Engineering Mechanics
Professor Manoj K Harbola
Department of Physics
Indian Institute of Technology Kanpur
Module 1
Lecture No 03
Multiplying Vectors

Next, we ask, can we define product of two vectors? The answer is yes but before doing that, I will take a slight digression and now we will go into vector algebra representing them by numbers in in in in a different way algebraically.

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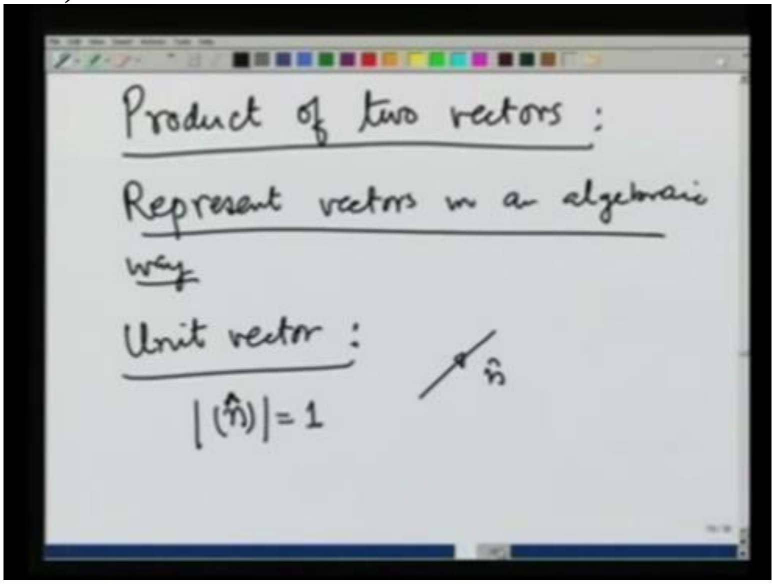

So 1st, let us represent vectors in an algebraic way and then we will be able to define the product of two vectors properly. Not only that, representing vectors in algebraic way would make addition and subtraction of two vectors easy when we have many many vectors. So for that, 1st I introduce a concept of unit vector. A unit vector is a vector of magnitude 1.

So let us say, a unit vector N and to show that it is a unit vector, we make a hat on the top of the vector. Its magnitude is going to be 1 and it could be any direction representing the vector in that particular direction.

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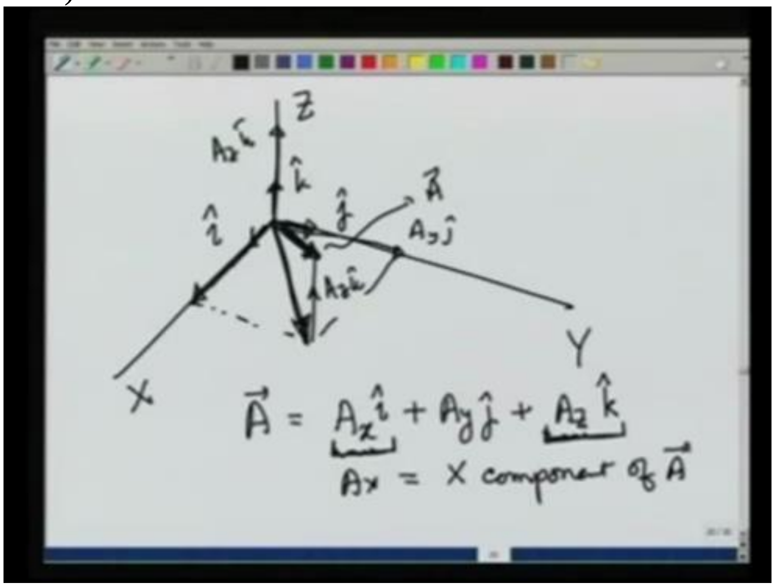

In particular now, we will define, take the X, Y and Z axis in Cartesian coordinate system and define unit vectors in X direction, call it I, a unit vector in Y direction, call it J and unit vector in the Z direction, call it K. Now any vector can be written as A what I call the X component of this vector times I + A the Y component of this vector J this AZ and Z component of this vector K.

So this is known as the Z component, Y component, X component. You can see that what we are doing is multiplying this unit vector by AX. So this will give me AX times this. A vector in this direction, let us say this AXI. And I add these to AYJ, so I draw it paralleling here. This will give me AXI + AYJ. And on top of it, we will add say AZK and move it here parallel.

So the net vector A would be something like this. This is AZK. Although in this case I added AX and AY 1st and then AZ and got this vector. It does not matter in which order you and these vectors because they have already proved that addition of vector space commutative. So I could have added AY and AZ 1st and then added AX, I will still end up with the same vector A.

So now, the meaning is quite clear. When I write A equals AXI + AYJ + AZK, I am writing a vector AX times I pointing in X direction, a vector AY times J of magnitude AY pointing in Y direction and a vector of magnitude AZ pointing in K direction and adding them all up. And that is the vector A. AX is called the X component of A and AY the Y component, AZ the Z component.

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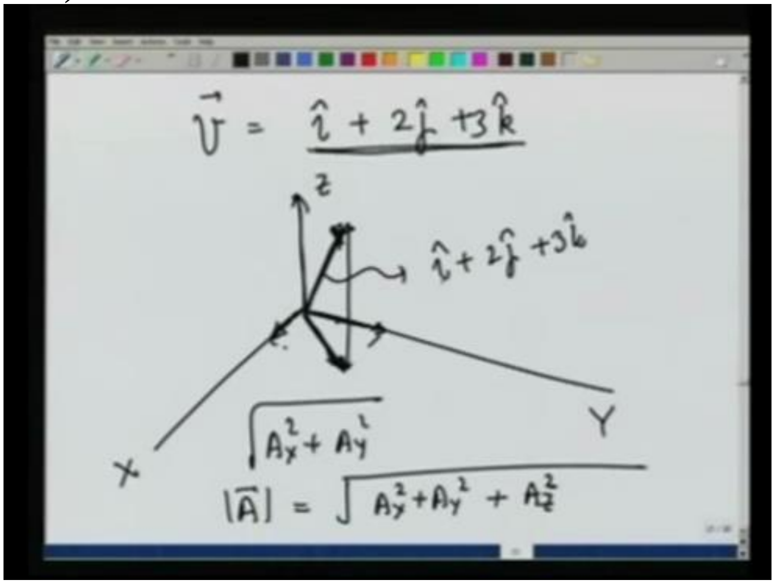

So let us see. Suppose I have, write a vector V is equal to I + 2J + let us say 3K. How would it look? X, Y, Z. So I would make vector I, unit vector in this direction + 2J. 2J would be a vector of magnitude 2 in this direction. So this would be I + 2J and 3K would be of magnitude 3 in this direction. So up to this point, let us make this vector here, this, then summation of all 3. This vector would be I + 2J + 3K.

How about the magnitude of this vector? You can see that I can 1st add these 2, get the magnitude of this. That would be AX Square + AY square square root and then again at a perpendicular vector to it which will be AZ. So the net magnitude would be square root of square of this component which is AX Square + AY square and then perpendicular component AZ, add them all up and then take the square root.

This is the magnitude of the vector and you can see the direction is easily seen and visualised by making the vector using these unit vectors.

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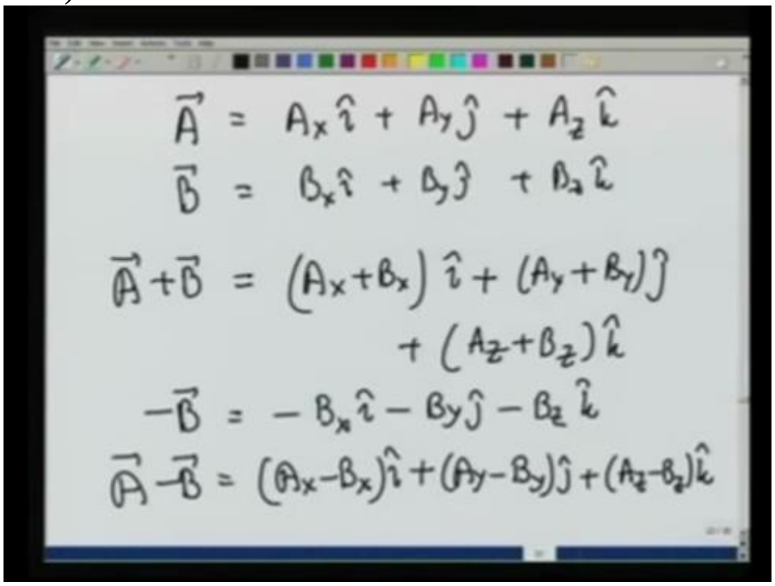

Having written a vector in its algebraic form, let us ask, how do I add 2 vectors? Suppose there is another vector, B which is BXI + BYJ + BZK. When I add the 2 vectors, A + B, you can see graphically and carry it out that the net vector would have X component which is the summation of the X components of individual vectors. Then I add the Y components of individual vectors and then the Z components of the individual vectors.

Similarly, - B would be nothing but each component of the B vector would become negative and therefore, A - B, this is nothing but adding A and - B, would be same as AX - BXI + AY - BYJ + AZ - BZ. Ok?

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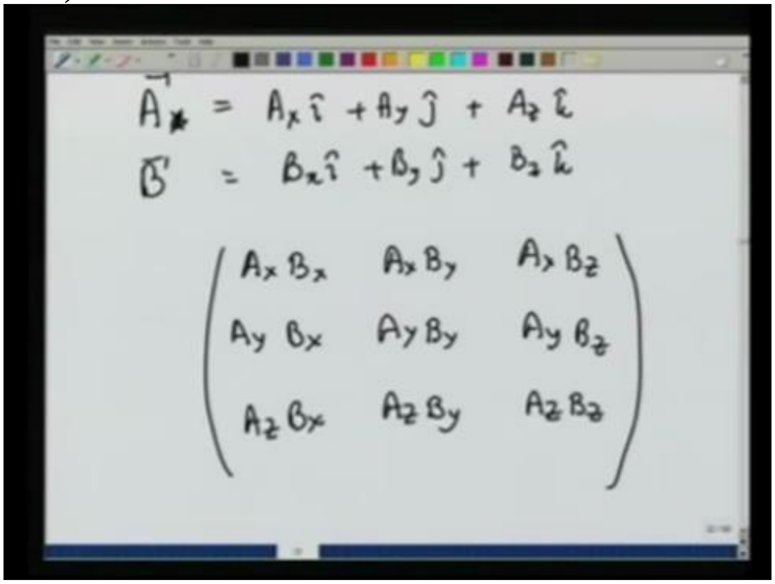

Having written the vectors A…AYJ + BZK and B as BXI + BYJ + BZK, we are now ready to write their product. All possible combinations of the product are AX times BX, AX times BY, AX times BZ. Similarly AY times BX, AY times BY, AY times BZ and AZ times BX, AZ times BY and AZ times BZ. These are all possible products of different components of the 2 vectors. Out of which, which ones do we take? Which ones do we define as vectors and scalars? Is the next task that we are going to address.

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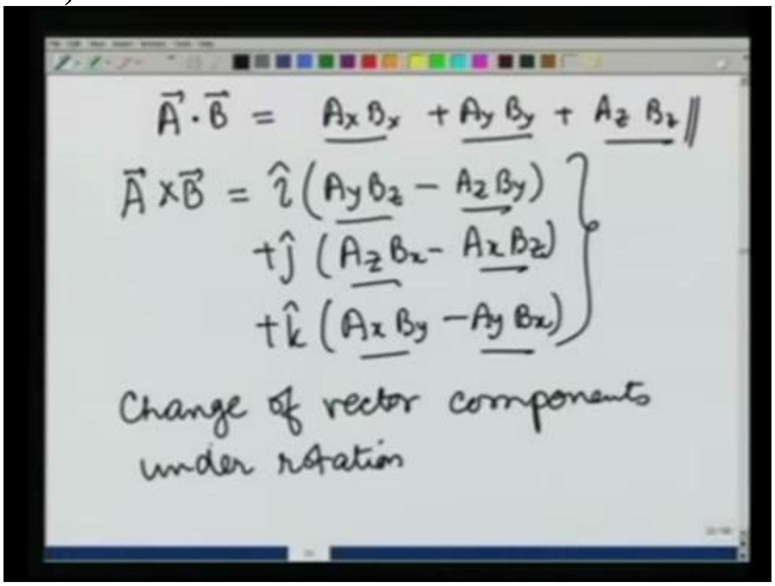

You already know from your intermediate 12th grade that there are 2 kinds of products, a dot product which is a scalar product which is given as a sum of AXBX + AYBY + AZBZ and a cross product, A cross B which is a vector quantity which is given as a component AYBZ - AZBY in X direction + J AZBX - AXBZ in Y direction + K the component in the direction is AXBY - AYBX. But as I showed you in the last slide, there are these all possible products which are being taken here, but why do we take them in this particular combination?

Why do I define this to be a scalar product? Why do I define this to be a vector product? Is the question that I want to address now so that we get a better understanding as to why the products of these 2 vectors are defined in such a manner. And for that I want to look at another property of vector quantities and that is the change of vector components under rotation.