Engineering Mechanics
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Module 04
Lecture No 35
Properties of plane surfaces – VI: Parallel axis transfer theorem for second moment and product of an area

Having defined these quantities, the 2nd moment of inertia and product of inertia, we now describe a relationship between the 2nd moment of an area about its set of axis passing through the centroid of the body and another set XY axis which are parallel to those passing through the centroid.

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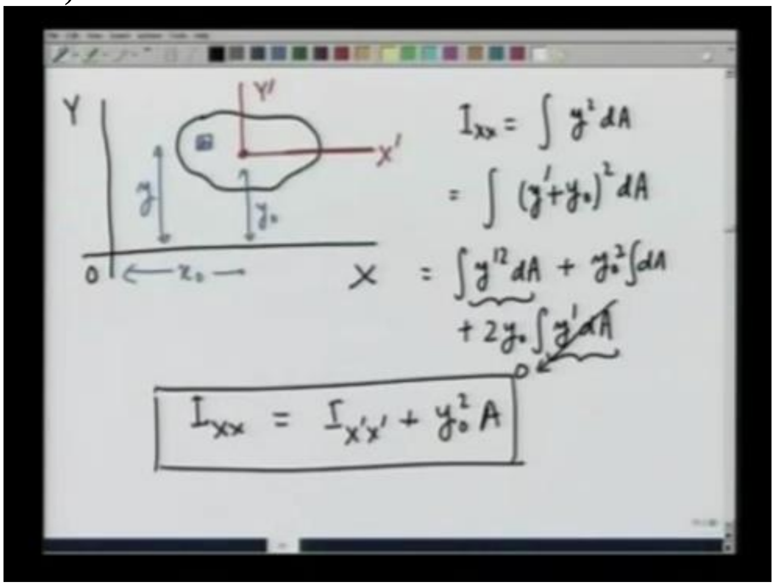

Thus suppose I have an area, the centroid with respect to a given set of axis X and Y with origin at O such that the coordinates of its centroid are X0 and Y0. 2 is another set of parallel axis X prime and Y prime passing through the centroid. X prime is parallel to X and Y prime is parallel to Y. And we want to now calculate, show that IXX IX prime X prime are related by a very simple theorem and so are the other moments.

Thus IXX is equal to Y Square DA. So suppose I have a small area DA here, this is Y which is also equal to Y is equal to Y prime where Y prime is the Y coordinate of the same area with respect to X prime and Y prime axis + Y0 square DA. And this is equal to Y prime square DA +

Y0 square DA + 2Y0 integral Y prime DA. Notice that Y prime square DA is the capital IX prime X prime. That is the 2nd moment of inertia about the X axis passing through the centroid.

And Y prime DA is nothing but area times the Y coordinate in the centroid frame. So this is going to be 0. If I calculate the centroid with the origin at the centroid the coordinates of the centroid are going to come to 0 and therefore I see that IXX is equal to Y prime square DA that is IX prime X prime + Y0 square times entire area. This way if I know the 2nd moment of inertia of a body about an axis passing through its centroid, I can easily calculate the 2nd moment of inertia of the same body with respect to an axis which is parallel to the 1st axis but displaced by amount Y0.

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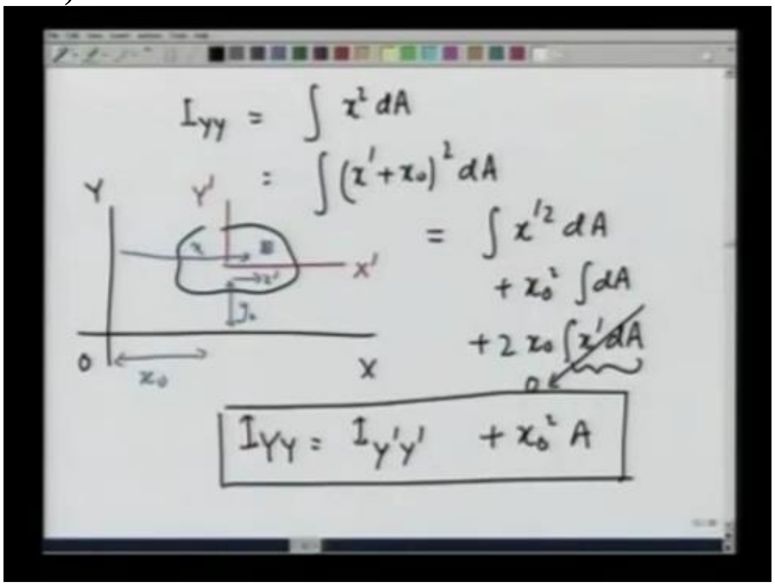

In the same manner we cannot calculate IYY which is equal to integral X Square DA which I can write as X prime + X0 square DA. Let me for convenience show in the picture again what we are talking about. This is the body. This is the centroid set of axis X prime, Y prime. This is X, Y, O, this is X0, this is Y0. So if I choose an area here, this is X and this is X prime.

X is X prime + X not. So this can be written as equal to integral X prime square DA + X0 square integral DA + 2 X0X prime DA. This again by the same logic as we applied earlier that this is the X coordinate of the centroid in the coordinate system which has its origin at the centroid itself. So this is 0. So I get IYY is equal to IY prime Y prime + X not square A.

So it is as if the entire area is concentrated at the centroid + whatever the 2nd moment of area is about the axis passing through the centroid.

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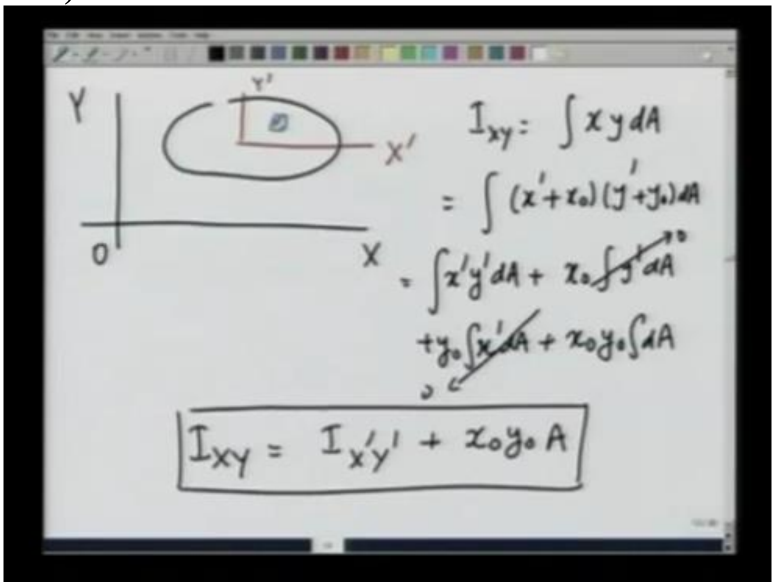

Next, let us calculate the product of area again making these axis X prime, Y prime. X, Y, O and I take an area here. The product is IXY which is equal to integral XYDA and I substitute for X and Y as X prime + X0 Y prime + Y0 DA which comes out to be X prime Y prime DA + X0 integral Y prime DA + Y0 integral X prime DA + X0Y0DA. Again by the arguments that we have used earlier, these 2 terms drop to 0.

And therefore IXY is equal to IX prime Y prime + X not Y0 times A for area. So what we have learned this if we know the 2nd moment of inertia and the product of inertia about a set of axis passing through the centroid, I can calculate about any other set of axis which are parallel to those passing through the centroid. Let us summarise these.

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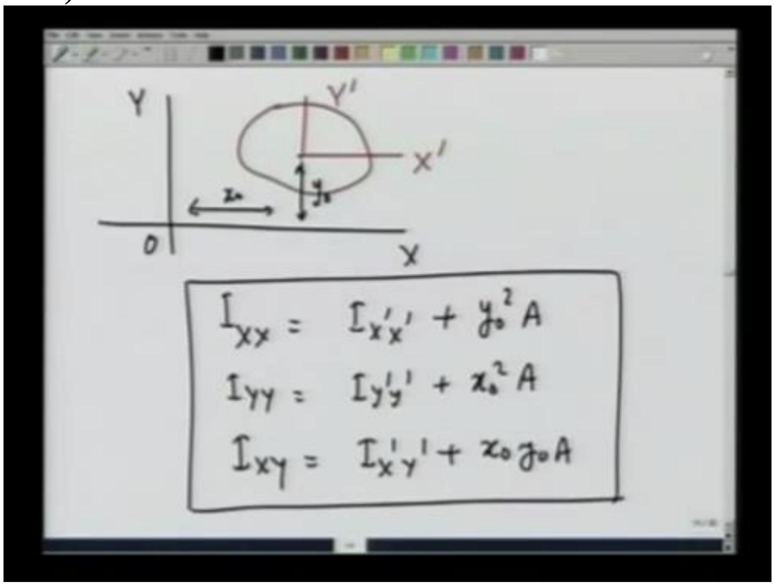

So for an area whose 2nd moment and product of area are known about the axis passing through the centroid, I have in general IXX equals IX prime X prime + Y0 square times the area where Y0 is the coordinate of the centroid. Y coordinate of the centroid IYY is equal to IY prime Y prime + X0 square times the whole area and IXY equals IX prime Y prime + X0Y0 times entire area. These are known as transfer theorem. Using these, I can transfer the moment of area or the product of area from one coordinate system to another.

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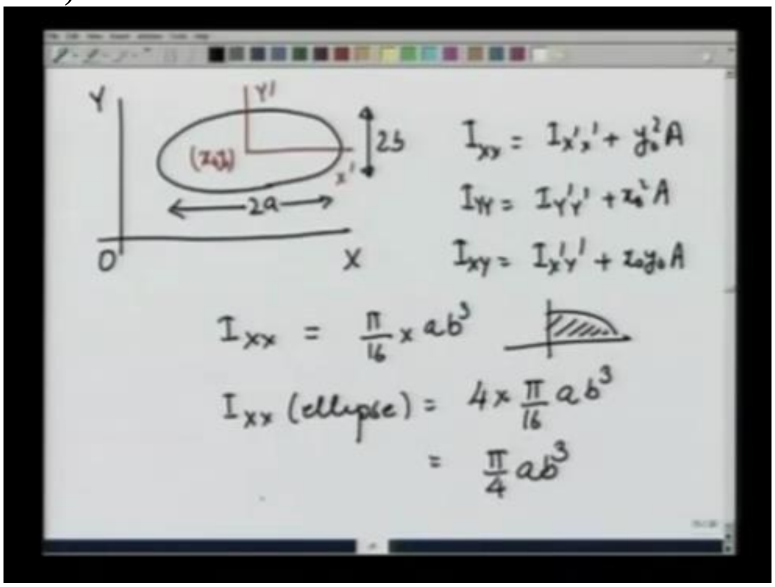

As an example of the application of transfer theorem, let us take case of an ellipse with its length being 2A and this being 2B, with its centroid at point X0, Y0 and calculate its moment of area, 2nd moment of area and product of area with respect to the XY axis shown here. So by transfer theorems I have IXX equals I X prime X prime + Y not square times the area of the ellipse. IYY similarly is IY prime Y prime + X not square times the area of the ellipse.

And IXY is equal to I X prime Y prime + X0Y0 times the area of the ellipse where IXX X prime X prime is the 2nd moment of area with respect to the X prime axis parallel to the x-axis passing through the centroid. IY prime Y prime is the 2nd moment of area with respect to the Y prime axis parallel to the Y axis and passing through the centroid. Previously we have calculated IXX as pi over 16 times AB cubed for a quarter of an ellipse like this.

So for the full ellipse and this I left as an exercise for you there, this is going to be 4 times as much. So this is going to be pi over 16 AB cubed which this pi over for A B cubed.

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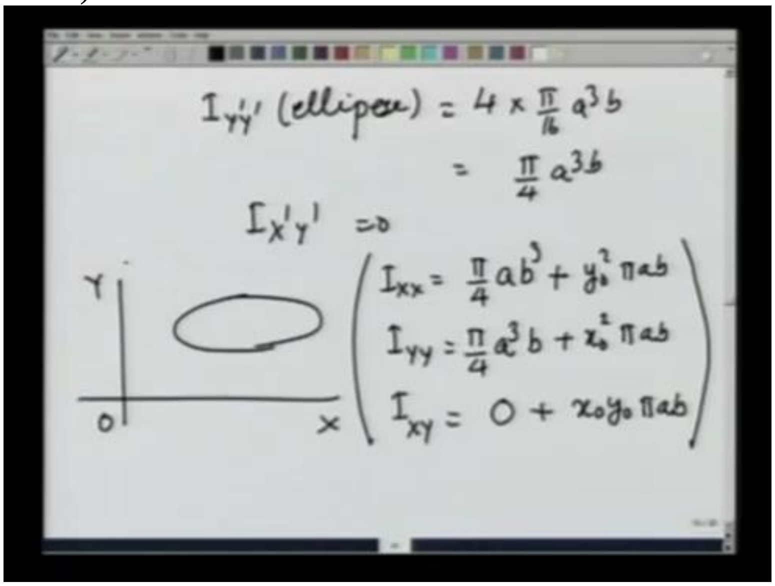

Similarly IYY prime for the ellipse is going to be 4 times pi over 16 A cubed B which is pi over 4A cubed B. And IX prime Y prime is 0. Therefore, for this ellipse, we will have IXX as pi by 4AB cubed which is the 2nd moment of area about the X prime axis passing through the centroid + Y0 square times pi AB because pi AB is the area of the ellipse. IYY is going to be pi over for A cubed B + X0 square pi AB.

And IXY is going to be 0 which is the IX prime Y prime. By symmetry, it is 0 for axis passing the centroid + X0Y0 pi AB. So using transfer theorems, we could calculate the 2nd moment of area and the product of area when it was given about the centroid.