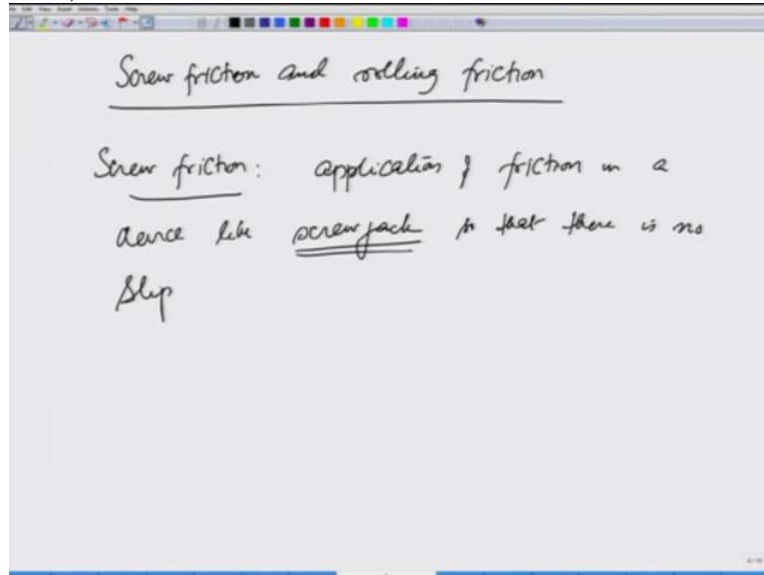


Engineering Mechanics
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Module 03
Lecture No 28
Screw friction and rolling friction

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In this lecture, we are going to talk about screw friction and rolling friction. We start with screw friction. Screw friction is nothing but a very specific application of friction in a device like screw jack so that there is no slip.

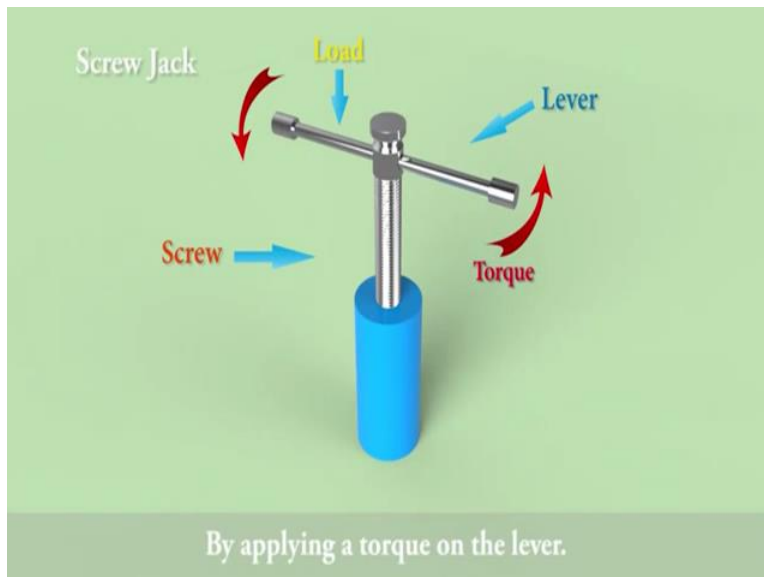
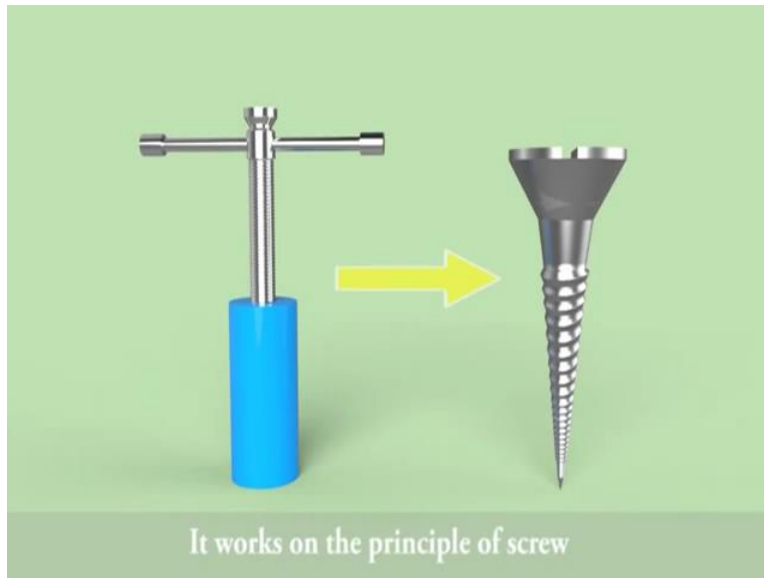
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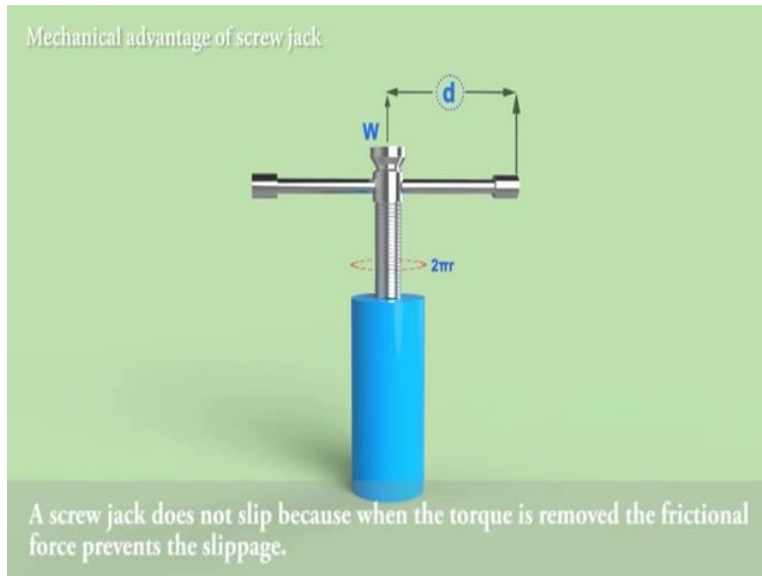
Screw Jack

Screw Jack



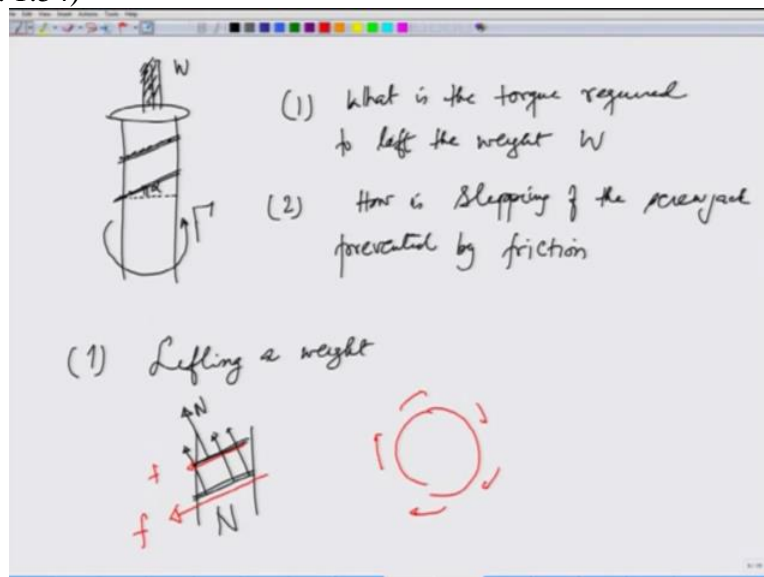
A screw jack is used to make it easy to lift heavy objects and hold them there.





Let me explain. A screw jack is used to make it easy to lift heavy objects and hold them there. It works on the principle of screw. That is, it moves up or down when it is rotated by applying a torque on the lever. A screw jack does not slip because when the torque is removed, the frictional force prevents the slippage. So in a screw jack, we take advantage of the friction to hold the weight where it has been lifted. In this lecture now, we analyse the physics behind a screw jack.

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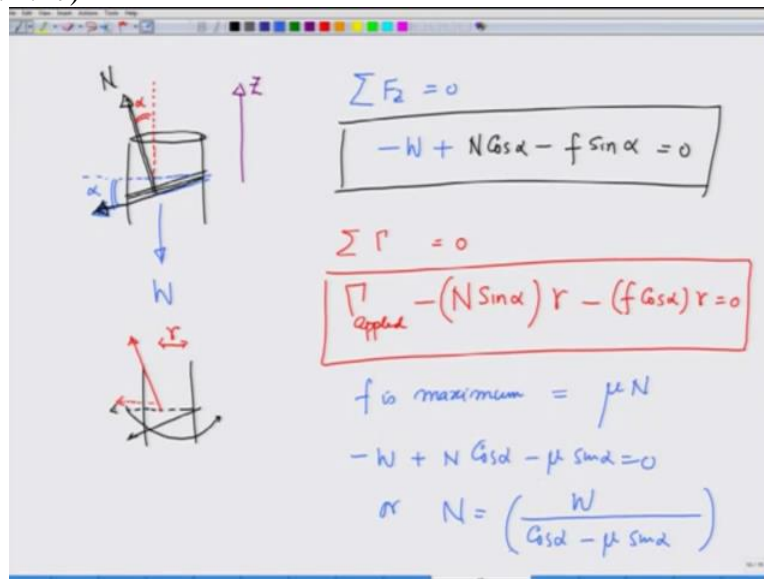
So as you saw in the animation, in a screw jack we have these threads that are at an angle α from the horizontal and we are suppose lifting a weight W that is sitting on the head of the screw

jack by applying a torque, let us call it gamma. And we wish to find out number one, what is the torque required to lift the weight W and two, how is slipping of the screw jack prevented by friction? So let us 1st look at lifting a weight.

Let us look at the forces that are on the thread or threads. This is in a casing. So casing applies normal reaction N. Let us call the sum of all these normal reactions, all the threads to be N. And if we are lifting it up then there is a frictional force opposing the movement of the screw. The frictional forces are necessary thing here and has to be accounted for. Frictional force is necessary because it is the friction that prevents slipping.

So all these, net of this frictional force we are going to call F. And if you look at it from the top, how does the frictional force work? It actually works all around the screen. Therefore the net force is 0 but the frictional force does provide a torque.

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So let us now analyse the relationship. So here is the screw when the screw jack which experiences a force N at an angle alpha and there is a frictional force F acting this way at an angle alpha from the horizontal. Let us call this axis z-axis. Since we already decided that the horizontal force is all around the periphery and therefore it cancels, the only force that matters for the equilibrium is the force in the Z direction.

So FZ should be 0 when we are just about to lift and therefore I have weight acting down which I will take to be the negative direction + the vertical component of N, this is N. So vertical component is going to be N cosine of alpha and the vertical component of the frictional force here which is acting down which is going to be - F sine of alpha which should be 0. That is condition 1.

And to lift the weight, the forces should be like this. I should also apply a torque because the frictional force, its horizontal component is producing a torque which is opposing the motion and the normal reaction which is there also has a component in the same direction which also opposes the motion. And therefore, summation over torque is equal to 0.

Just when the screw is about to move up, is going to give me that the torque applied which I will take to be the positive direction - the horizontal component of the normal reaction is going to be N sine of alpha and let the radius of the screw be R. So this is R out here. - F cosine of alpha times R should be 0. That is my equation number 2.

Now it is very easy to calculate the torque applied because when the whole thing is about to move, F is maximum which is equal to myu times N where myu is the coefficient of friction between the casing and the threats. And therefore I have - W + N cosine of alpha - myu sine of alpha is equal to 0 or N equals the weight divided by cosine of alpha - myu sine of alpha. That is the normal reaction if the weight is to be balanced.

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The image shows a handwritten derivation on a whiteboard. It starts with the torque equation:

$$\Gamma = (N \sin \alpha + f \cos \alpha) r$$

$$= N (\sin \alpha + \mu \cos \alpha) r$$

$$= \frac{W}{\cos \alpha - \mu \sin \alpha} \cdot (\sin \alpha + \mu \cos \alpha) r$$

The final equation for torque is boxed:

$$\Gamma = W r \left(\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} \right)$$

Below this, there is a small diagram of a screw thread with a force F applied tangentially at a radius d . The corresponding force equation is:

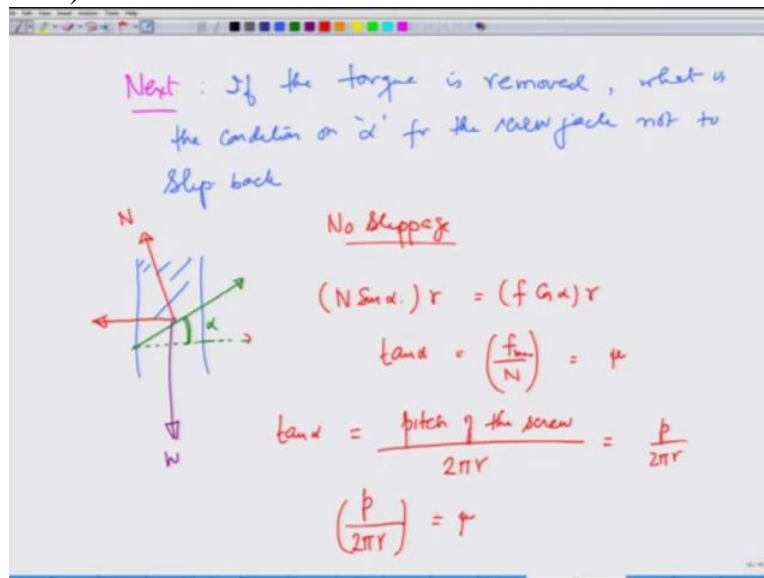
$$F = \frac{\Gamma}{d} = \frac{W r}{d} \left(\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} \right)$$

And therefore the torque which is equal to from the previous page this is the equation $N \sin \alpha + F \cos \alpha = R$ which is equal to $\sin \alpha + \mu \cos \alpha = \frac{R}{N}$ and we have already calculated N which is equal to $\frac{W}{\cos \alpha - \mu \sin \alpha}$ which comes out to be $\frac{WR \sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$.

This is the torque required to lift weight W . Now as you saw in the animation, this torque is applied by a lever. So here is the screw and I have a lever on which I apply a force, horizontal force F and this distance is D , then the force required would be equal to torque over D which is equal to $\frac{WR}{D \sin \alpha + \mu \cos \alpha}$.

You can see the μ if the μ the frictional coefficient was 0, then the torque required or the force required will be less. But then we cannot have μ to be 0 because in that case the screw will slip back and therefore μ is necessary there.

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Now the next what we want to check is if the torque is removed, what is the condition on α for the screw jack not to slip back? It should not just unwind and come down. Right? So in that case again let us analyse the forces. In that case what is happening is, there is still a normal reaction N , there is a weight W and now if the screw is slipping back, the frictional force will be in direction opposite at an angle α .

And N has a component in this direction. So now what is happening is the normal reaction is trying to pull it down. On the other hand, the frictional force provides are talking the opposite direction and for it not to slip, the two torques should balance. And therefore what I am going to have for no slippage, N sine of alpha times R should be equal to F cosine of alpha R irrespective of what weight there is.

And therefore that gives me Tan alpha is equal to F over N which if I am just at the point where it made the slip, then I should have F Max here and this becomes equal to myu. Tangent of alpha is equal to the pitch of the screw divided by 2 pi R. So this is equal to P over 2 pi R. P over 2 pi R, pitch means in one rotation how much it moves, is maximum allowed is myu. Right? So that is the condition for it not to slip back.

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$$\tan \alpha = \mu$$

should $\tan \alpha > \mu$ or $\tan \alpha < \mu$

$$\underline{f r \cos \alpha} \geq \underline{N r \sin \alpha}$$

$$\frac{f}{N} \geq \tan \alpha$$

$$\mu \geq \tan \alpha = \frac{p}{2\pi r}$$

$$\boxed{2\pi r \mu > p}$$

We have derived the condition that tangent alpha equal to should be equal to myu when the torque due to the friction and the normal reaction just balance each other. But the question is, should Tan alpha be greater than myu or Tan alpha be myu for proper design of a screw jack. So let us understand that. So we said that the frictional force FR cosine of alpha, the torque due to this and the torque due to normal reaction is NR sine of alpha.

And this makes the screw slip. This prevents, the fictional term prevents the screw from slipping and therefore the term that prevents should be greater than or equal to the term that makes it slip.

And this immediately gives you that F over N should be greater than or equal to tangent of α or μ should be greater than or equal to tangent of α which is equal to P over $2\pi R$.

So for a screw of radius R , we should have P the pitch less than $2\pi R \mu$ in order that the screw jack does not slip back or frictional coefficient should be large enough so that the screw does not slip back. Let us solve an example of this.

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Example (Book by Harbola)

- Screw of radius 2mm
- Coeff of friction $\mu = 0.15$

$$\mu \cdot 2\pi R > p$$

$$0.15 \times 6 \times 2 > p$$

$$1.8 \text{ mm} > p$$

$$\frac{10\text{cm}}{1.8\text{mm}} = \frac{10}{1.8} \approx 6 \text{ threads per cm}$$

Number of threads $= \frac{1}{p}$ $p <$

$$> \frac{1}{1.8} = c$$

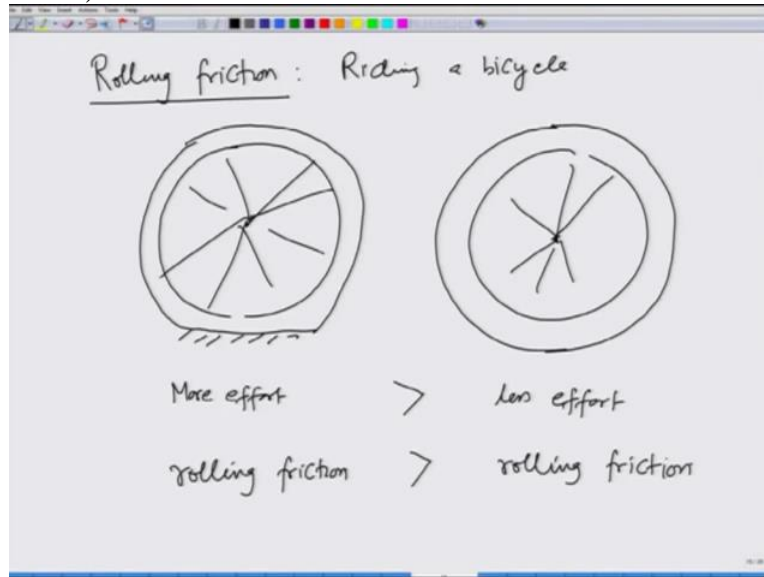
This is taken from the book by Harbola and the example is, sometimes you have seen, if you have a refrigerator or some big item, sometimes a screw jack is put at the bottom. Next time you look at the refrigerator, check that. There is a screw that goes up and down to make the level horizontal properly. And this screw should be such that no matter what the weight of the refrigerator is, does not slip back by itself.

Otherwise, the refrigerator would not hold. So the question is that if you have a screw of radius 2 millimetre and the coefficient of friction is 0.15, what should be the pitch of the screw? So we have just seen that μ times $2\pi R$ should be greater than P . And therefore 0.15 times roughly 6 times 2 should be greater than P . So pitch should be less than 1.8 mm. okay?

And therefore we should have about 1 cm divided by 1.8 mm which comes out to be 10 divided by 1.8, roughly 6 threads per centimetre or more because pitch should be smaller. So minimum 6 threads per centimetre should be there so that the screw holds there and does not slip back. So

the number of threads is equal to $1/P$ and since P is less than 1.8, so number of threads should be greater than $1/1.8$ which is equal to 6 and that is what we said earlier.

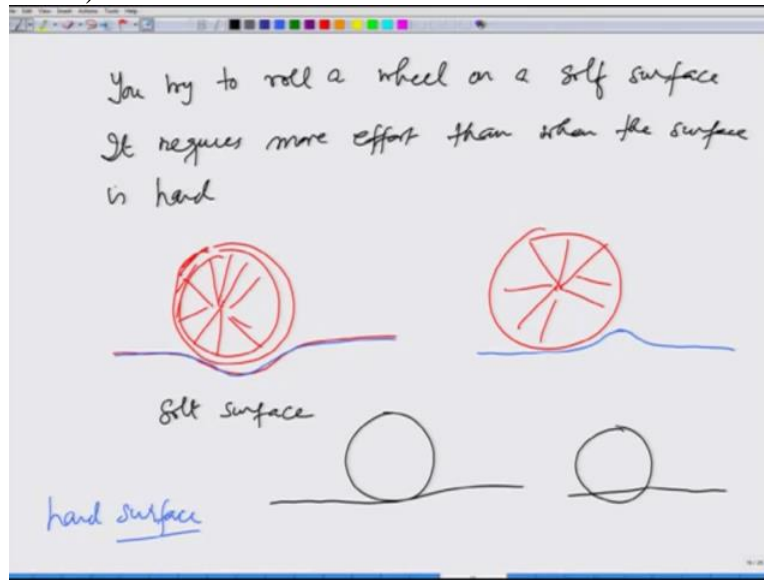
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Next, I want to discuss rolling friction. And to motivate this, let us look around and see what where does the rolling friction arise from? You may have noticed that if you are riding a bicycle, you require more effort to ride it air is less in the wheels. If wheels are really pressurised, then you do not require that much effort. And let us look at the true situation.

This is the bicycle wheel. If the air is not much, then it deforms at the bottom and more effort. Then, a situation, we make the wheel properly like this. Then a situation where the wheel is highly pressurised, less effort. So naturally what we are going to say is that rolling friction in the case on the left-hand side is greater than rolling friction when the pressure is more.

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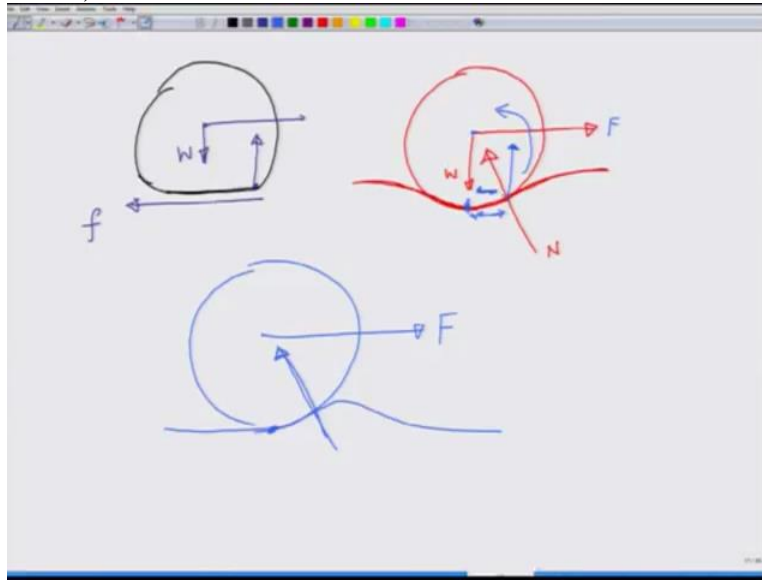


Similarly, notice other cases, if you try to roll the wheel on a soft surface, it requires more effort than when the surface is hard. What is the difference between the two surfaces? Let us look at soft surface. What happens on a soft surface? If there is a wheel on the soft surface, then the surface deforms. Let me make it by a different colour. The surface deforms like this.

Or it may so happen that the surface may deform by forming a bump in front of the wheel. On the other hand, if the surface is hard, on hard surface, the deformation may not be that much. So rolling friction has something to do with the deformation of the surface. And that is precisely how we calculate it.

We can very easily understand if the surface is getting deformed. In the deformation and then coming back to its original position. The surface would generate some energy and that is where the energy is lost and that is what is the loss of energy due to rolling friction. So let us calculate this.

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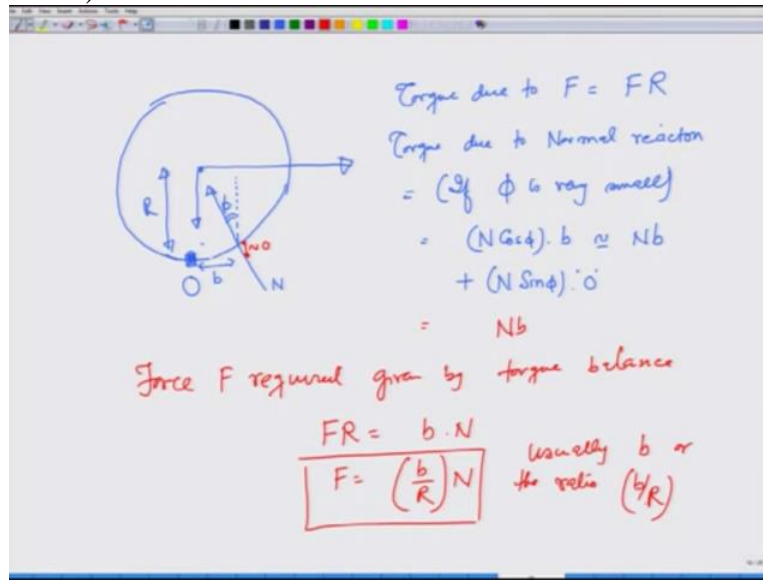
Let us look at the 3 situation. In situation where the tyre kind of becomes flat, that is one situation. 2nd situation where the tyre goes into a kind of lower surface or the 3rd situation in which case there is a bump rising at one of the tyres. What happens when I try to pull this wheel forward? There is a normal reaction acting at this point and there is the frictional force preventing the sliding. There is obviously the weight acting.

In the 2nd situation, if I am trying to pull the wheel, there is a normal reaction acting here like this and there is a weight to pulling it down. Similarly here also there will be normal reaction acting like this. Right? So what this normal reaction does is its away from this point. What this force is trying to do is rotate the wheel about this point.

What this normal reaction does is vertical component if this distance is very small, it is trying to rotate it backwards. If this distance is small, we can neglect this comparable to the normal reaction as I will show and it is this normal reaction which is acting slightly away from the point of contact that prevents that has to be come over when the wheel is being pulled.

And that is what coming over it requires a little bit of force, force F and that is what gives rise to wheel working against this rolling friction. And this is how the rolling friction is calculated. So let us do one calculation.

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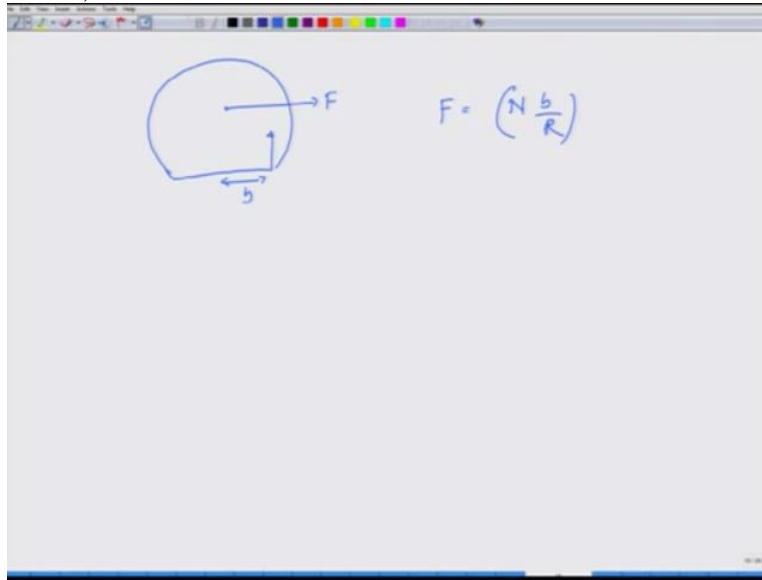


Let us take the case where there is a deformation on the surface and therefore there is a normal reaction N . There is a force F which is trying to make the wheel roll over. This point is stationary and this is weight W . So let us take the torque about this stationary point O , then the torque due to F is equal to F times R where R is the radius. Let us say this normal reaction acts at a distance B , this angle is ϕ , then the torque due to normal reaction is equal to, N is almost vertical if ϕ is very small.

Then this going to be $M \cos$ of ϕ times B which is roughly equal to Nb . And the other torque is going to be $+ N \sin$ of ϕ which is almost at a 0 distance from O . B is very small. This is almost a 0 distance. And therefore this is going to be Nb . So the force required is given by torque balance and that gives me FR equals B times N or F equals B over R times N .

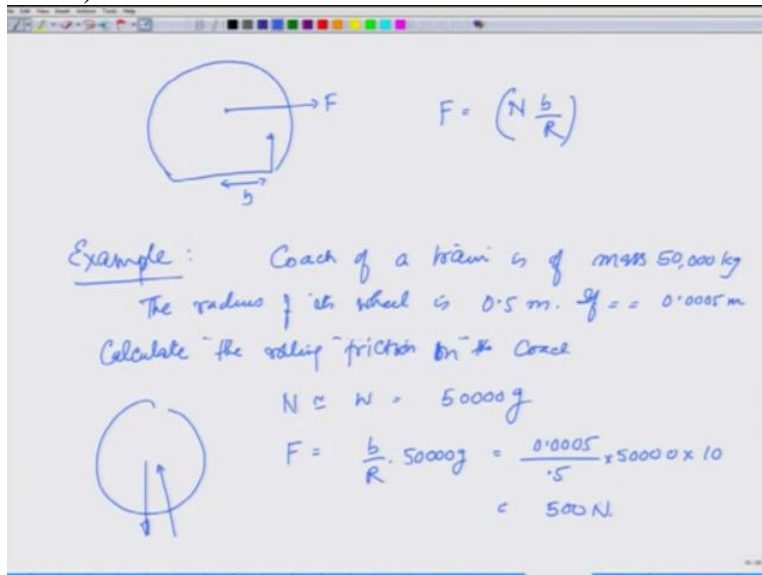
So if I want to specify rolling friction, usually B ratio B by R is specified. And that gives me the value of rolling friction.

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In the same manner, when I look at the deformed wheel, the normal reaction is acting here again at a distance, this is F . So again I am going to get F equals Nb by R . So you understand how the rolling friction arises. It arises because when 2 surfaces or wheel and comes in a contact of rolling surface or surface on which it is rolling, there is deformation. And that deformation gives rise to normal reaction which is not acting at the point of contact and that opposes the motion. To overcome that, I have to apply a force. And that is what is **is** overcoming rolling friction.

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Let me solve an example of rolling friction. This is to do with train. A coach of passenger train. A coach of a train is of mass 50,000 kg. Its radius, the radius of wheel is 0.5 m. If B equals 0.0005 m, calculate the rolling friction on the coach. So here is the coach. Here is N which is roughly vertical. It balances W . So N is roughly equal to W which is 50,000G. And therefore the rolling friction F is going to be B by R times 50,000G which is 0.0005 divided by 0.5 times 50,000. I take G to be 10 and it comes out to be 500 newtons.