Engineering Mechanics

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Module 03

Lecture No 27

Dry friction III: Drive thrust bearing and belt friction with demonstration

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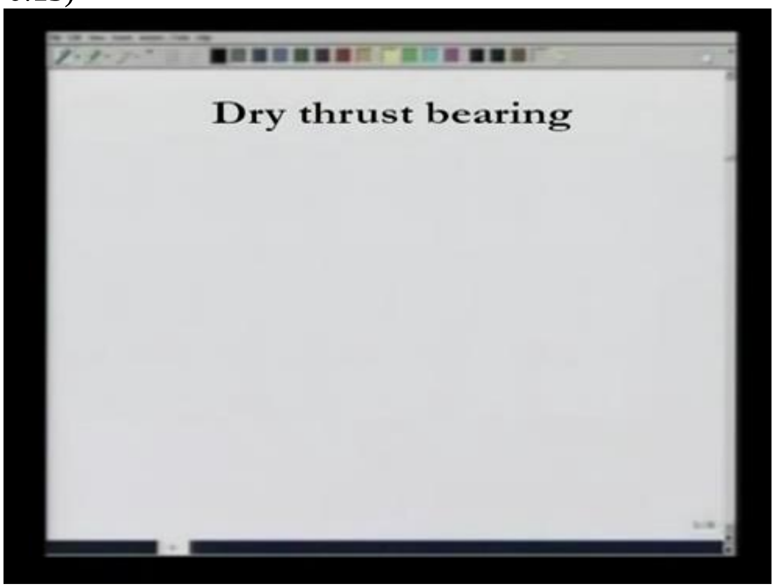

One particular situation that I want to look at is what we call a dry thrust bearing.

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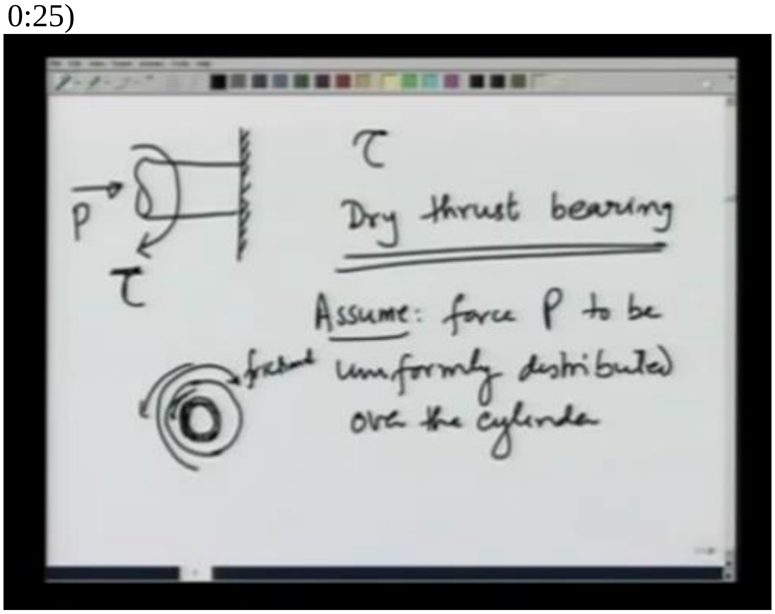

What this is is nothing but a cylinder against a wall. The cylinder is being used by a force P and we apply a torque on this. So I would like to know for what value of Torque Tao, can the cylinder remain static? This is known as what I said earlier, is dry thrust bearing. So let us see what is happening.

If the cylinder is being pushed against the wall and if I look at the cross-section at the wall, the cylinder experiences a normal force because of which it is in equilibrium. If we assume force P that is pushing the cylinder to be uniformly distributed over the cylinder, then each ring I can calculate the force in each ring of the cylinder that I am making here.

This ring because of the Torque being applied tents to move and this motion is opposed by the frictional force. However, frictional force has a maximum possible value. So only up to that maximum possible value can one oppose the motion of can can the cylinder oppose the motion and after that it will start rotating. So let us do that calculation.

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$$N = \frac{P}{\pi R^2} \cdot 2\pi r\,dr$$

$$= \frac{2P}{R^2} r\,dr$$

$$f_{max} = \mu_s \times \frac{2P}{R^2} r\,dr$$

$$d\tau = \frac{2\mu_s P}{R^2} \cdot r \cdot r\,dr$$

$$\tau = \frac{2\mu_s P}{R^2} \int_0^R r^2 dr = \frac{2}{3} \mu_s P R$$

Look at this cylinder. If I look at this ring, at a distance R, the normal reaction of the wall on this ring is going to be force per unit area where R is the radius of the cylinder times 2 pi R DR. Pi cancels and I get this to be 2P over R square RDR. And therefore the maximum possible friction on this ring is going to be myu S Times to be over R square RDR.

The Torque D Tao due to this friction is going to be myu S 2 myu S P over R squared times R times RDR. And I integrate to get the total Torque, it is going to be 2 myu S P over R square integration R square DR 0 to R. It comes out to be two thirds myu S PR. So that is the maximum possible Torque that can be generated by the friction.

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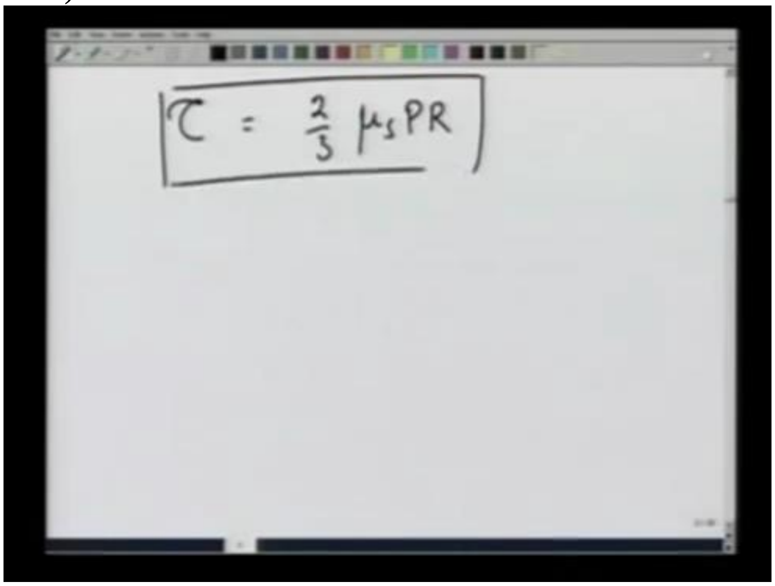

And therefore the maximum possible Torque Tao that I can require without moving the cylinder is going to be two thirds myu SPR.

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Next, we consider a different kind of friction, a case which is known as belt friction. In this case we consider if a belt or a rope goes over an object which is rough so that there is a fictional force possible between them, what is the maximum possible value of this frictional force? For simplicity, we take a pulley, a fixed pulley and let a row go around at some angle so that angle from here to here is theta 0. In this case, maybe the contact angle here is theta 0.

If the coefficient of fiction between the two is myu S, what is the maximum possible value between these 2?

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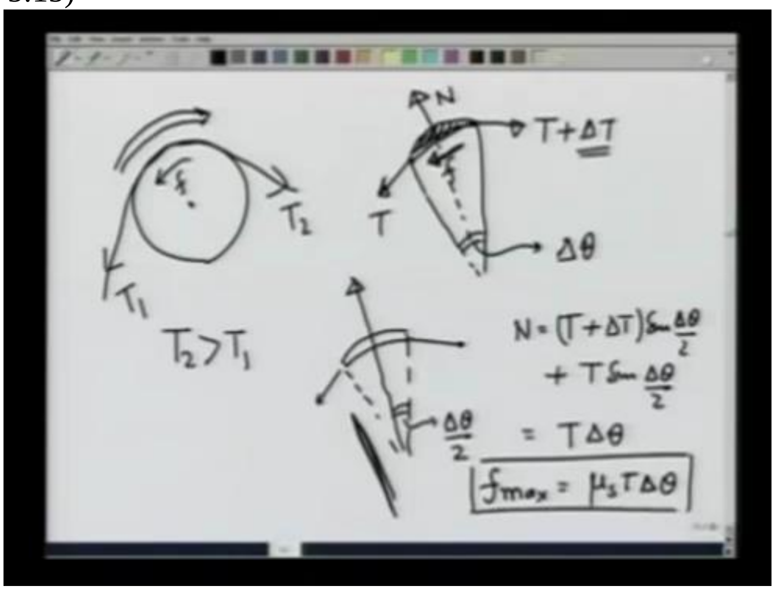

So for this, let us take a small portion of the rope going over the pulley. Let this small portion have an angle delta theta. Let the rope be pulled by tension T1 in this direction and T2 in this direction and without loss of generality, I take T2 to be greater than T1 so that the rope has a tendency to move clockwise and the force, frictional force is opposing it. So here in this section of the rope, frictional force is in this direction.

There is a normal reaction of the pulley on the rope. The tension in this direction is T + some delta T, there is a friction pulling it back is T. Now we want to relate what maximum possible value of delta T would prevent would still keep the rope from moving and therefore that will give me the maximum possible friction value.

Now see if I calculate the normal reaction, let me make this picture again let us draw this line through the centre of this section. This will then be delta theta divided by 2. Since the force in this direction would be balanced, that will give me N is equal to T + delta T sine of delta theta by 2 + T sine of delta theta by 2. And keeping only the 1st order term, I get N to be T delta theta. So that is the value of normal reaction. Therefore F maximum possible is going to be equal to myu S T delta theta.

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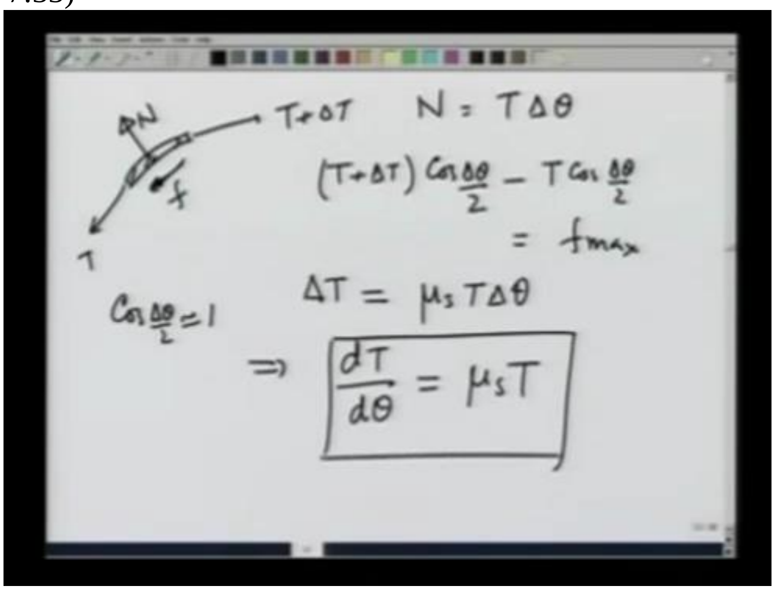

So again making this picture, I had tension T this way, tension T + delta T this way, frictional force this way, normal reaction this way and I have found value of N to be T Delta theta . And the maximum possible delta T that I can have is maximum possible value of friction. So therefore, T + delta T cosine delta theta by 2 - T cosine delta theta by 2 would give me F Max.

And this gives me delta T is equal to F Max which is myu S T delta theta because T cosine delta theta by 2 cancels and cosine delta theta by 2 is roughly equal to 1. And this gives me the equation for the relation between tension and angle is is equal to T. The maximum possible value of fiction is myu S Times N and this gives me the relationship between the maximum possible difference of tension that will still keep the rope in equilibrium, will keep the rope from moving under the different tensions and under this friction.

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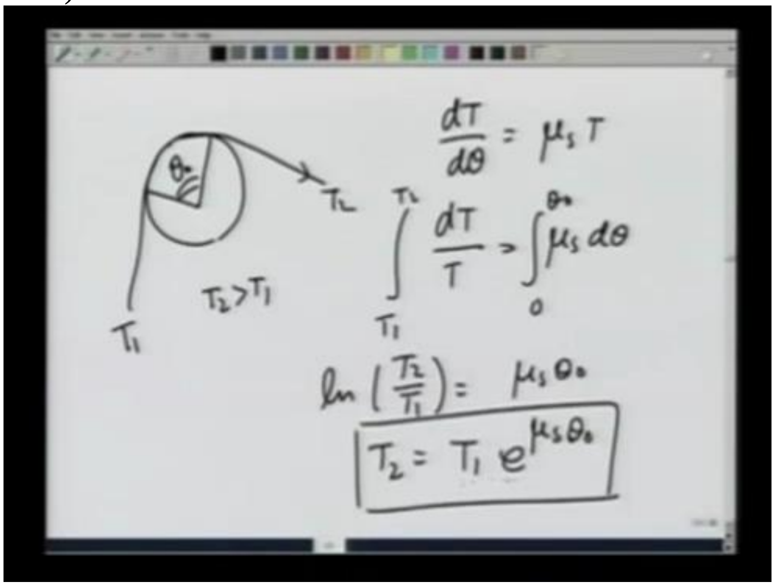

If I calculate now, as I said earlier that there is a pulley and the rope makes an angle theta 0 from T1 T2 such that T2 is greater than T1. And the equation is DT over D theta equals myu S T. If I solve this equation by writing DT over T equals myu S D theta, integrate this from T1 to T2, integrate this from 0 to theta 0, I get log of T2 over T1 is equal to myu S theta 0 or T2 equals T1 E raised to myu S theta 0.

Thus, if I apply T2, this tension which is T1 times this number E raised to myu S theta 0, the rope would remain in equilibrium, it will not move. You see because of this exponential, the force difference is quite large event for very small myu S. Let me show this to you by demonstration.

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Here you see what I have done is taken a bottle filled with water. This is about a litre of water. So this is about 1 kg and I am going to balance this with a very small mass on the other hand which is just just these bunch of keys and you will see that the 2 will balance the cost of the friction. Let me take this pen and if I just put the rope for the string over the pen once, you see the bottle is not balanced. It goes down.

Let me increase the number of turns. I make it 2 more turns and you will see because of the friction now the bottle is still going down but slightly lesser. If I make it one more round, that means I am basically increasing theta, the bottle is still going down, not balanced. Now therefore, let me increase the turns quite a lot. And you will see now that with the small mass, maybe 100 grams maximum, I am balancing a weight of 1 kg on the other side.

And it is all because the tension difference between these 2 would be as large as E raised to now for each turn, I get a a 2 Pi turn and therefore B raised to that many turns times 2 Pi times myu and that can balance the force the last tension on the side with a very small tension on this side.

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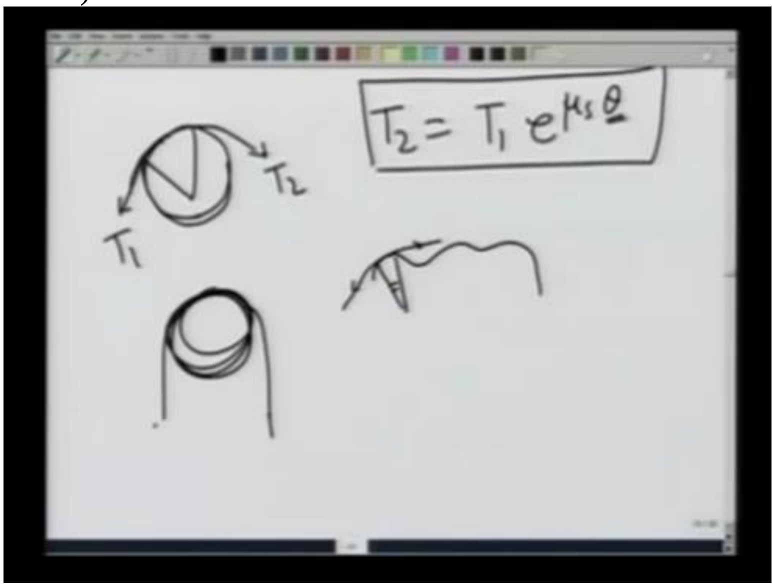

So in this example, what we have therefore seen is that if rope is passing over a rough object, then the tension between the 2 sides maximum would differ by T1 E raised to myu S theta. So as we keep increasing theta, the frictional force keeps going up and a larger and larger difference of T1 and T2 can be balanced. I would like to point out one thing, although for simplicity, we took a round object here, this theta could be over any shape of object.

For example, I could have a shape like this, a rope going over this and this edge, making an angle theta. Then also the difference in the tension between the 2 sides will be equal to E raised to myu theta. It does not depend on the shape of the object. It does not have to be spherical. As long as this, there is an angle theta over which the rope is passing over the rough surface, the tension difference between the 2 maximum would be this. Let us now solve one example using this.

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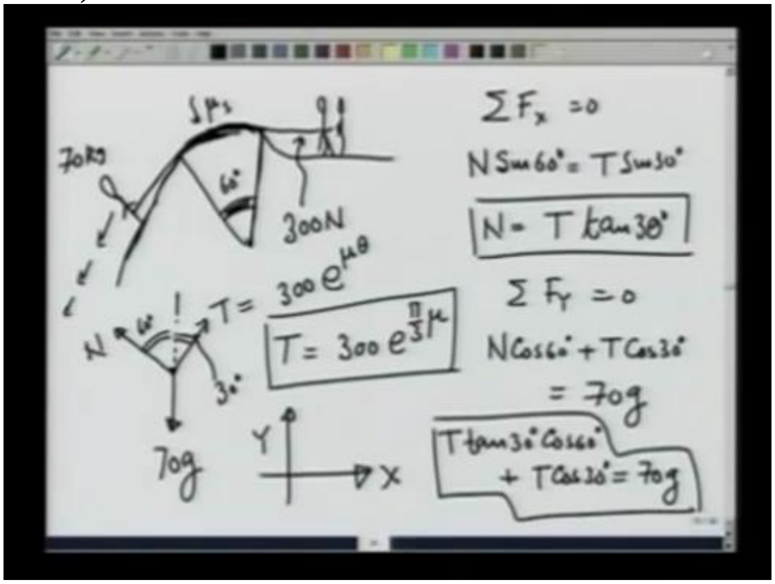

Let us take a mountaineer who is climbing down a cliff of 60 degrees. His rope is holding here. He is making an angle of contact angle of 60 degrees with this mountain cliff. Two of his friends are holding the rope on this side and the mountaineer is slowly going down. His mass is 70 kg and these friends are applying a force of 300 newtons on this side. What we would like to know is, what is the coefficient of friction myu S here.

Everything is sort of in equilibrium and the mountaineer is slowly going down. So let us look at the free body diagram of this mountaineer. He is being pulled up by tension T. There is a normal reaction of the cliff and his weight down is 70g. The normal reaction will be making an angle 60 degrees with the vertical, just like this angle here and tension T which is parallel to the mountainside is making an angle 30 degrees with the vertical.

This T is going to be equal to 300 times E raised to myu theta where theta is Pi by 3. So the tension T on the rope on the left side of this mountain top is 300 E raised to Pi by 3 myu. And now under this tension and the normal reaction and its own weight, the mountaineer is in equilibrium.

If I take this to be the x-axis and distribute the y-axis, I can write summation FX isequal to 0 and that gives me N sine of 60 degrees is equal to T sine of 30 degrees and therefore N equals T tangent of 30 degrees. Summation FY is equal to 0 gives me N cosine of 60 degrees + T cosine of 30 degrees is equal to 70 G. What we have already calculated what N is. Putting that value X get T tangent of 30 degrees cosine of 60 degrees + T cosine of 30 degrees is equal to 70 G.

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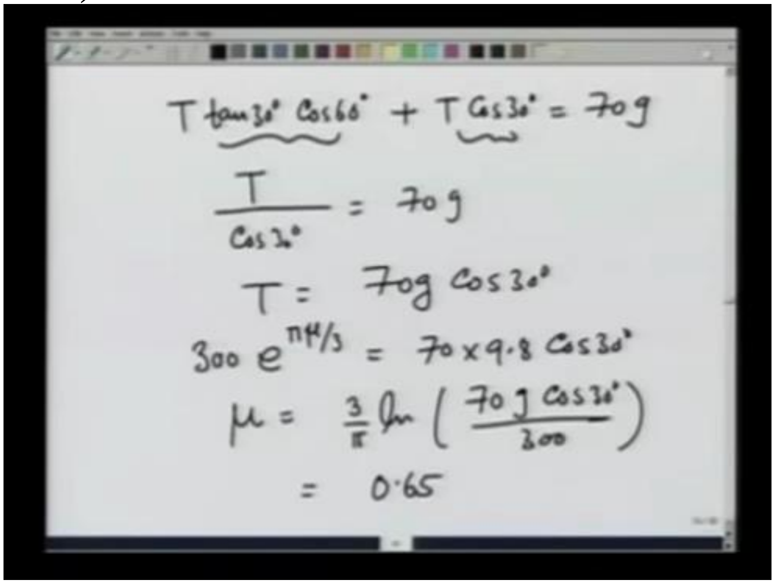

So the answer we have is T tangent 30 degrees cosine of 60 degrees + T cosine of 30 degrees is equal to 70 G. You can simplify by trigonometry manipulations and you get T sine square 30 degrees + cosine square 30 degrees divided by cosine of 30 degrees is equal to 70 G. And T therefore is 70 G cosine of 30 degrees. T we have already calculated is 300 newtons times E raised to Pi myu divided by 3 is equal to 70 times 9.8 cosine of 30 degrees.

And therefore myu comes out to be 3 over pi log of 70 G cosine 30 degrees over 300. You calculate this number and this comes out to be 0.65. And therefore with this 300 N, a 70 kg mountaineer can slowly go down as if he is in equilibrium. The coefficient of friction between the rope and the mountain top is 0.65.