Engineering Mechanics
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Module 03
Lecture No 29
Dry friction V: Examples

In this session, we are going to solve a few problems related to dry friction.

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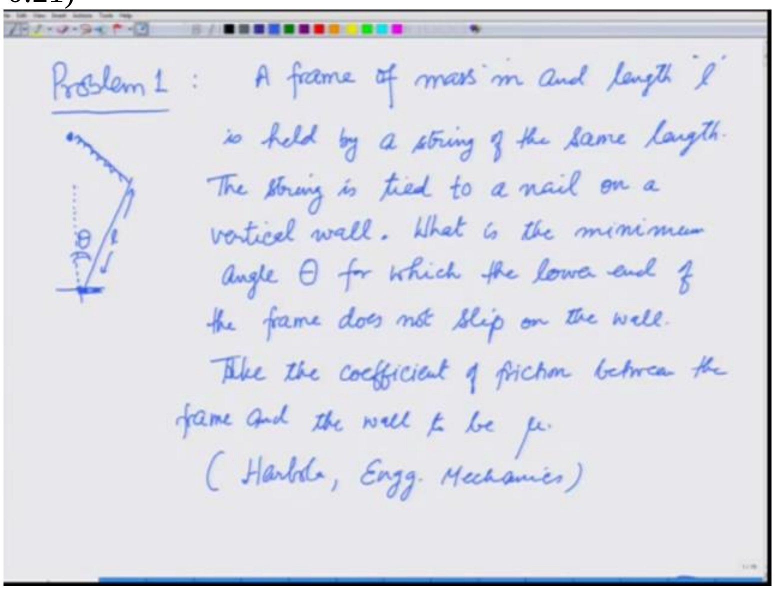

So problem number 1 is you may have noticed that at home sometimes the photo frames are hanged by a string. This is supposed to be a string and this is a photo frame. By putting a nail here and on this side, the lower end of the frame rest on the wall, usually one puts a kind of support here so that the frame does not slip down or does not go down. The problem that we are making is we do not want to have this support and see if the friction can hold the frame.

So the problem is, a frame of mass M and length L, this length is L, is held by a string of the same length. Width of the frame does not matter. The string is tied to a nail on a vertical wall. What is the minimum angle, let us call it theta and let me make it here. Theta for which the lower end of the frame does not slip on the wall.

Take the coefficient of friction between the frame and the wall to be myu. This problem is taken from my book and so are the other 2 problems that I am going to do.

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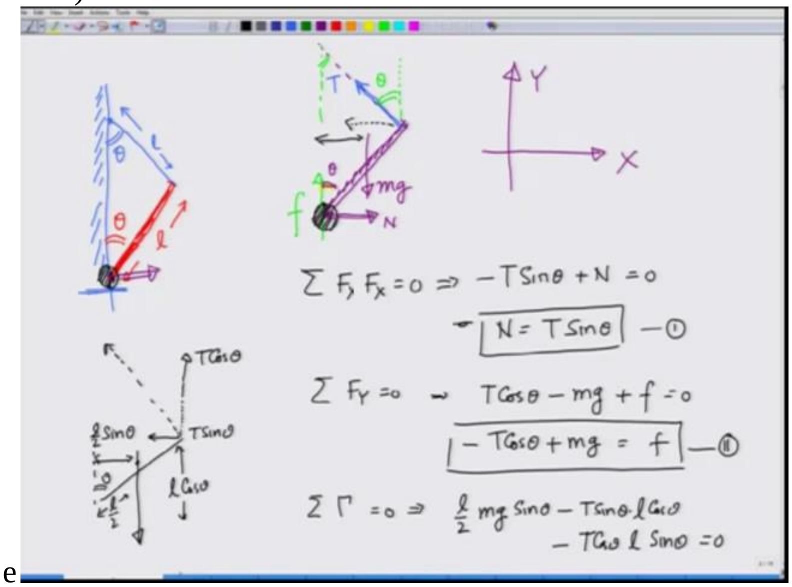

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So let us see and analyse what we are doing. What we are doing is, this is the wall. Here is the nail and this is the string and frame I am going to make with a different colour. This is the frame. I will make it a little thicker. Both are of the same length L and this is also L. Therefore whatever is the angle out here, theta the same angle is up here also. And without any support at the bottom out here, we want this frame to be held so that it does not slip and what is the minimum angle for that to happen?

What you can see is that the friction arises because this frame has a normal reaction on it due to the wall out here and that normal reaction gives rise to the friction. So let us look at the forces on the frame. The forces on the frame and let me make this here are the normal reaction, the weight MG and the tension let me show that by blue here, T by the string. Finally because of normal direction, there is a frictional force. Let us call it F.

The the frame is in balance due to all these forces. The angles are going to be theta because this is theta here and theta here. Let me show it by purple, theta here. Let me take this direction to be X, this direction to be Y. Then for equilibrium, summation FX equal to 0 gives me T, the tension has a component in the direction opposite to X. So it is going to be - T sine theta + N is equal to 0. N is in the going in the positive direction and this gives me N equals T sine theta. That is equation 1.

Summation FY is equal to 0 implies the vertical component of T is T cosine of theta in the positive Y direction - MG and sense the lower end has a tendency to slip down, I have already made the frictional force taking it up. So + F is equal to 0. And therefore, T cosine theta + MG is equal to F. That is my equation number 2. And 3rd is that summation of torque about any point is 0. We will take torque about the lower end.

So let me indicate that here on the picture. We will take torque about this end here or this end here. All right? Now the torque due to N and F vanishes here because they are passing through the point about which we are taking the torque. The torque arises only due to the weight of the frame and due to the tension. So this is going to give me the perpendicular distance of MG from the point is going to be, let me make this picture on the left here again.

This is T. Here is MG. This angle is theta and therefore this distance is L by 2. This is L by 2 sine theta and this has a tendency to make the frame rotate in a clockwise direction. So let me take that to be positive. So it is going to be L by 2 times MG sine of theta. And T I can easily calculate, take its 2 components.T cosine of theta and T sine of theta. T sine of theta has a tendency to make the frame rotate counterclockwise and so does T cosine of theta.

They are going to be with a - sign. - T sine theta times this distance from the point is L cosine of theta. L cosine of theta - T cosine of theta times L sine of theta is equal to 0.

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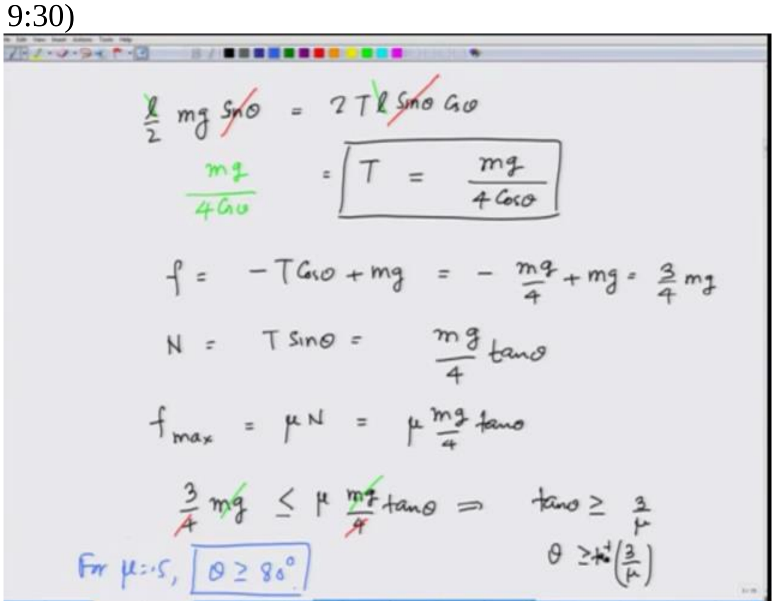

And this gives me L by 2 MG sine of theta is equal to 2TL sine theta cosine of theta. sine theta cancels. So T comes out to be, so the L also cancels. T comes out to be MG over 4 cosine of theta. Let me write this in black. MG over 4 cosine of theta. What we are more interested in is the frictional force and let me look at the equation. The force was F equals - T cos theta + MG.

And we have F equals - T cosine of theta + MG which is equal to - MG by 4 + MG which is three quarters MG. And N which from the 1st equation here is T sine theta is equal to T sine theta which is going to be MG over 4 tangent of theta. So we have found the frictional force and the normal reaction. Now frictional force maximum would be equal to myu times N.

So frictional force maximum is equal to MG by 4 tangent theta times myu. So 3 by 4 MG which is the frictional force required for the frame to be in equilibrium should be less than or equal to myu MG by 4 tangent of theta. Let us cancel terms. MG cancels, so does 4. And this immediately gives me tangent theta is greater than or equal to 3 by myu or theta should be greater than or equal to myu tan inverse.

And that is my answer. This is angle is not very small. For myu equals 0.5, this theta comes out to be greater than or equal to about 80 degrees. That is problem number 1.

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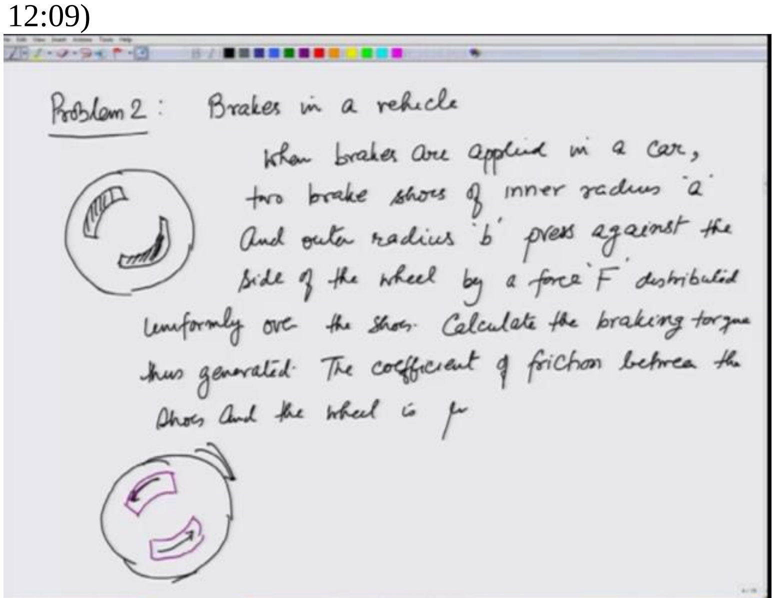

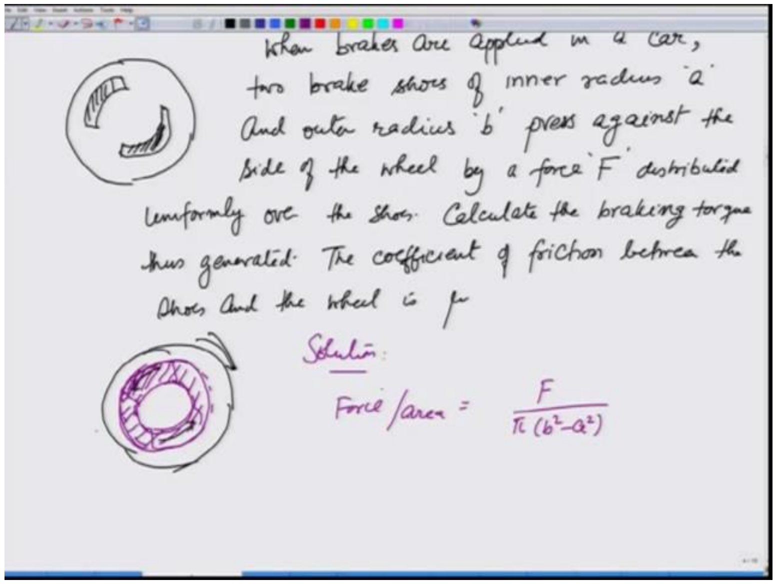

Problem number 2 concerns dry thrust bearing and it is to do with brakes in a vehicle. Next time when you go out, notice that when you apply a brake on a moving car or a scooter, what happens is, there are two brake shoes that press against the moving wheel, the inner part of the moving wheel. When they press, they apply a force and that force generates a frictional force and that frictional force oppose the motion and thereby slowing the wheel down.

So the problem is when brakes are applied in a car two brake shoes of inner radius A and outer radius B press against the side of wheel by a force F distributed uniformly over the shoes. Calculate the braking torque thus generated. Coefficient of friction between the shoes and the wheel is myu. So you understand what is happening. This wheel is rotating and then we press it hard. And therefore the frictional force that is generated, creates a torque. Now let us look at this.

Here is the wheel and let me just make the shoes like this. And they are generating force, when the wheel is rotating like this, they are generating a force in the opposite direction. You can see, the forces would cancel each other. Therefore the only thing that is happening is it provides a torque. Now force is distributed over the entire area.

Doing the solution, the force per unit area is equal to F divided by Pi B square - A square. If we take the shoes to be roughly completely circular so that there is no area gap, we are assuming that the area becomes, area of the shoes is roughly equal to this circular region.

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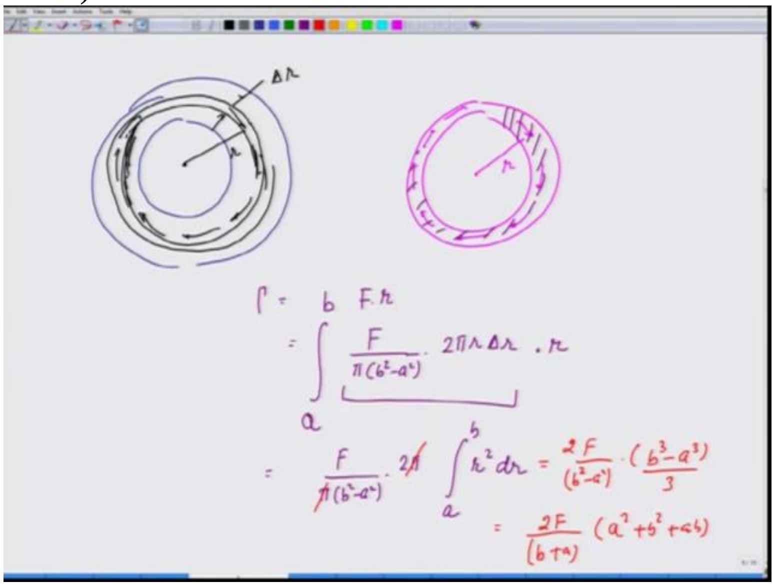

Therefore when I calculate the frictional torque, I will take a small section here of radius R and width delta R. The force here, frictional force that is being generated is acting like this in the opposite direction. So let me make it again. If I look at this section, in this, this force is being generated at a distance R.

So the torque provided by this force is going to be the force times R. Force is the net force divided by pi B square - A square times 2 pi R delta R which is the area of this region. That is the force that is acting. Times R becomes the torque and I integrate it from R and goes up to B. That will be the answer. So this we work out F over pi B square - A square times 2 pi integration R square DR A to B which is an easy integral to perform.

This pi cancels and I get this equal to F over B square - A square times B cubed - A cubed divided by 3. And there is a 2 also here. So the answer becomes 2F over B + A times A square + B Square + AB and that is the answer.

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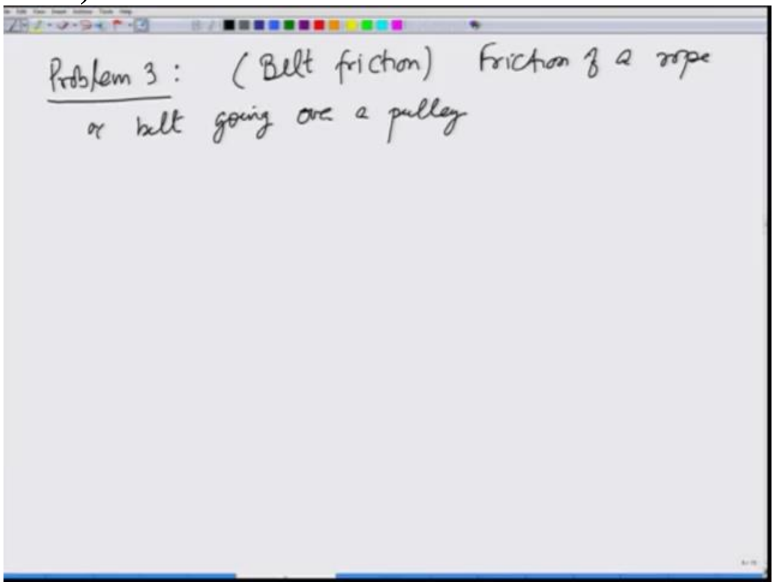

The 3rd problem that I am going to take now has to do with the belt friction or the friction of a rope or belt going over a pulley. So to motivate this problem, let me show you something which you see everyday in the bathrooms. Suppose there is a towel rod. Right? And you see if I hang the towel with roughly equal distances on the 2 sides, the towel holds in there very nicely. If I increase the length, length on one side is larger, it still holds.

If I increased further, it still holds. I increase it further, it holds. And further, it slips. And we want to find out when would this slip?

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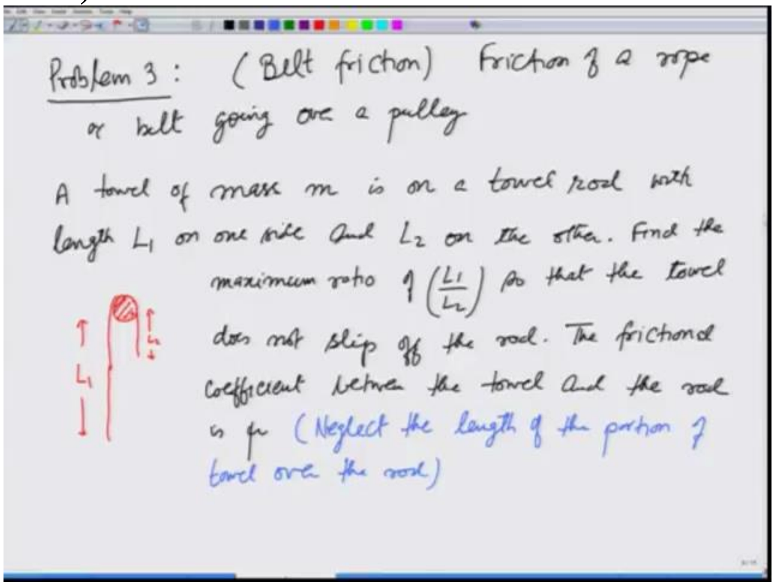

So the problem is this. A towel of mass M is on a towel rod with length L1 on one side and L2 on the other. So let me explain that by picture. Here is the towel rod and here is the towel. Let us call this L1 and let us call this L2. Find the maximum ratio of L1 by L2 so that the towel does not slip of the rod which you saw just now, if I increase the length L1, it slips off.

The frictional coefficient between the towel and the rod is myu and further what we are saying is neglect the length of the portion of towel over the rod. As just now saw, rod is really very small in area. So that length is really negligible. So let us see what is happening.

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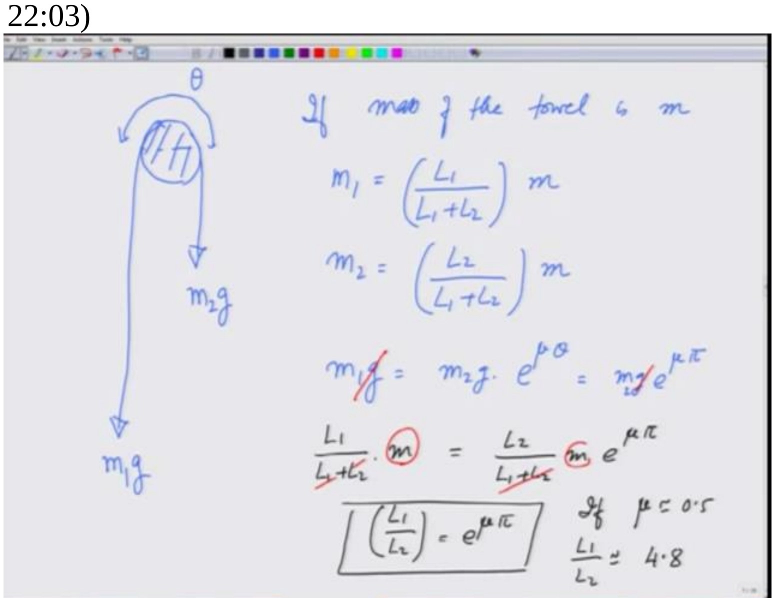

What is happening is in this situation, I will make a bigger picture. Here is the towel on one side, here is the towel on the other. This is being pulled down here by the force M let us call it 1G. And this is being pulled down by the force M2G. If mass of the towel is M then M1 is L1 over L1 + L2 times M and M2 is L2 over L1 + L2 times M. And if the towel should not slip then the force M1G has to be equal to M2G times E raised to myu theta where theta is this angle of contact between the towel and the rod.

And this we know from this is nothing but pi. So this has to be M2G E raised to myu pi. Now we can cancel G and we have M1 which is L1 over L1 + L2 times M is equal to L2 over L1 + L2 times M E raised to myu pi. And this immediately implies, because this term cancels, so does M, the answer comes out to be L1 over L2 is equal to E raised to myu pi. If we take myu equals roughly 0.5 then L1 over L2 comes out to be about 4.8 and that is the answer.

So we have solved 3 problems each involving different concept. In one problem, we solved the frictional force due to a surface. In the 2^{nd} problem, we looked at what is known as dry thrust bearing and 3^{rd} problem, we looked at the belt friction.