Engineering Mechanics
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Module 3
Lecture No 32
Properties of plane surfaces – III: Centroid of a distributed force and its relation with centre of gravity

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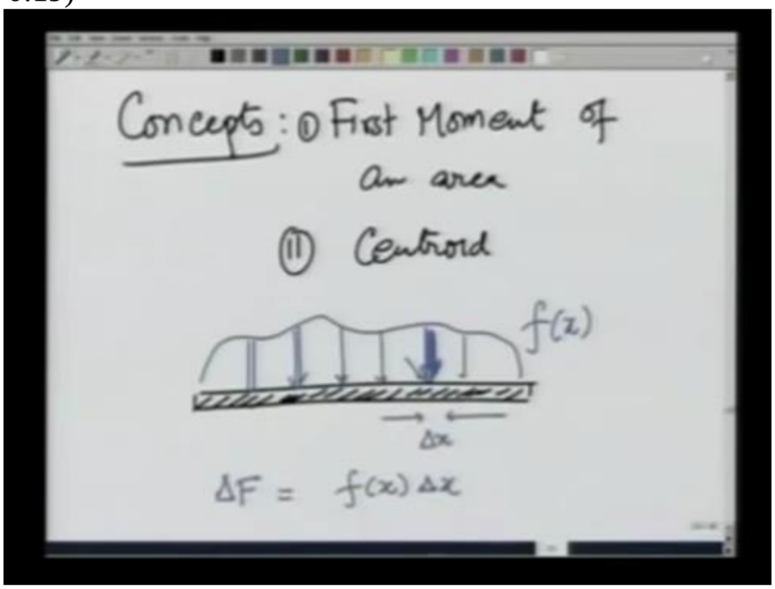

Having given you the concepts of first moment of an area and the concept of centroid, we are now ready to apply these concepts to mechanics. So the question we ask is suppose I have a beam on which I do not apply a force at a certain given point but the force on it is distributed. So there is some force per unit area. Let me show this by a graph. Sorry per unit length.

So there is a force acting at each point. Let me call this function FX so that if I take a distance a small section of length delta X here, the net force acting on this point delta F is going to be given as F at X delta X. What we wish to now find is what is the net force acting on this team and where does it effectively act? That means if I were to suppose apply in opposing force to balance it, where should I apply it so that this force is balanced?

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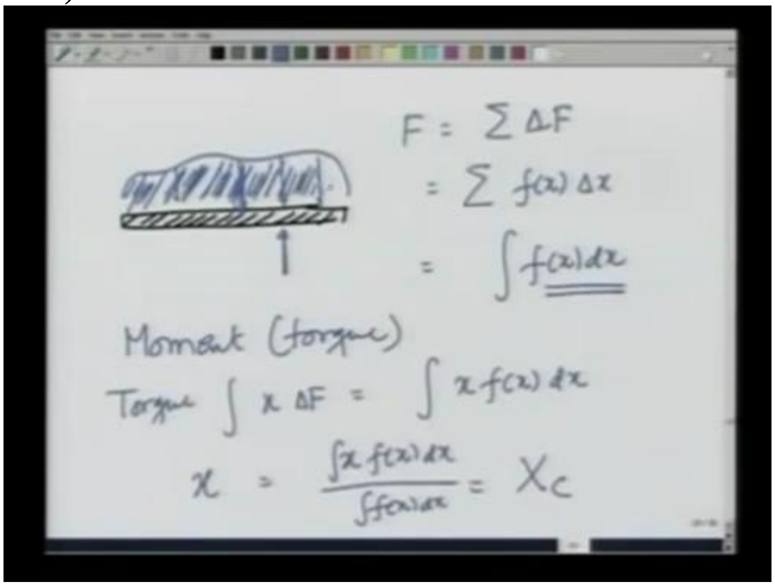

The answer comes in terms of the quantities just defined. So if there is this force FX, distributed force acting on it, the beam, the net force F is going to be summation delta F which is nothing but summation, delta F is nothing but FX delta X and this is nothing but integration FXDX which is the area under this curve. That gives me the total force for a distributed force per unit length given. How about the moment or the torque created by this force?

That is obviously going to be X delta F integrated which is nothing but XFXDX. This is the torque. If I want to calculate, apply an opposing torque with a point force. That force has to be equal to the net force which is being applied which is the area of this curve and it has to be at a point X which is nothing but integration XFXDX divided by integration FXDX which is nothing but the centroid of this curve that describes force per unit length.

So we can say that effectively the force acts at the centroid of this given area which describes the force distribution. Absolutely immediately the connection the first moment or the centroid that we defined with a problem in mechanics.

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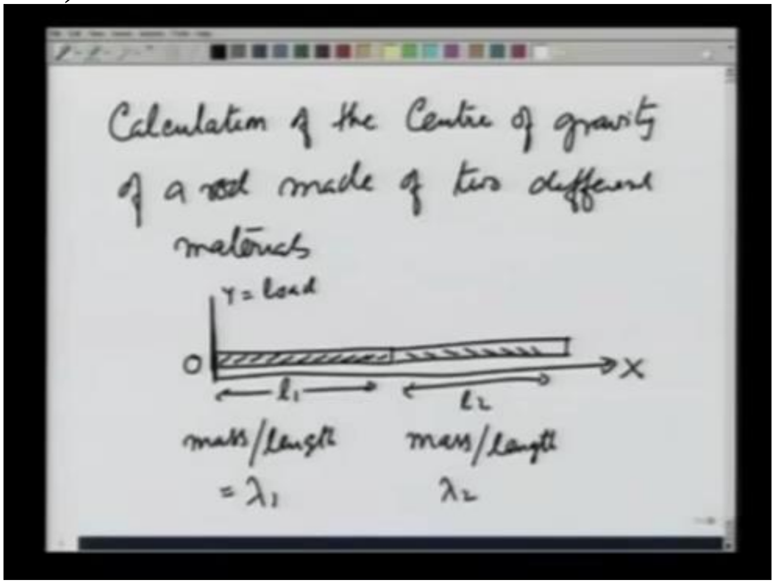

As an example of things to come, let me end this lecture by solving a problem of calculation of the centre of gravity of a rod made of two different materials. So we take a rod which is made of two different materials. One material here is of length L1 and its mass per unit length is lambda 1. The other rod here is of length L2 and its mass per unit length is lambda 2. We wish to calculate its centre of gravity.

Starting from this point let us say this is our axis so that this is our X axis, this is over Y axis or on the Y axis I will be showing the load and this is X equal to 0.

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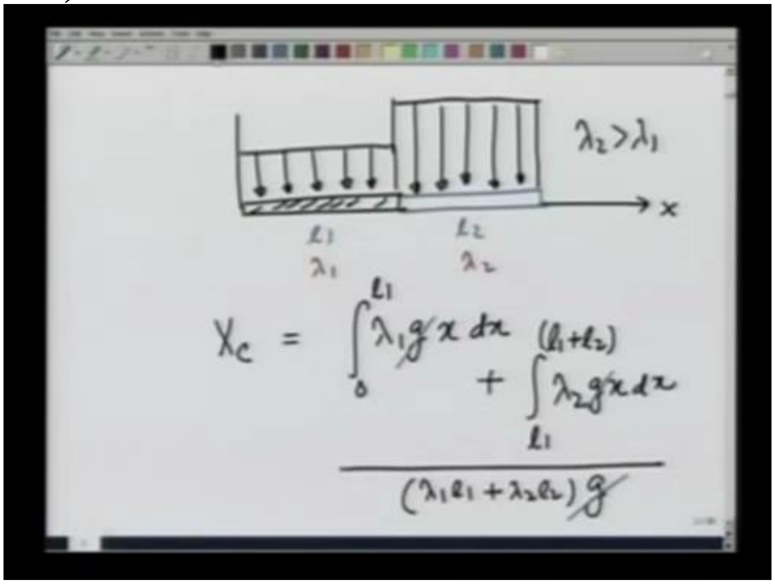

Let me go to next page and show this rod, black here of length L1 and blue part for the other length L2 L1. Mass per unit length is lambda 1 here, lambda 2 here. My axis is starting from here, going this way. If I plot the load curve, it will be constant lambda 1G per unit length here and if we assume lambda 2 to be greater than lambda 1, lambda 2 greater than lambda 1 and I wish to calculate the centre of gravity. By definition, centre of gravity is the centroid of this load curve.

So this is going to be lambda 1G is the load per unit length in the 1^{st} part times XDX integration 0 to L1 + integration L1 to L1 + L2 lambda 2 GX DX divided by the total area which is going to be lambda 1 L1 + lambda 2 L2 times G. You see, G cancels from the denominator and the numerator.

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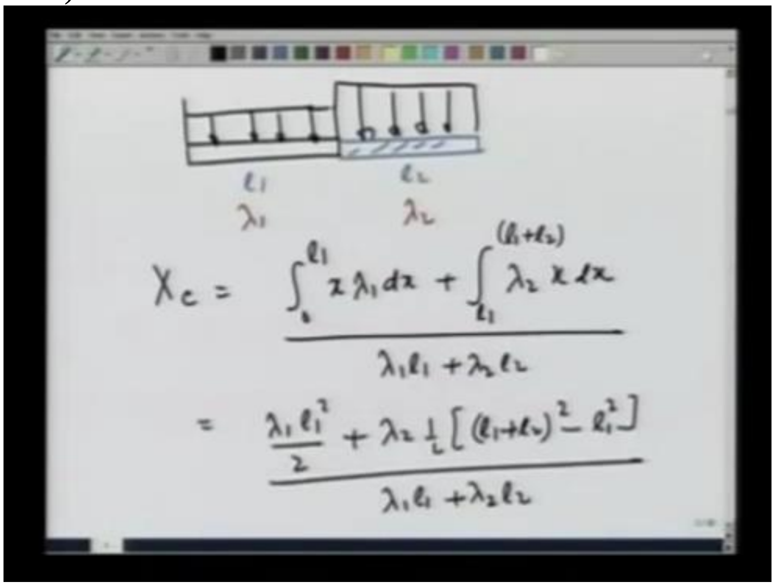

And I am left with the centre of gravity of this rod L2, L1, lambda 1, lambda 2 and let me again sure you, the load curve is something like this. This, the centre of gravity is 0 to L1 X lambda 1 DX + L1 to L1 + L2 lambda 2 XDX. G we have already cancelled over lambda 1 L1 + lambda 2 L2 and this comes out to be lambda 1 L1 square divided by 2 + lambda 1 over 2 L1 + L2 whole square - L1 square divided by lambda 1 L1 + lambda 2 L2.

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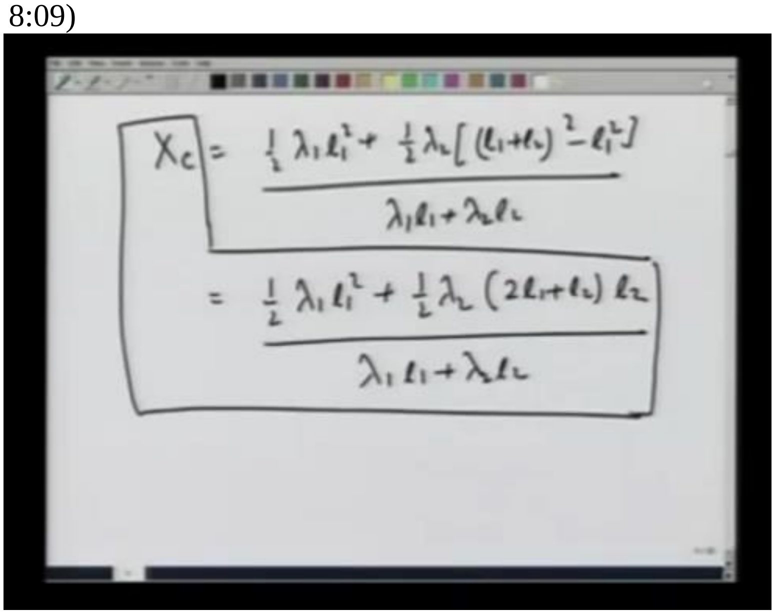

You simplify this and you get XC which we just calculated, it will be one half lambda 1 L1 square + one half lambda 2 L1 + L2 Square - L1 square over lambda 1 L1 + lambda 2 L2 to be one half lambda 1 L1 square + one half lambda 2. I can simplify this. This will come out to be 2L1 + L2 times L2 divided by lambda 1 L1 + lambda 2 L2. So from the left-hand side end of the rod, this is where the centre of gravity lies.

We can also calculate the centre of gravity by using our observation earlier that if there are 2 different areas given, the centre of gravity can be calculated from those 2 if I know the centre of gravity of each one of them.

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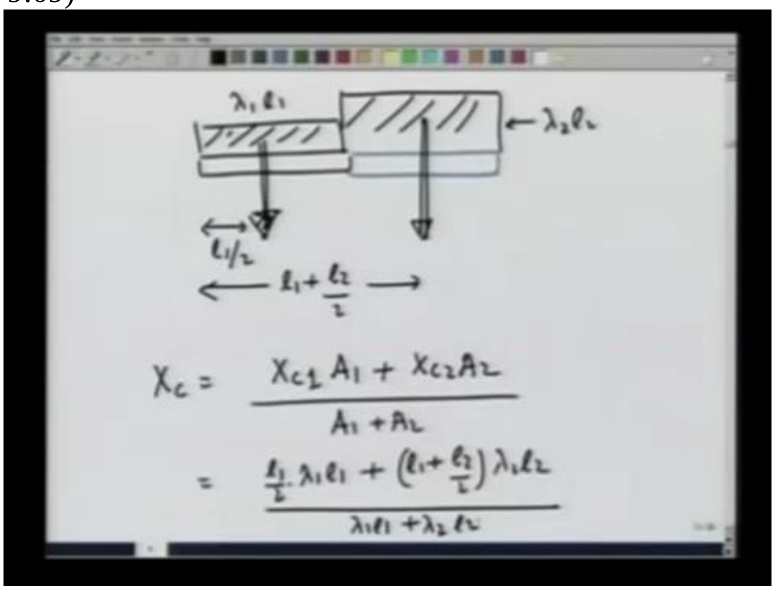

So if I were to use that, you see the load curve on the rod is something like this. I know the centre of gravity of this load which is like here at a distance of L1 by 2 and centre of gravity of the other load is here which is at a distance of L1 + L2 divided by 2. The total area here is lambda 1 L1, total area here is lambda 2 L2. So XC is going to be XC 1, that is the 1st part times area 1 + XC 2 area 2 divided by area 1 + area 2.

And that is going to be L1 over 2 times lambda 1 L1. G would cancel from the top and the bottom. + XC2 which is L1 + L2 divided by 2 lambda 2 L2 over lambda 1 L1 + lambda 2 L2.

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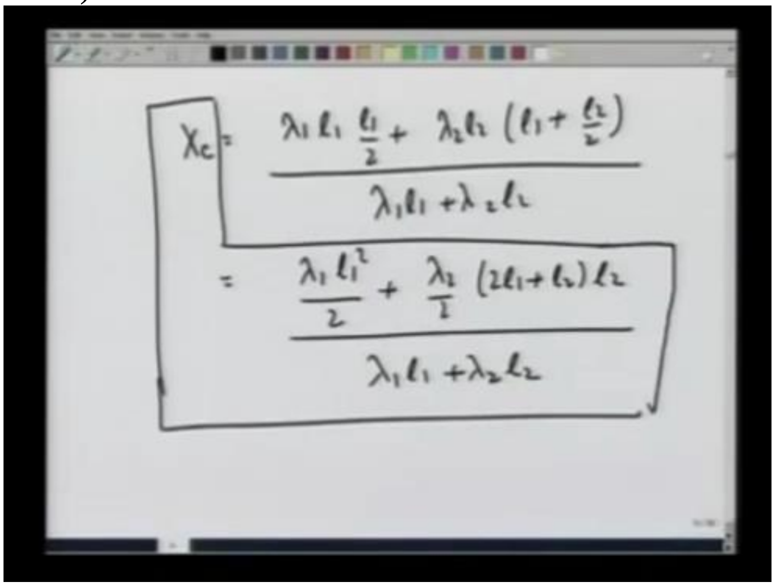

So we have calculated XC to be lambda 1 L1 L1 over 2 + lambda 2 L2 L1 + L2 over 2 divided by lambda 1 L1 + lambda 2 L2 which is same as lambda 1 L1 square divided by 2 + lambda 2 over 2 2 L1 + L2 times L2 divided by lambda 1 L1 + lambda 2 L2 which is the same answer as we obtained earlier when we calculated the centre of gravity by integrating over the entire area. So this is a simple example of how centre of gravity is related to the centroid of the load curve. In the next lecture, we will be doing more such examples.