Engineering Mechanics

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Module 03
Lecture No 31

Properties of plane surfaces – II: Centroid of an area made by joining several plane surfaces

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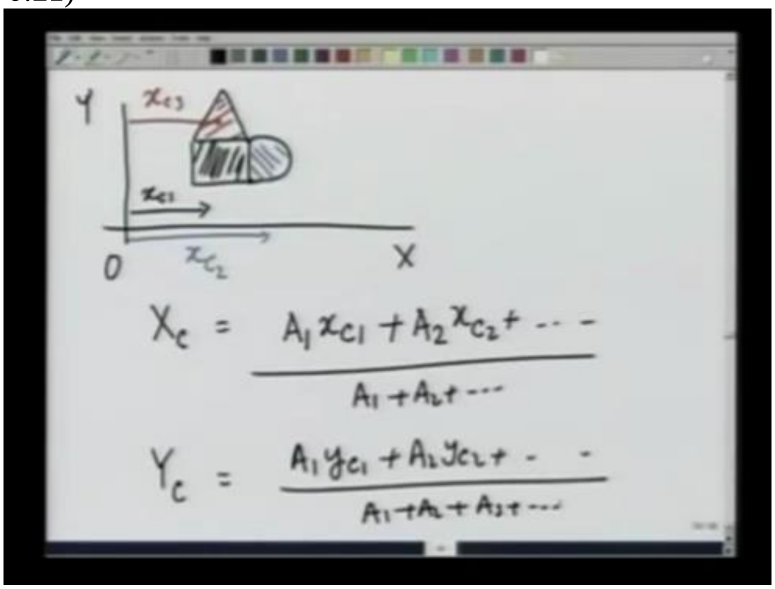

As an important observation about centroid, let us now take any area which is made up of two or three simple plane areas. Let us say a square, a triangle and semicircular planar area. So there is one kind of area, there is another kind of area and here is the third kind of area but all of them are joined together. Then it is very simple to prove that if I want to calculate the centroid of this whole system, then XC of the entire system would be equal to the area of system 1 times its centroid + area of the second area XC 2 + and so on divided by the total area and so on.

That means in this case if I consider the square to be the area 1, then XC1 is known. Similarly if I take semicircle to be area 2 then XC 2 is also known because we already calculated it. And XC 3 for the triangle is also known. I also know the formula for 3 shapes and therefore can easily calculate the centroid for the entire system similarly, YC, the total area is going to be A1 times YC1 + A2 times YC2 + so on divided by A1 + A2 + A2 + A3 and so on.

This comes in handy when we know several area that are joined together and I know the formula for different areas but wish to calculate the centroid for the joined area. Let us do a couple of examples using this. I leave a simple proof of this for you to do.

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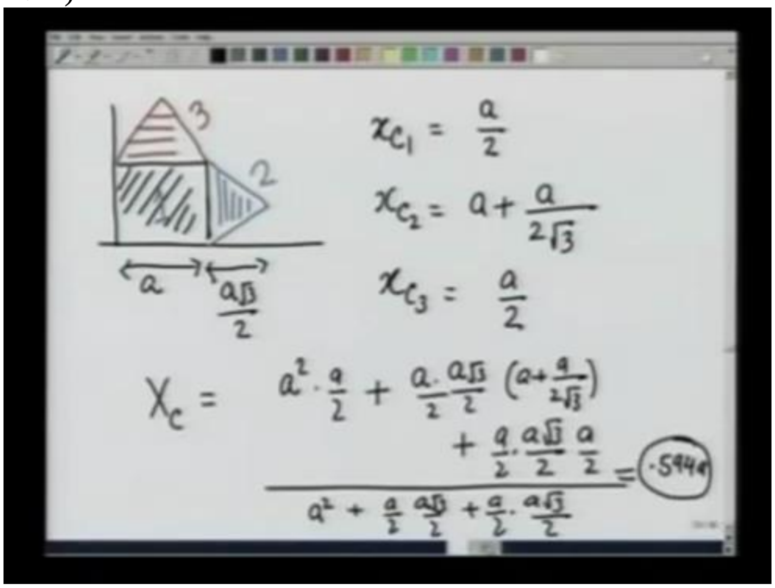

So as the first example I take a square and joined on a two triangles, equal triangles. One here and one here. I wish to calculate the centroid of this assembly. Let me call this number 1, let me call the triangle here, number 2 and let me call the red triangle number 3. Then I know that XC1 that is the X coordinate of the centroid of this square is going to be if this side is A, it is going to be A divided by 2.

Similarly, the height of this triangle because the site is A, is going to be A root 3 by 2 and therefore XC2 is going to be equal to A + the distance of the centroid from the base which is nothing but one third the height and therefore A divided by 2 root 3. And XC3 that is the X coordinate of the centroid of the third triangle is going to be right on the dividing axis and which is going to be A by 2 again.

And therefore, XC of the entire assembly is going to be equal to area of the first square which is A square times A by 2 + area of the second one which is going to be equal to the base which is A times the height which is A root 3 by 2 divided by 2, that is the area times XC2 which is A + A divided by 2 root 3 + the area of the third one which is A by 2 A root 3 divided by 2 times A by

2 divided by the total area which is A square + A by 2 times A root 3 by 2 + A by 2 times A root 3 by 2.

That is the formula for XC. And we calculate this, this comes out to be about 0.594 times A. By symmetry, YC would also come out to be the same.

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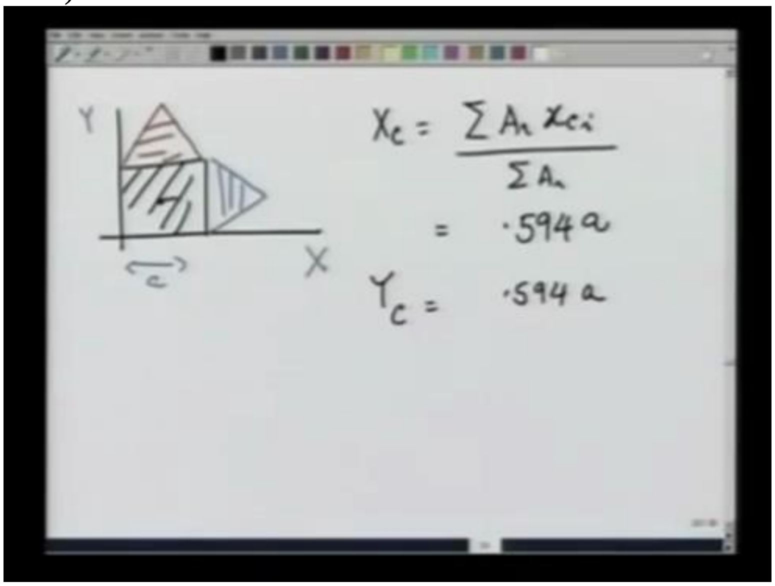

And therefore for this assembly where I have a square of side A, a triangle, equilateral triangle on top and on the side. This is A, X, Y. I get XC to be which is summation AI XCI, I am writing a general formula divide by summation AI which comes out to be 0.594A slightly to the right of the centroid of the square which is expected. Because of the area added onto the right, it shifts to the right. And similarly, you can calculate YC is going to be come out to be again 0.594A.

So this is how you use this observation that if I want to calculate the centroid for an assembly of plane areas for which I know individually where the centroid is going to be, I can calculate the centroid for the total assembly.

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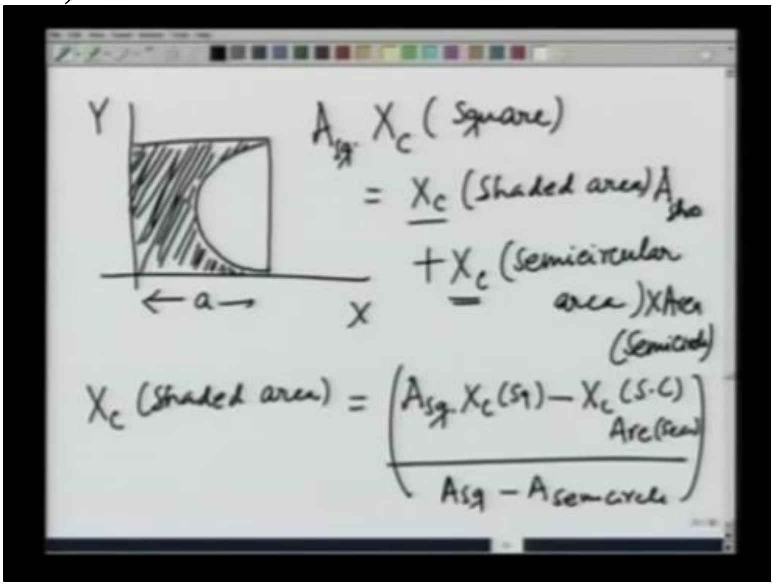

Let me take another example where I have an area formed by taking away a semicircular area from a square of side A. So I wish to calculate the centroid of the shaded area where the shaded area has been formed by taking away a semicircular area on a square of side A. Again, I can use the same theorem but in a different way. Because now what I am going to say is that XC of this square which I already know is going to be right in the middle, is going equal to X centroid of the shaded area + X centroid of the semicircle.

This is area of the square area of the square is going to be equal to XC of the shaded area times area of the shaded times the area of the semicircle. So area of the square times the centroid of the square is going to be equal to a area of the shaded area times the centroid of the shaded area + XC centroid of the semicircular area times the area of the semicircle.

And therefore XC of the shaded area is going to be equal to A square times XC of the square - XC of the semicircle times the area of the semicircle divided by the area of the shaded portion which is going to be the area of the square - the area of the semicircle. And that is what we wish to calculate.

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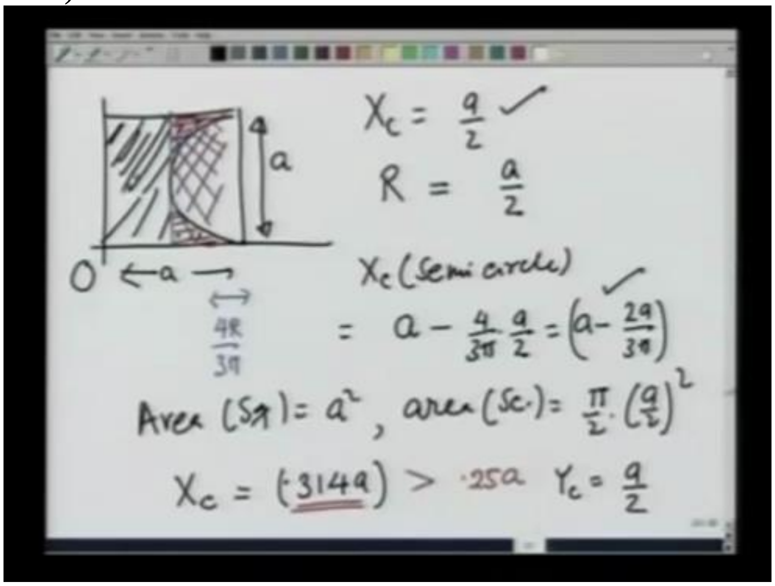

We have already calculated that for a square XC is going to be A by 2 where A is the side. The radius of the semicircle is equal to A divided by 2 and I know from the base we already calculated this, the centroid is at a distance of 4R divided by 3 pi. And therefore XC with respect to this origin of the semicircle is going to be equal to A - 4 over 3 pi times A by 2 which is A - 2 A by 3 pi.

This is the XC of the semicircle. So now we are all set. I know XC for the square, I know XC for the semicircle and area of both. Area of square square is equal to A square, area of semicircle is equal to pi by 2 times A by 2 whole square. And therefore I can calculate XC for this shaded region from the formula that we derived earlier and XC comes out to be, we plug in the numbers, 314A.

Notice that if I had removed the entire, this half. Let me make it with blue, if I had removed this half blue of this square, XC would have been here at 0.2 5A but because of this extra area which I am showing here to the right, the centroid shifts slightly to the right and is at 0.314A which is greater than 0.25A. That would have been the case if I had removed this entire half of the circle. Y coordinate obviously because of the symmetry remains at A by 2.

So I have given you two examples of how to use the observation that the centroid of a given area is actually a weighted combination of centroids of the areas making this joint area to calculate centroids of complicated shapes.