Engineering Mechanics
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Module 03
Lecture No 33
Properties of plane surfaces – IV: solved examples of calculation of first moment and centroid of distributed forces

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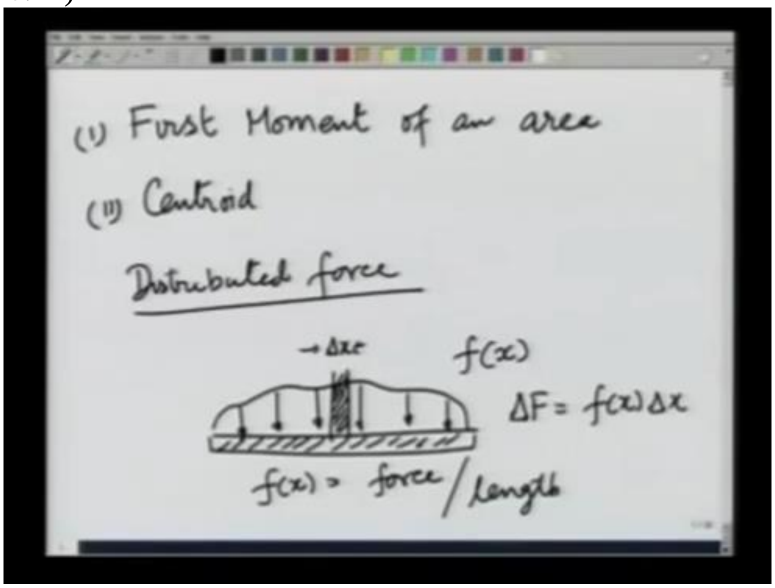

In the previous lecture, we had defined quantities like the first moment of an area and centroid mathematically and said that these are related to problems in mechanics. In this lecture, we are going to solve some problems using these concepts. As I had indicated towards the end of my previous lecture on this topic, the place where we use these concepts is where we have a distributed force. What do we mean by that?

For example, if I have a beam and there is some mass on top of it so that it applies a force on the beam, the force maybe described by a function FX so that if I take a small section here delta X length, the force on this section, delta F is equal to Fx times delta X. So Fx is nothing but force per unit length. It is in dealing with such distributed forces that the concepts developed in the previous lecture are going to be handy.

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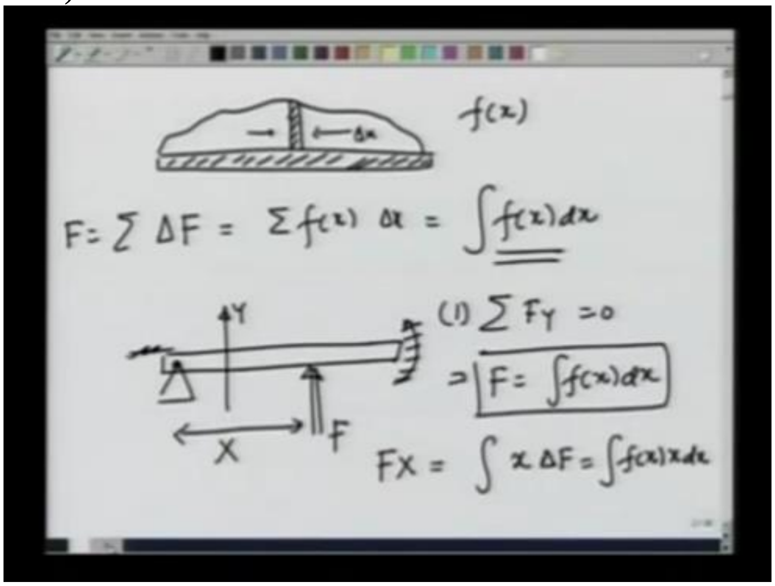

So the question we ask is given this distribution of force FX what is the total force on the system and where effectively is it acting? Let me explain that a little further. The total force is going to be formation of that delta F that is acting on a small section of length delta X and this is going to be summation Fx delta X which in the limit is going to be integration Fx DX, that is the net force.

And when we say where effectively is it acting, that means what moment or torque should I apply to this beam in order to keep it in equilibrium. For example, if I have this beam fixed at this point or let me put a pin joint here, what torque should I apply here in order that this beam is in equilibrium or equivalently at which point should I apply this net force F that I have calculated above here so that the effect of this force both the torque as well as the net force is nullified?

To do that, one we require that summation FY where Y is this direction be 0 and that gives me the net force F should be equal to Fx DX. The 2nd equilibrium condition is that the torque about this pin joint vanish and that requires that the distance of this force that I am applying, call this X be such that it nullifies the torque generated by this force Fx. So F times X should be equal to summation X delta F which is nothing but integration Fx X DX.

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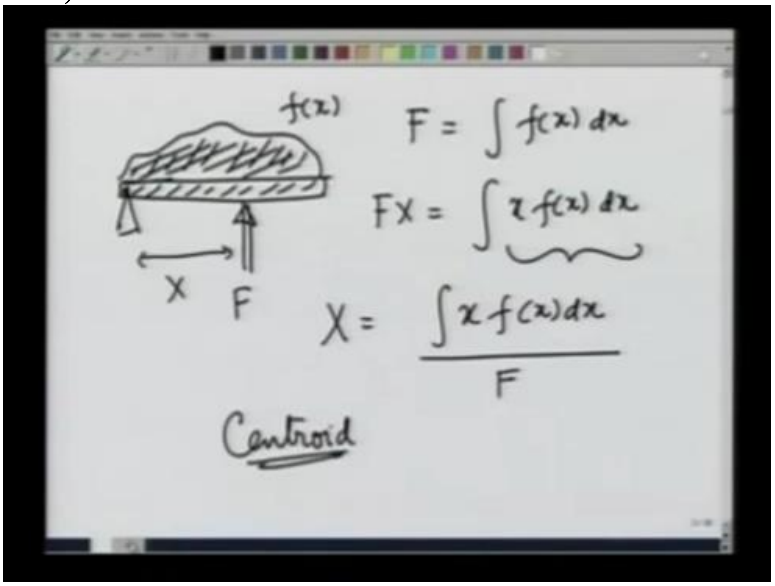

So on this beam if there is a force distribution Fx then I have the net force F that I am supposed to apply this is about this pin joint to equilibriate the beam is going to be Fx DX and I should have F times X where X is the distance at which the force is being applied equal to integration X FX DX which is nothing but the moment generated by the force distribution Fx. And therefore X equals integration X Fx DX over F.

And this by definition is the definition of centroid. Therefore the net force is the area of this force distribution curve and the point at which effectively this force acts is the centroid of this area formed by the beam and this force distribution curve. That is how we use the concept of the first moment or the centroid. I must point out that that when the total force F is applied at the centroid, no other force is needed to support the beam.

That is, in that situation, the force applied by the pin joint will be 0. As such, in the case of Fx being the gravitational force, the centroid gives the position of the centre of gravity. Let us take some examples.

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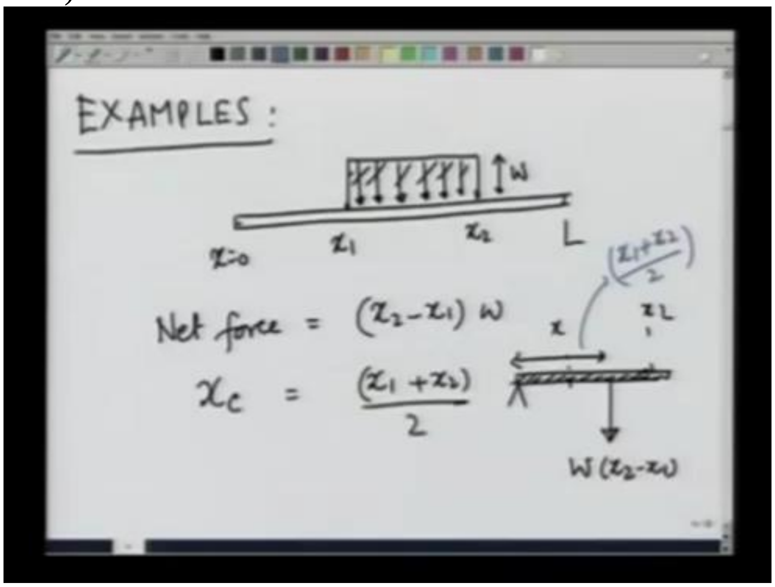

Suppose I have a beam. Let us call this point X equal to 0 of length L and there is a force distribution of the form of a rectangle from point X1 to X2. In that case, one can easily see, suppose this magnitude is W, one can easily see that the net force that this applies is the area of the rectangle which is going to be X2 - X1 times W. And where does it act? It acts at the centroid of the area formed by this force distribution and the beam.

An X centroid is nothing but X1 + X2 divided by 2. Therefore I can replace this entire force or represent this entire force like this. This is a beam, this is where it is hinged. The net force is of the amount W X2 - X1 acting at a distance this is X1, this is X2 at a distance let me write it in blue, this is X1 + X2 divided by 2. So that is one example.

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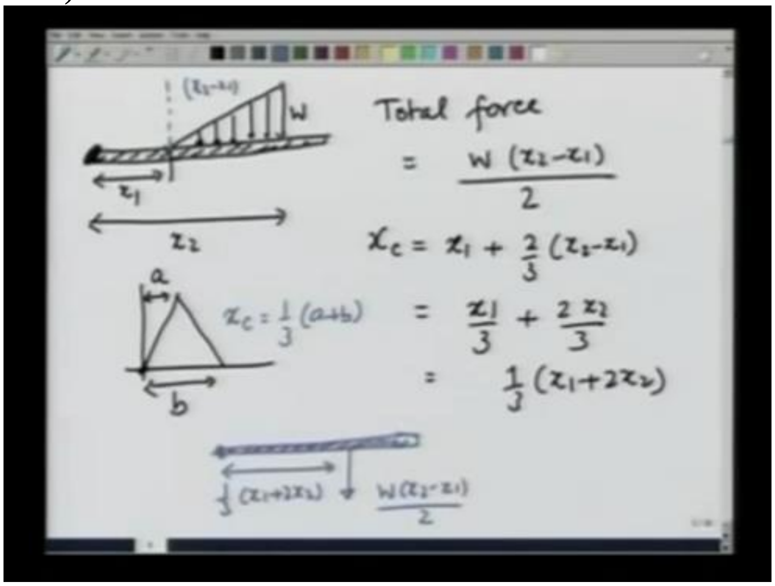

Next, we consider triangular loading. In that I have a beam of some length L and it is loaded like this where the maximum load is per unit length is W. Load starts at a distance X1 and goes all the way up to X2 and what we want to figure out is how much is the total force acting on the beam and very effectively is it acting? So total force is going to be the area of this triangle which is W X2 - X1 divided by 2 and where it acts is the centroid of this triangle.

Recall from the previous lecture that if I am given a triangle then with respect to one of these corners, if this distance is A and this is B, then the centroid XC is given as one third A + B. In the present case, the 2 points are at a distance of X2 - X1. Both the points are distance of X2 - X1 from this corner. And therefore, the centroid XC is going to be at a distance from this point the hinge point here X1 + two thirds X2 - X1 or this comes out to be X1 over 3 + 2X2 over 3 is equal to one third + 2X2.

So if I were to look at this load effectively how it is working if this is the beam then the load can be effectively replaced by a force of W X2 - X1 divided by 2 acting at a distance of one third X1 + 2X2 from this point. This is another example of how we apply the concept of first moment and the centroid in mechanics.

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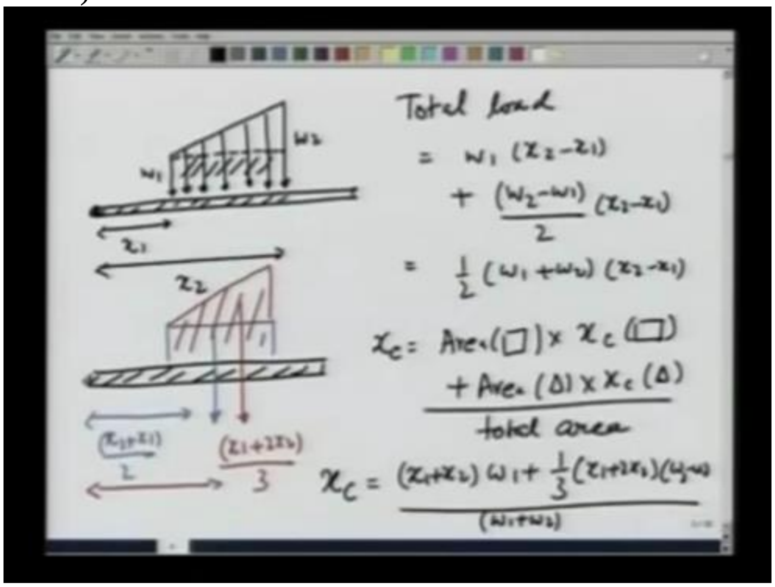

Next, I will consider one more example where the loading is trapezoidal. That means, the loading starts at X1 but it has a finite amount W1. Then it goes up to W2 per unit length and distance X2. Along the way this is how the load is distributed. Again, with respect to this point where the beam is hinged, I want to find out what is the total load and where effectively is it acting?

So the total load is going to be the area of this trapezoid I for later reference is divide this area into 2 areas, one which is corresponding to this rectangle of height W1 and width X2 - X1 and another one, the triangle.

So the total area is going to be the area of the rectangle which is W1X2 - X1 + the area of the triangle is going to be W2 - W1 divided by 2 times X2 - X1 which comes out to be the area of trapezoid with is nothing but one half W1 + W2 times X2 - X1. So this is the net load which is working on this. To calculate where it acts, I am going to use a an observation that we made last time.

I know that this rectangle where load acts right in the middle at a distance of X2 + X1 divided by 2 from this point. Similarly the triangular loading which we just calculated from the previous slide acts as X1 + 2X2 divided by 3 distance from this point. So I take these 2 loads and then calculate what will be the effective centroid for this entire area and that we know from our previous lectures is going to be XC equals area of the rectangle times XC of the rectangle that is making it symbolically.

+ area of the triangle times XC of the triangle divided by the total area. Total area which is the net force we have already calculated. We also know the positions of the centroid of the rectangle. We also know the position of the centroid of the triangle and we now the area for both. And therefore we can calculate XC. You do a quick calculation and the answer you get is XC equals X1 + X2 times W1 + one third X1 + 2X2 W2 - W1 divided by W1 + W2.

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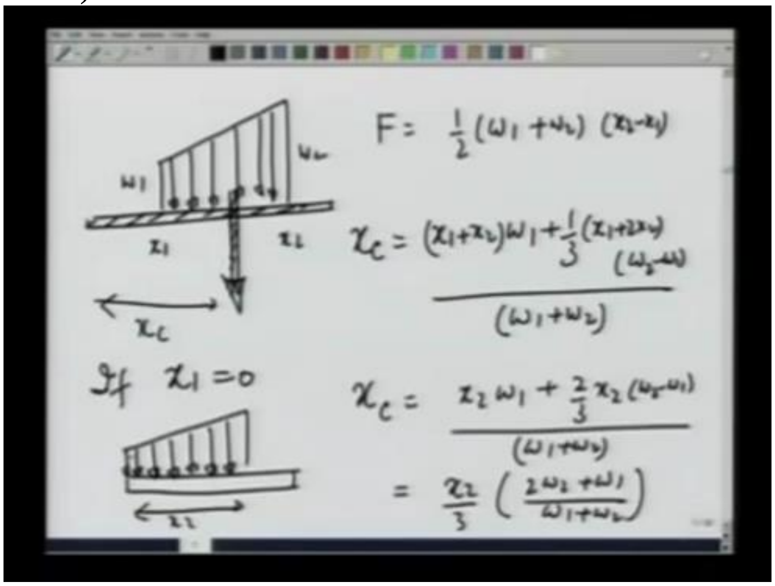

Let me write again. So what we are considering is this beam which is loaded from point X1 to point X2 with this trapezoidal load. Then the total load force it comes out to be one half W1 + W2 times X2 - X1 and the point at which it acts is at a distance XC which is equal to as I wrote in the previous slide, X1 + X2 times W1 + one third X1 + 2X2 times W2 - W1 divided by W1 + W2.

If I take X1 to be 0 that is the load starts right here and goes up to X2 then the centroid comes out to be X2W1 + two thirds X2W2 - W1 divided by W1 + W2 which is nothing but X2 over 3 2W2 + W1 divided by W1 + W2. So that is the example for trapezoidal loading.

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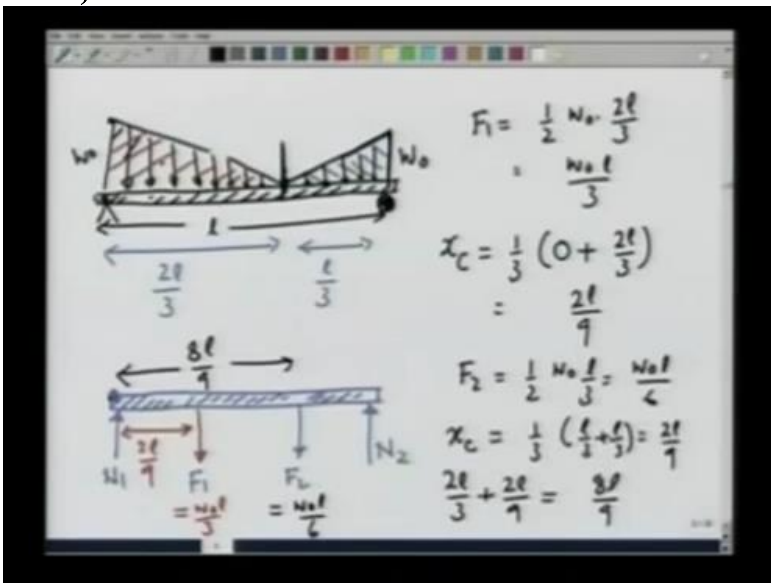

Next, let us now solve a problem with a particular loading. So suppose I have a beam of length L. It is on a roller on this side and on a pin joint on this side and it is loaded with a triangular loading like this with these being W0, W0. This length being 2L by 3 in this length therefore being L by 3. And I wish to calculate the reactions at the pin joint and at the roller. So for that, what I need to do is take this beam and make a free body diagram.

So we will assume that reaction at this point is N2, reaction at this point is N1. And the 2 forces due to the loading okay the forces due to the loading I will split them into 2, 1 due to this triangle to the left and one due to this triangle to the right. I will take the triangle on the left acting like this at a distance which we have to calculate. And similarly, the triangle to the right also applies a load, let us call this F1 and F2 at a distance from this point.

Now what we did earlier I can take F1 to be acting at the centroid of this 1st triangle. F1 is going to be equal to one half W0 times 2L by 3. So this is going to be W0 L by 3. And where does it act? It acts at the centroid of this triangle. The centroid of this triangle XC is going to be equal to one third. A in this case is 0. This is the point where A is and B is 2L divided by 3.

So this is acting at 2L by 9. So this distance is 2L by 9 and F1 is equal to W0L by 3. Similarly for the other triangle, the force F2 is going to be the area of that triangle which is going to be one half W0 L by 3 which is W0 L by 6. So this force is equal to W0 L by 6. And it acts at the centroid of the right hand triangle.

The centroid with respect to this point, the point in the middle is going to be at a distance of one third of L by 3 + L by 3 which is one third of 2L by 3, 2L by 9. So the distance from this point the corner is going to be 2L by 3 + 2L by 9 which is nothing but 8L by 9. So the force here acts at 8L by 9. We are now ready to solve the problem.

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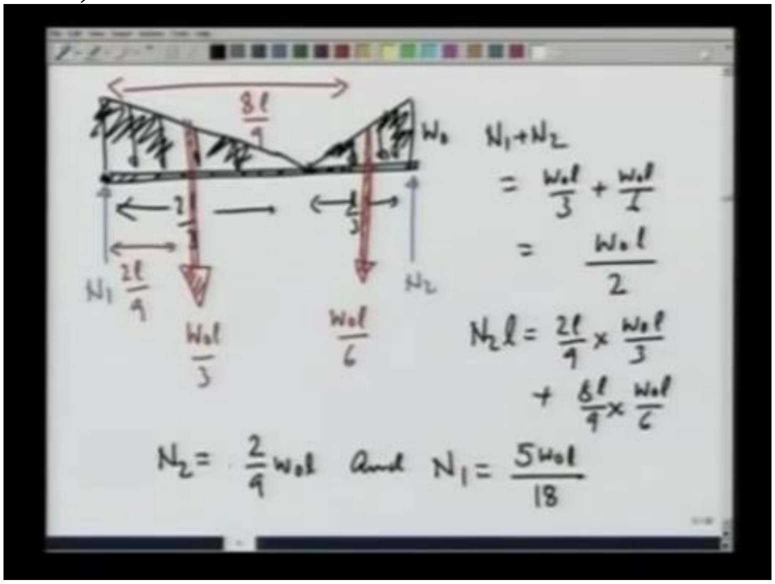

So what we have done effectively is took this beam which is loaded like this with this being W0, this distance being 2L by 3, this distance being L by 3. Then we said, reactions at these points are going to be N1 and N2. And the 2 triangles we replace by one force acting downwards which is W0 L by 3 acting at a distance of 2L by 9.

And the other force acting here whose magnitude is given by W0 L by 6 and it is acting at a distance of 8L by 9. So I can forget about these triangles now and instead focus on these forces and do my calculations as we have been doing in the beginning of the 1st few lectures of this course.

A beam is loaded with these 2 forces and there are these are 2 reactions, simple N1 + N2 being equal to W0 L by 3 + W0 L by 6 which comes out to be W0 L by 2. And then we balance the moments about this point. All the distances are known and when we solve so it is going to be N2 times L is going to be equal to 2L by 9 times W0 L by 3 + 8L by 9 times W0L by 6.

When I solve the 2 equations, I get N2 to be equal to two ninths W0L and N1 to be equal to 5W0 L over 18. So that is how we have used the concept of centroid in finding out where effectively the force given by a distribution acts, what is its moment and then applying a regular point force diagrams to calculate the reaction forces and so on.