and this tells this tells you that the work done by all the external forces together on each particle is 0. If the work done on each particle is 0, the net force therefore summation over I is also 0. Implies delta W, the virtual work external is 0. We have proved this necessary part of the principle of virtual work. How about the sufficient conditions?

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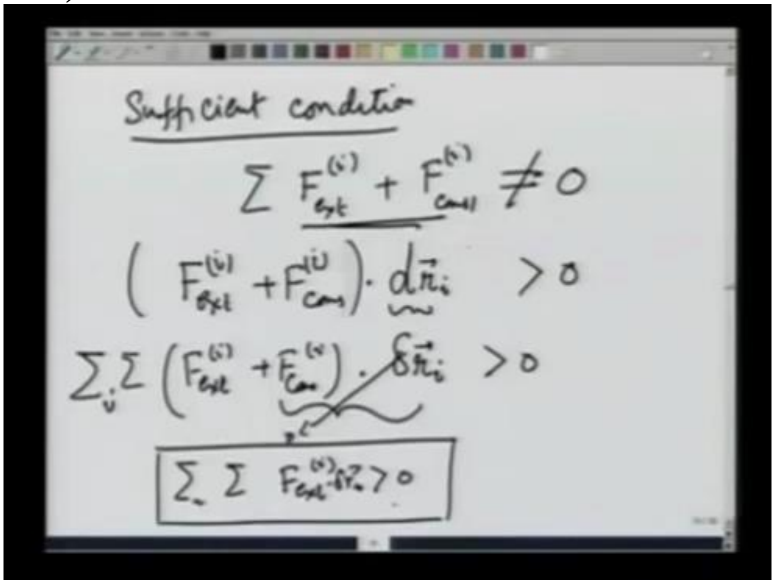

As I said, it is also a sufficient condition. So if the forces on all the particles FI external + F constraint I suppose they do not vanish, if they do not vanish system or each particle would tend to move in the direction along which this force is and therefore F external I + F constraint I dot to distinguish from virtual displacement I am writing this as DRI for each particle would be greater than 0.

It is greater than 0 because force would make the particle move in the same direction in which it has the net direction. However now in virtual work, I can replace I can imagine the system being displaced virtually by delta RI. So F external I + F constraint I virtual will also be greater than 0 for each particle. If this is greater than 0, for each particle, the summation over I would also be greater than 0.

So the work cannot vanish. If the work vanishes, that means that this some must be 0. Again we are going to use the fact that the work due to constraint forces is 0 and therefore again summation I summation F external I is greater than 0. Applying the same argument, if this is nonzero under the condition that system is not in equilibrium, if this is 0, system must be in equilibrium. So that proves that it is also a sufficient condition.

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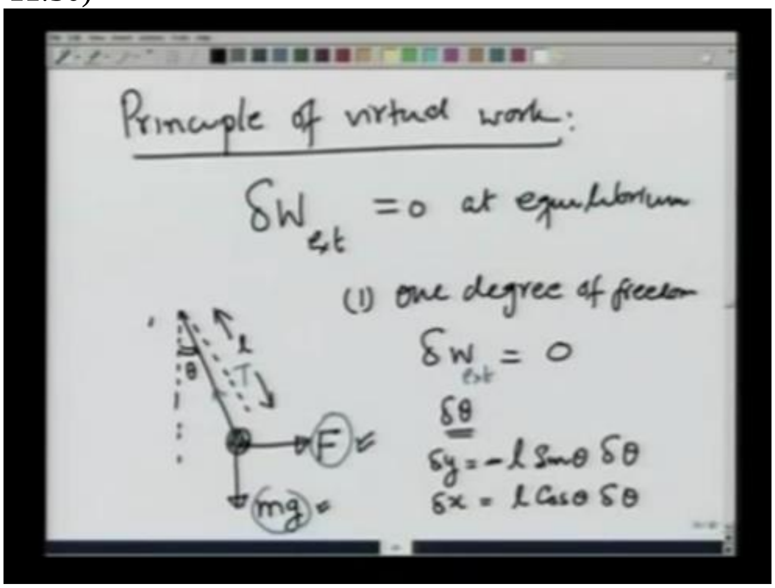

So now we are going to take a thumb there that the principle of virtual work is I will write this in symbols the virtual work done by external forces is equal to 0 at equilibrium. And that is sufficient to determine the forces and so on. Let us take an example. I will start with the pendulum example. This angle is theta.

This pendulum is supposed to be in equilibrium under the applied force F and the weight of the bob MG. This has 1 degree of freedom. So I can make a virtual displacement changing that theta only here. The principle says that delta W only external forces is equal to 0. The only external force that are working on the system are F and MG with T being the constraint force which does 0 work.

So this is only external. Let us make a virtual displacement of delta theta. If I make a virtual displacement of delta theta let us calculate the work done virtual work done by force F and force MG. When I make this displacement, if the length of the pendulum is L then the bob moves up let us call it delta Y by an amount L sine theta delta theta.

And it moves in the horizontal direction by L cosine theta delta theta. In fact I should put this negative to show that it is moving up.

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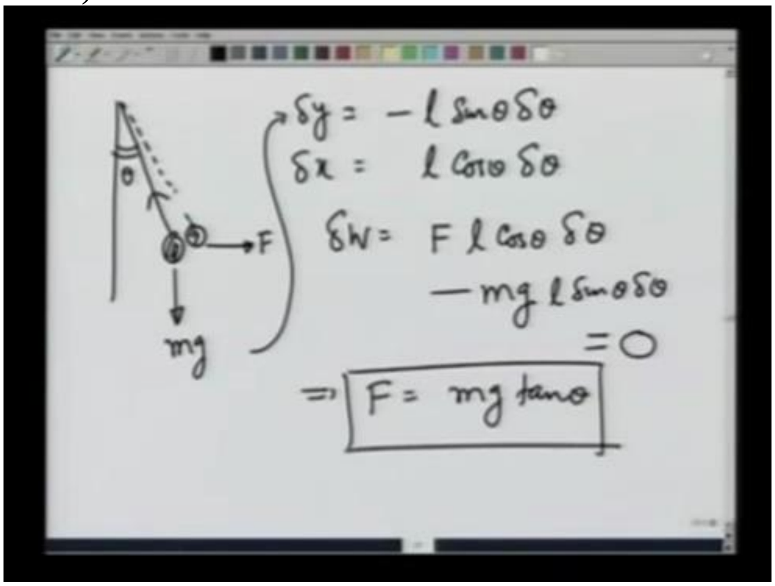

So the virtual work done is going to be let me make the picture again. This is the bob at an angle theta. I changed this to a virtual position theta + delta theta. Then Delta Y is moving up - L sine theta delta theta. Delta X is L cosine theta delta theta. The force this way is F, force this way is MG. So this - sign, delta Y actually the first 2 this being opposite to MG.

So delta W is going to be FL cosine theta delta theta - MGL sine theta Delta theta. And by principle of virtual work this should be 0. And that gives me F equals MG tangent theta straightaway. You see, I did not have to go to calculate tension whereby I would have written T cosine theta is equal to MG and T sine theta is equal to F and therefore F equals MG tangent of theta.

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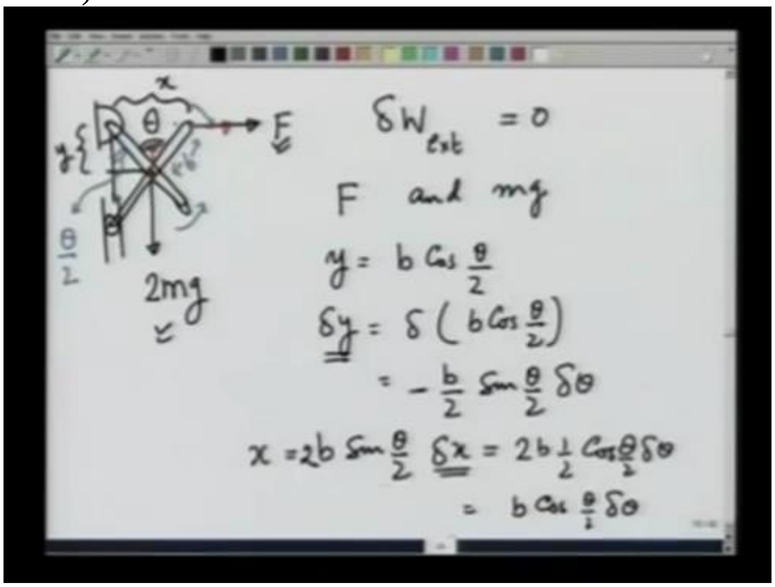

Let us take the other example that we took in the beginning with this mechanism here which is in equilibrium under this applied force F in its own weight here 2 MG. And the only angle that matters here for the degree of freedom is this theta. Each side, each half of the bar is B. The constraint forces again do not do any work and therefore delta W external forces only should be 0. So 2 external forces that are there are F and MG.

Let us make a virtual displacement of the system. What would a virtual displacement be? Virtual displacement be will be to make this angle larger. If I make this angle larger, what I will be doing is pulling this point out. Let me show it by red. This point will be going out and this point would be moving up. By how much amount? Let us calculate that.

So the distance here is nothing but let us call this Y and let this distance from here to here be X. Then you see that Y is equal to B. This angle is going to be theta by 2. So Y is B cosine of theta by 2. If I change theta slightly, delta Y is going to be delta of B cosine of theta by 2 and that is going to be B over 2 sine of theta by 2 delta theta. This is the virtual displacement of the centre point if I change the angle from theta to delta theta.

Similarly X is equal to B sine of theta by 2, this entire distance. And therefore Delta X is going to be 2B 1 by 2 cosine of theta by 2 delta theta which comes out to be B cosine of theta by 2 delta theta. Why we are calculating these delta X and delta Y is so that I can calculate the virtual work done by the force as and 2 MG.

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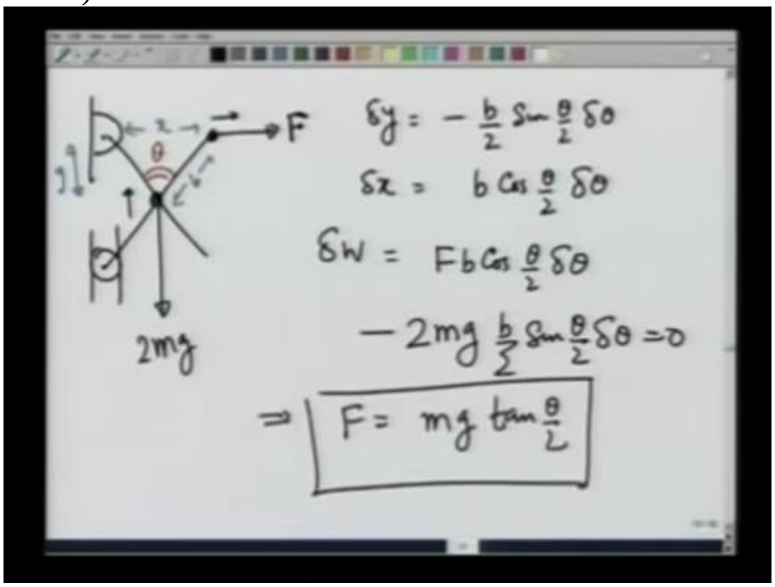

Let me make the picture again schematically and we have calculated that delta Y is nothing but - B over 2 sine theta by 2 delta theta and delta X is equal to B cosine of theta by 2 delta theta. By - sign, we mean that Y is decreasing or the point here is moving up. So Delta W is going to be equal to the work done by force, virtual work done by force F.

Since X has increased by B cosine theta by 2 delta theta, work done by F is going to be FB cosine of theta by 2 delta theta positive because the point here has moved in the same direction as the force. And work done by the force MG or the weight is going to be - 2 MG B over 2 sine of theta by 2 delta theta and this should be 0.

You calculate this and you get F equals MG tangent theta by 2 and that is your answer that we had obtained earlier. Notice I did not have to go through calculating the normal forces and things like those which we avoided and bypassed because of the work being, virtual work being done by those normal forces or constrained forces 0.