Engineering Mechanics
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Module 4
Lecture No 38
Method of virtual work-I: degrees of freedom, constraints and constraint forces

In this lecture we are going to deal with the method of handling systems, equilibrium of those systems which are large and have a large number of subsystems.

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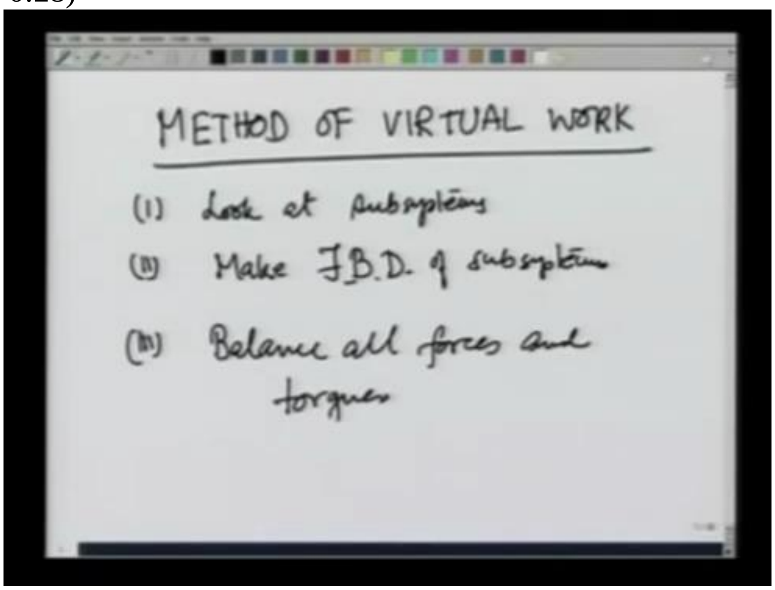

The method is known as method of virtual work. To motivate this, let us recall how we dealt with equilibrium of system such as particles, trusses and so on. Our strategy in those cases was to look at subsystems and make free body diagram of subsystems replacing the constraints or the the contact with other subsystems by the forces applied by them and balance all forces and torques.

However, as the system size grows, there are number of forces, number of subsystems may grow quite large and in such systems, the method that we use is the method of virtual work. The method focuses directly on the external force that they want to calculate.

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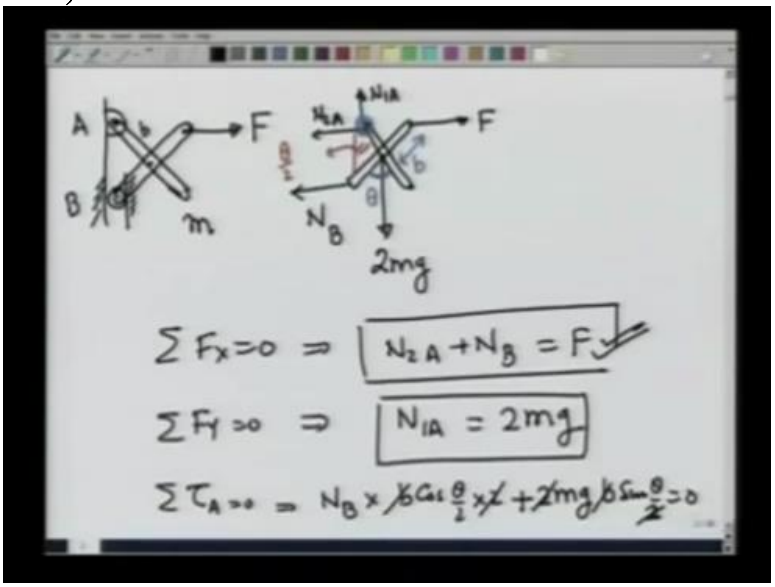

To motivate it further, let us take an example. Suppose I have a mechanism whereby made up of 2 bars whereby it is scissors like mechanism with a pin joint here and a roller here which is free to move in this between these 2 walls. Each bar has a mass M and this is kept in equilibrium by a force F at this point. If we want to calculate the force F, our strategy is we 1st look at this mechanism.

On this point, there can be only one force. Let us call it NB. This is point B, this is point A. This is a pin joint. So there could be a force N1A and N2A. The force F acts here and the overall weight 2MG pulls it down. If we were to bring this in equilibrium by summation FX equal to 0 would give me that N2A + NB equals F. That is one equation.

Similarly summation FY equal to 0 would give me N1A equals 2MG. Remember what we are after. We are after calculating this force F required to keep the entire system in equilibrium. 3rd, the torque equation. For the torque equation I will take torque about point A here. Let me make it by blue. I will take torque about this point so that torques due to N1A, N2A and F all vanish.

So if I take torque about A to be 0, I get if the length of half of the bar is B. Right? So I am taking this half length to be B and let this angle, entire angle be theta then while calculating torque, I am going to get NB times this angle here is going to be theta by 2. So I am going to get B cosine theta by 2 times 2. That is this arm. + 2 MGB sine theta by 2 is equal to 0. And that gives me NB. B drops out. This 2 I know, so this 2 is there.

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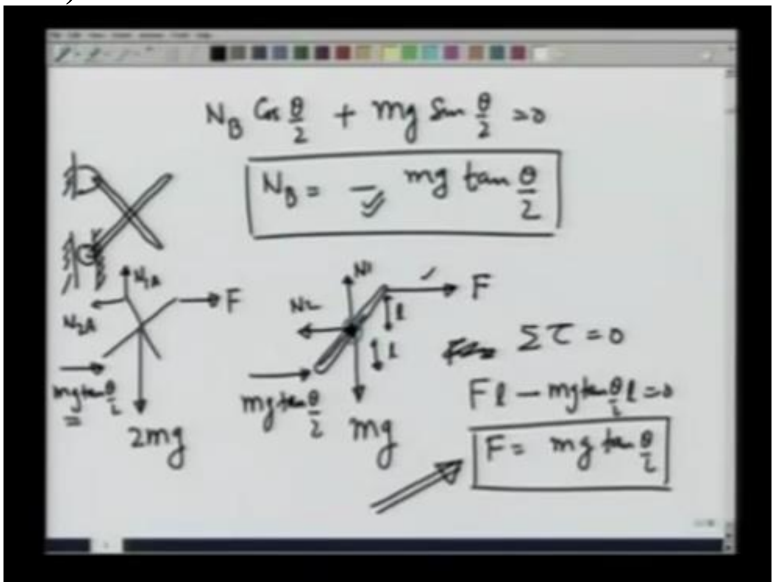

This 2 drops out and therefore I get NB to be I had NB cosine of theta by 2 + MG sine of theta by 2 equal 0 or I get NB to be - MG tangent of theta by 2. Therefore if I look at this mechanism that we had made earlier, this is a pin joint here, this is on a roller that is free to move and this I have a force F acting this way, 2 MG pulling it down, NB we have just calculated is MG tangent theta by 2.

Notice that because of this negative sign, I have already corrected the direction of MG and there is N1A and N2A. Now to calculate F, I look at one of the rods individually, this one. There is a force F pulling it this way, it is own weight MG pulling it down, there is a force, normal reaction here which we have just calculated, MG tangent theta by 2. And now they are going to be normal forces due to the pin out here.

Let us call this N1 and N2. If I am just interested in calculating force F, then you can see that I can take the torque about this middle point, the pin here, let me show this in blue about this point and calculate F to be equal to by taking the torque about this point let me call summation torque length is about this point is 0.

And therefore F times whatever this length is call it L. L - same going to be here MG tangent theta by 2L equals 0 and F equals MG tangent of theta by 2. So we have found the force F. If you are interested, you can also calculate N1, N2 and other normal forces. But our main focus was to calculate F and that is what we have found. Notice that we had to take several steps in getting to this answer. If we use method of virtual work as I will show later, all these steps are bypassed.

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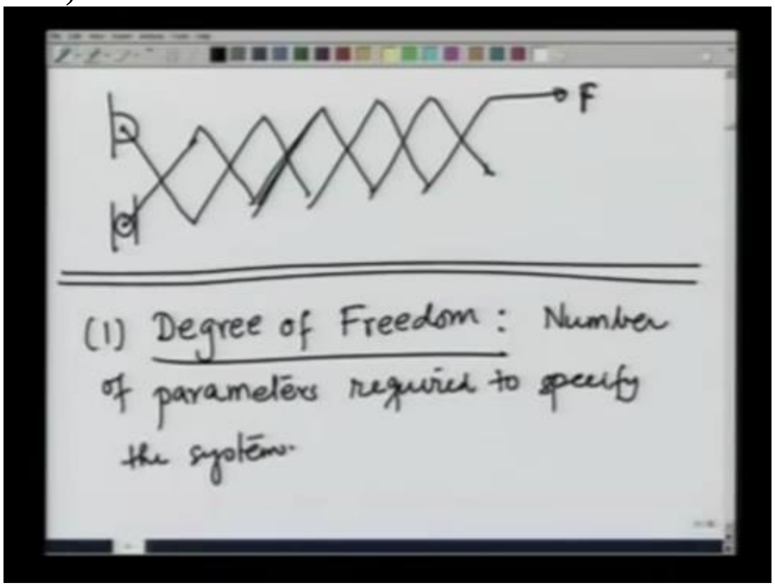

Not only that, if I were to solve the same problem about the same mechanism but make it slightly more complicated, in that now I have several of these links and I apply a force here. Each link, each of these bar has mass M and length 2B and I want to calculate the force F to keep this whole thing in equilibrium. You see we go the conventional way, I have to build up the solution step-by-step-by-step.

With method of virtual work, we will be able to do it in a much more easy manner. So to develop the method, we need certain terminologies. Let me now go over that. So now we are going to go over some terminologies that are going to be used in the method of virtual work. 1st, let us define degree of freedom of a system.

The degree of freedom of a system is the number of parameters that will be clear when I give an example, required to specify the position, the orientation and all that, the system. So when I say specify the system, I mean precisely that, what is its position? What is its orientation? At what angle is it from vertical and so on.

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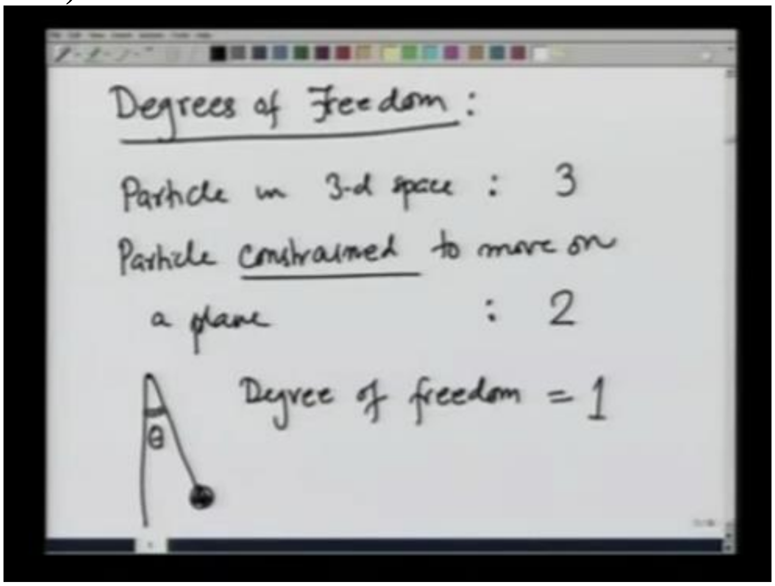

So for example if I take so we are talking about degrees of freedom. Again number of parameters required to specify a system, for example if I take a particle in 3-D space, number of degrees of freedom is 3. If I take a particle constrained to move only on a plane, constrained and I will make this definition constrained clearer little later. Constrained to move on a plane than the degrees of freedom are 2.

Suppose I take a pendulum so that it swings in a plane. The only thing that I need to specify where the pendulum is, is this angle. Then the degree of freedom is 1.

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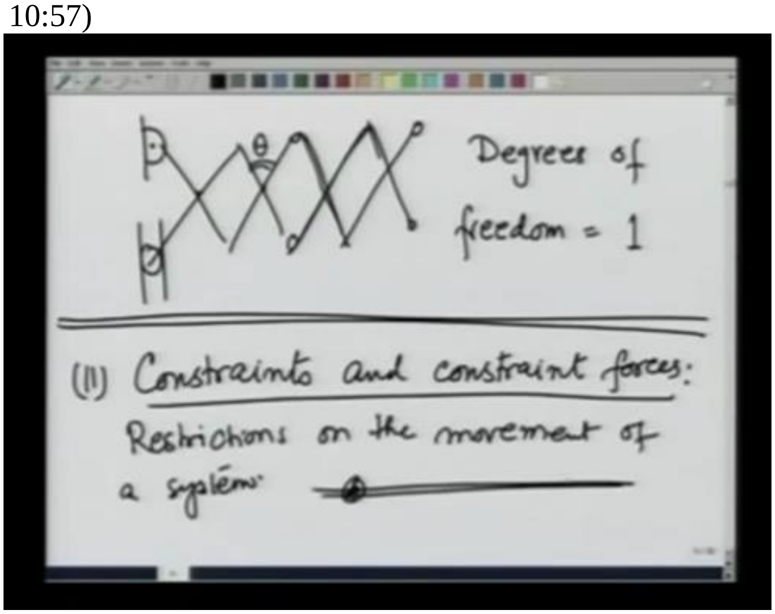

Even in the complicated example that we took previously for that mechanism whereby I had these cross-links like this. The only thing that I need to specify the orientation or specific position of the system is this angle theta. Nothing else. The moment I specify this theta, because the lengths of these bars are fixed I specify exactly how the system is. So degrees of freedom in this case is also 1.

So this is one concept we are going to use. The degree of freedom specifies the parameter used to describe a system and the combat city of the problem. Less the number of degrees of freedom, less complex the system is. Concept number 2. Constraints. This we have been talking about while talking about equilibrium but let me go over it in a formal manner now.

Constraints and constraint forces. By constraints, we mean the restrictions on the movement of the system. For example if I take a particle and constraint it to move only along say a wire. Then that is a constraint that is put in that it can move only along a straight line.

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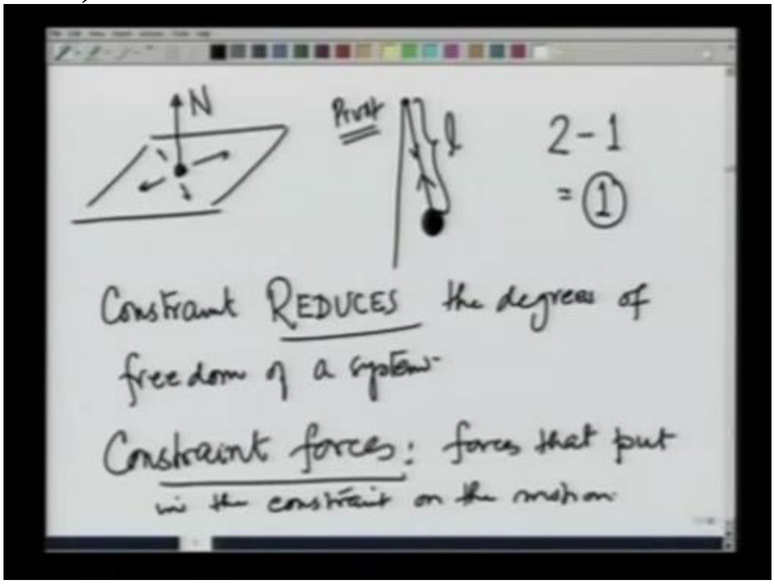

Or if I take a particle, put it on a plane and constrain it to move only on a plane, this puts a constraint on the particle to move only in this plane. Similarly if I take this pendulum example that I took, the constraint is that the distance of the Bob here of the pendulum is fixed from the pivot point. So you see, we are putting constraints whereby we restrict the motion. And what does a constraint do?

A constraint reduces the degrees of freedom of a system. For example, in the case of a pendulum, number one, the pendulum is to move in a plane. So degrees of freedom are 2. Then I put constrain that the distance of this bob from the pivot point is fixed. So I reduce the degrees of freedom by one by putting one constraint and finally the degree of freedom therefore comes out to be 1.

How do we put the constraints on our system? We put constraints on a system by applying certain forces. For example here a force that is related with the constraint is the tension in the string. In a motion on two-dimensional plane, the constraint force is the normal reaction on the particle due to the surface.

It would not that normal reaction does not let the particle fall below and go make a motion in the direction perpendicular to the surface. So these are known as constraint forces. These are forces that put in the constraint on the motion.

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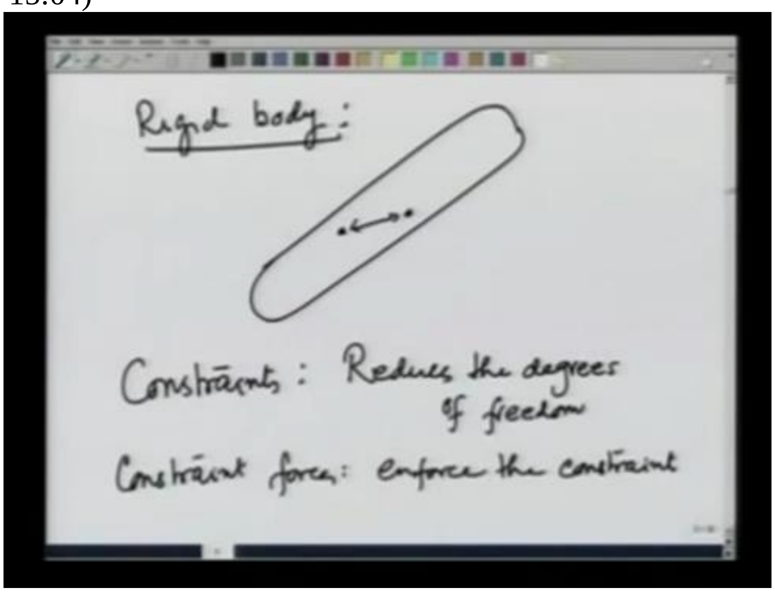

One of the very familiar constraint that you have come across time and again is the rigid body. This is a concept we keep using. In a rigid body, the constraint is that distance between any 2 points always remains fixed. And what is the constraint force? The constraint force is that internal force that keeps the distance fixed. All right?

So the concept that we have come up with other constraints okay this result is that reduces the degrees of freedom. And constraint forces they enforce the constraint.

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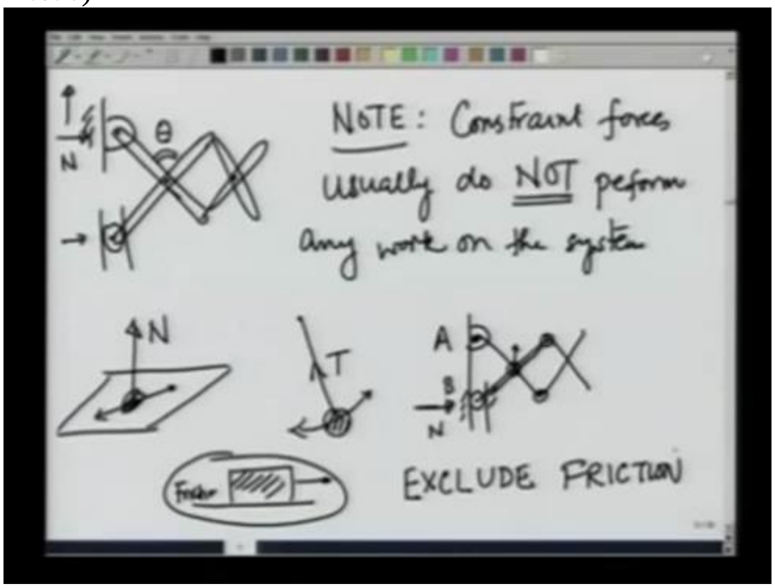

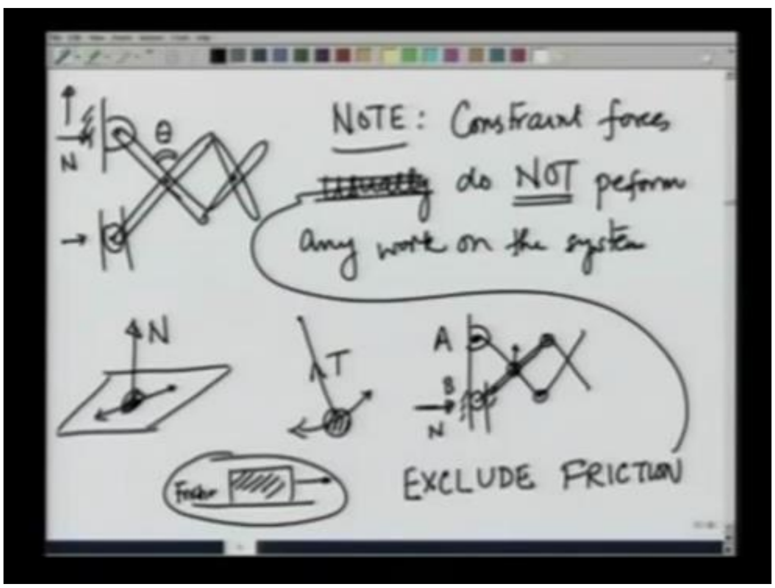

Let us go back to that example of that mechanism whereby the bars making the mechanism and so on, what are the constraint forces here? The constraint forces are the normal reaction here, both in vertical and horizontal direction, the normal reaction here. And the forces that are applied by the spring. It is these forces that make the system move in a particular way.

It reduces the degrees of freedom and degrees of freedom finally is only 1 given by this angle theta. Note and this is very important to develop the method of virtual work is that constraint forces usually do not perform any work on the system. Why do I say usually? Because I am going to exclude a particular constraint force little later.

So take the example of a particle on a two-dimensional surface. A constraint force, normal reaction is perpendicular to the surface. The particle moves along the surface and 2 are perpendicular. So normal does not do any work on this. Take the example of the pendulum. The constraint force, tension is always perpendicular to the direction of the motion of the Bob and therefore this also does not do any work.

How about this mechanism? The point here, A point does not move at all and therefore no work is done by the normal forces at this point. The normal reaction at this point B is perpendicular to the direction of motion and therefore that also does not do any work. How about the forces due to the pins here? You see, the pins apply a force on one particular bar in one direction and exactly opposite force on the other bar.

However, the pins move by exactly the same amount. So bars also move exactly in the same direction but the forces are opposite. The result is that the network done because of this constraint forces is 0. It is positive in one bar, negative in the other bar, add it up, it gives me 0. So you see, I have given you 3 examples where you see that constraint forces usually are not performing any work.

Why I say usual is, this is if we have a fictional force as the constraint force, that does perform work when the system moves. Therefore if we include friction, then I would say constraint forces usually do not perform any work on the other hand if I exclude friction, then I will take you here and cut this off. Then I would say that constraint forces do not perform any work on the system.

In my development in the development now, we will be excluding friction and deal with those constraint forces that do not perform any work on the system and it is this observation that makes dealing with the system very easy.