Engineering Mechanics
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Module 03
Lecture No 26
Dry friction II: A solved example

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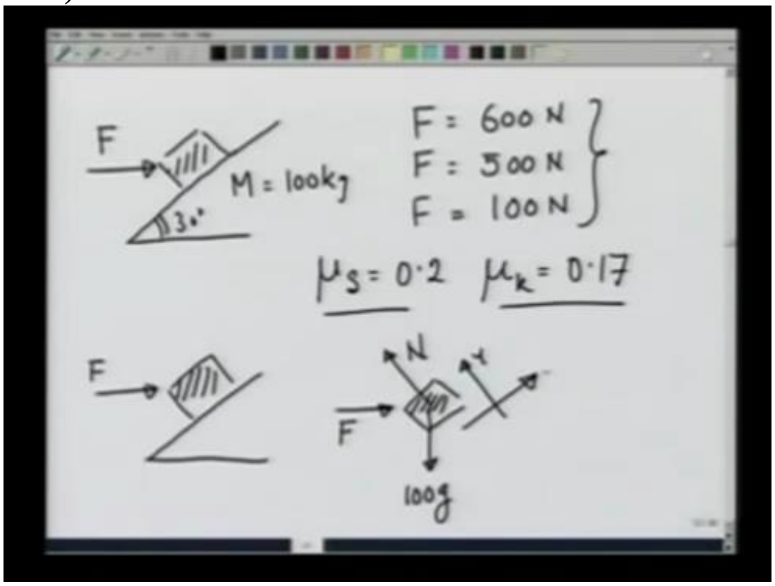

As a 2nd example, let us take the case where again I take a ramp at 30 degrees from the horizontal, put a block here of mass M equal to 100 kg and apply a force in this direction F. I would like to know, what happens to the block if F equals 600 newtons, if F equals 500 newtons and if F equals 100 newtons. I want to know in these 3 cases, does the block move? Does it remain stationary and so on?

This is an interesting problem because in this case, number 1 I do not know whether the block would be moving or would remain static. Number 2, I do not know a priory whether the block has a tendency to move up or it has a tendency to move down. So we have to check all these possibilities. The parameter I did not give you earlier is myu static is equal to 0.2 and myu kinetic which is less than the static coefficient of friction is 0.17.

So what we should really check for is whether the block is moving or not moving and calculate friction accordingly using either myu K or myu S. 70 static, does the friction act up the slope or does the friction and down the slope? These questions we have to answer. There are 2 different ways this can be done in. I will do it in one particular way and leave the other way for you to work out.

To see which way does the block have a tendency to move under an applied force F, I will 1st is that there is no friction and calculate the force F that is required to keep the block in equilibrium. If I increase the force then the force would have a larger component of the plane and therefore the plot would have a tendency to move up. If I decrease the force below that equilibrium force, it will have a tendency to move down.

Let us then proceed. If I make a free body diagram of the block, the force is acting this way. There is MG 100G acting down and normal reaction acting this way. Let me in this case take again the X axis to be in this direction and Y axis in this direction.

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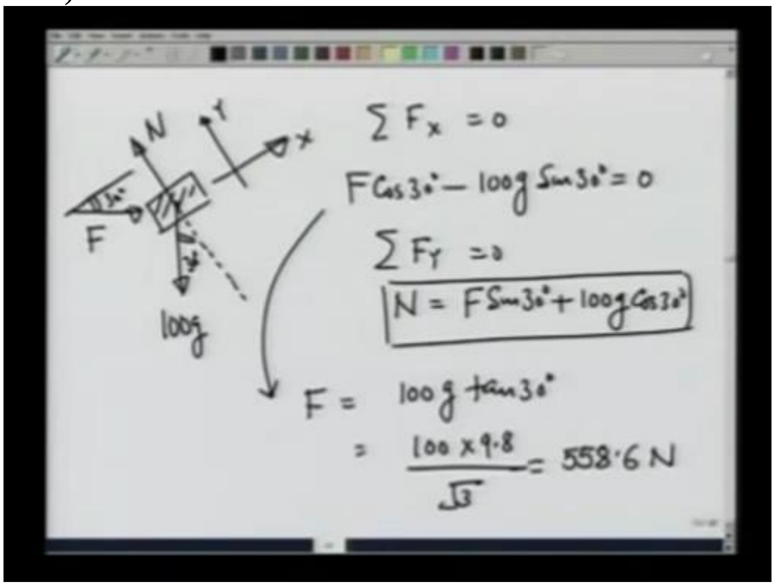

What I have is this block 100 G pulling it down, N acting this way and F in this way. Let me take this to be the x-axis and just to be the y-axis. Summation Fx is equal to 0 gives me let me make the angles, this is 30 degrees and so is this. So it gives me F cosine of 30 degrees - 100 G sine of 30 degrees is equal to 0. The balance in Y direction gives me N is equal to F sine of 30 degrees + 100 G cosine of 30 degrees.

This equation I do not need to calculate F. I will calculate F straight from this equation which gives me F is equal to 100 G tangent of 30 degrees which is 100 times 9.8 divided by square root of 3. And you can calculate this number. This comes out to be 558.6 newtons. So without friction, 558.6 newtons gives me a force that will keep the block in equilibrium.

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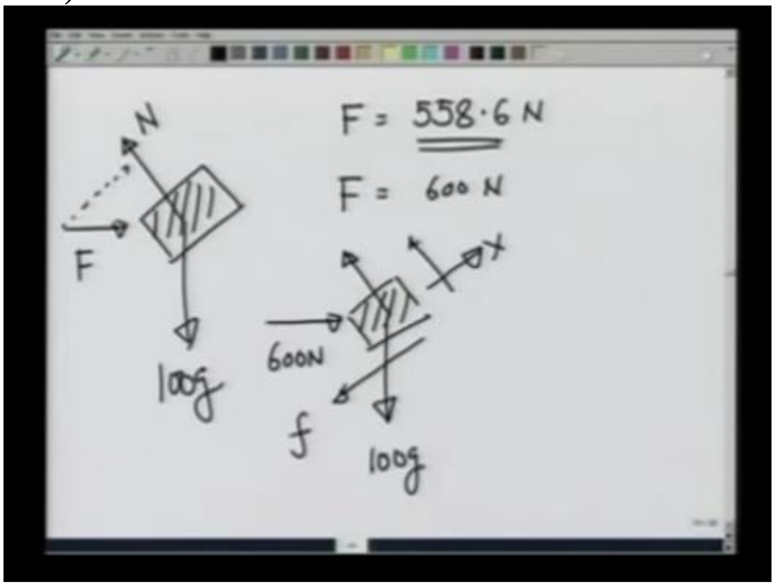

So let me make this block again. 100 G, normal reaction, the force F. If S is equal to 558.6 newtons, the block is in equilibrium and fictional force is 0. I can say that because with this, I do not need any frictional force to have the block in equilibrium. If I go beyond 558.6 newtons, this force would have a larger component up the plane. And therefore, the block would have a tendency to move up.

And therefore, for F equals 600 newtons, the frictional force on the block would be acting down. If I make a free body diagram for F equals 600 newtons, I would have 600 newtons acting this way, 100 G pulling it down, normal reaction M and frictional force down the plane. This will be the free body diagram.

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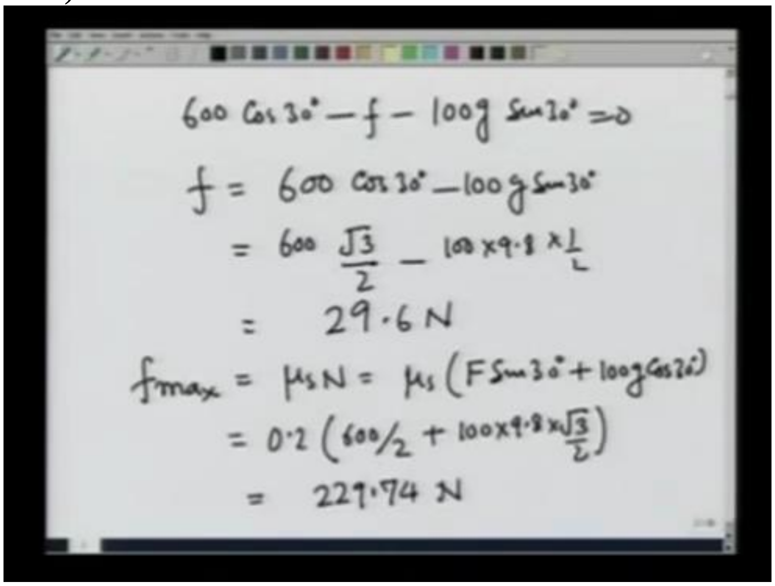

And therefore again taking x-axis in this direction and y-axis in this direction if we now look at the equation of equilibrium, it will look like 600 cosine of 30 degrees - S - 100 G sine of degrees equal to 0 or F equals 600 cosine of 30 degrees - 100 sine of 30 degrees. This number comes out to be 600 square root of 3 by 2 - 100 times 9.8 times 1 over 2 which is equal to 29.6 newtons.

So for equilibrium, we require a frictional force of 29.6 newtons. If the maximum frictional force that can be generated is greater than this, the block will be in equilibrium. If not, then the block will start moving. So let us see what is the maximum possible value of the frictional force. This is myu S N and we have already calculated that N is equal to F sine of 30 degrees + 100 G cosine of 30 degrees.

So in this case, this will come out to be 0.2 times 600 divided by 2 + 100 times 9.8 times square root of 3 by 2. This number you can calculate, comes out to be 229.74 newtons. So you see when I apply a force of 600 newtons, the maximum possible friction that can exist is 229.7 newtons and the frictional force that I required to keep the block in equilibrium is only 29.6 newtons. And therefore the block will remain in equilibrium.

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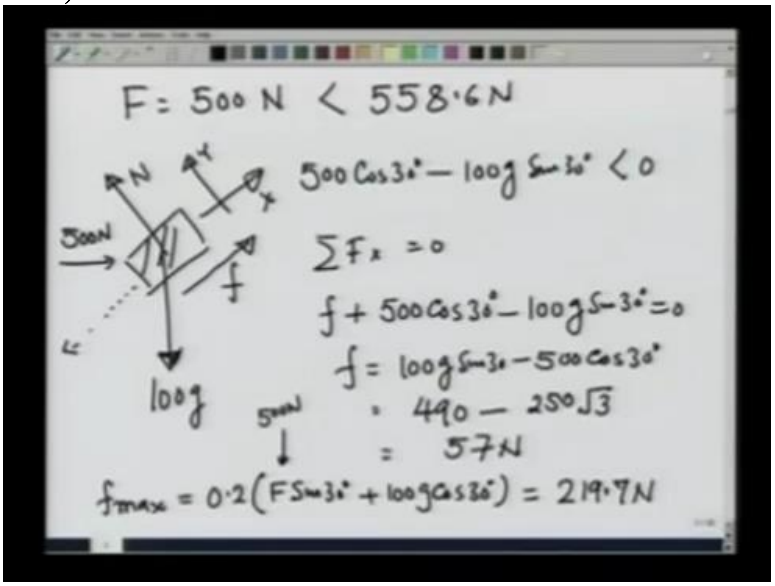

Let us now take the 2nd case of F equals 500 newtons and I had earlier calculated that for 0 frictional force, the force required to keep the block in equilibrium is 558.6 newtons. So this is less than that. And therefore, in this block, when I apply a force of 500 newtons, normal reaction, the component of this 500 newon force up the plane is going to be less than the component of 100 G down the plane.

That means 500 cosine of 30 degrees - 100 G sine of 30 degrees is less than 0, is negative. And therefore, the block will have a tendency to slide down the plane because now the component of 100 G down the plane would be winning. In this case, the frictional force would be acting up because the block has a tendency to slide down. Again, the equation for equilibrium, summation Fx equal to 0 gives me taking this to be the X direction and Y to be the perpendicular direction to the plane, F + 500 cosine of 30 degrees - 100G sine of 30 degrees is equal to 0.

Or F equals 100 G sine of 30 degrees - 500 cosine of 30 degrees. I plug in the numbers, I get 490 - 250 root 3 which comes out to be 57 newtons. So the frictional force that I required to keep the block in equilibrium is 57 in newtons. Let us see what is the maximum possible frictional force that I can have. So F max in this case is again going to be myu S 0.2 times the normal reaction and I have been calculating it. So F sine 30 degrees + 100 G cosine of 30 degrees.

F in this case is 500 newtons. You calculate this number and it comes out to be 219.7 newtons. So again I see that the maximum possible friction that can be generated when I apply a force of

500 newtons in this direction is 219 degrees and it is much much greater than the 57 newton force that I require to keep the blog in equilibrium. And therefore the block will remain in equilibrium.

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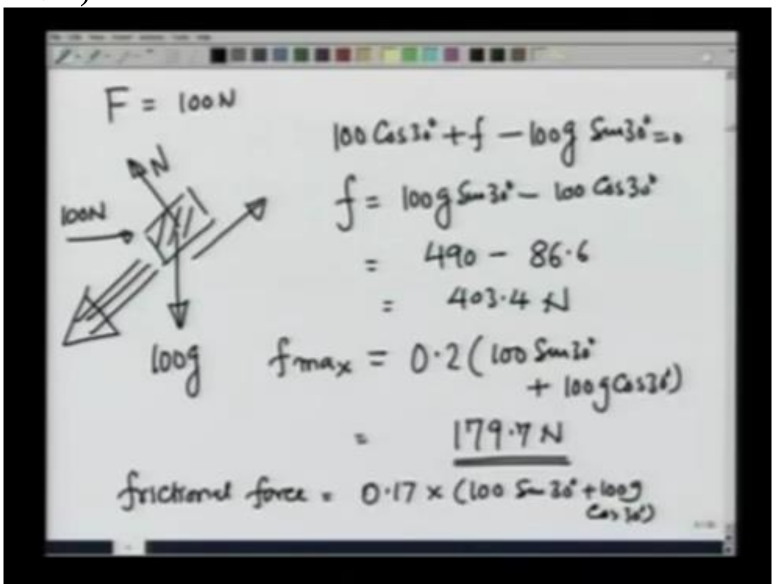

Let us take the 3rd case of F equals 100 newtons. I am applying a force of 100 newtons in this direction. 100 G is pulling it down. There is a normal reaction N and again 100 newtons is much smaller than 558. So frictional force would be acting up the plane. To keep the block in equilibrium, I should again have that 100 cosine of 30 degrees + F - 100 G sine of 30 degrees should be 0.

Or the frictional force should be equal to 100G sine of 30 degrees - 100 cosine of 30 degrees and this comes out to be 490 - 86.6 which is nothing but 403.4 newtons. So the 4 that I required to keep the block in equilibrium is 403.4 newtons. Let us see what is F maximum possible value. F maximum possible value is 0.2, the static friction 100 sine of 30 degrees + 100 G cosine of 30 degree.

We plug in the numbers, sine 30 is one half, cosine 30 is root 3 by 2 and the answer you get is 179.7 newtons. Thus the maximum frictional force that is possible in this case when I apply 100 newton force is less than the fictional force required to keep the block in equilibrium. And therefore the block would start sliding down. If the block starts sliding down, that means the frictional force will know more be static and fictional force in that case would be the kinetic friction which will be 0.17 times 100 sine 30 degrees + 100 G cosine of 30 degrees.

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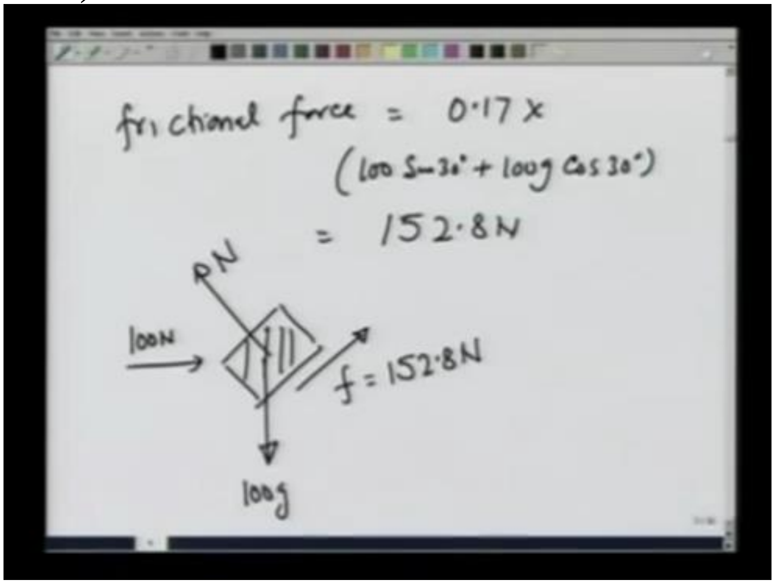

You calculate this number and this comes out to be the frictional force let me write it again. Because now the block a sliding down, is going to be 0.17 times 100 sine 30 degrees + 100G cosine of 30 degree . We calculate this number, it comes out to be 152.8 newtons.

So therefore the frictional force will be slightly less this time and the block would be moving down the plane. Here is 100 newtons. Here is 100G. This is the normal reaction and this is the frictional force of 152.8 newtons.

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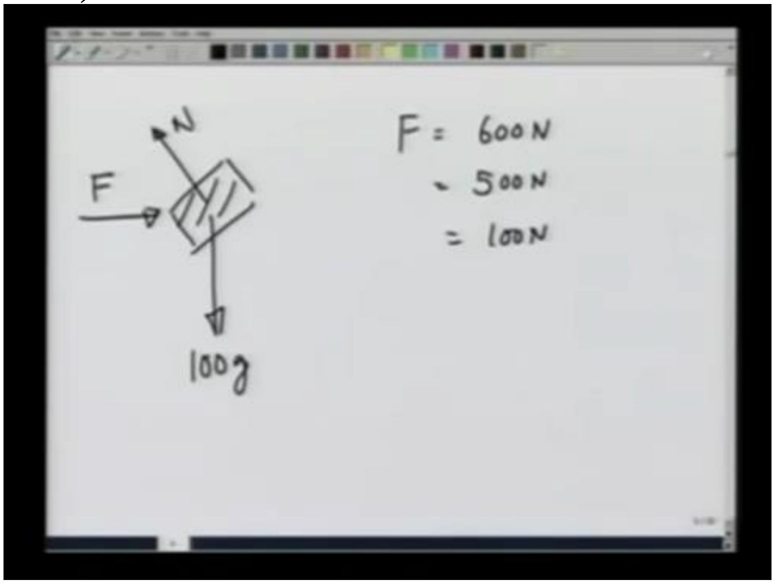

As I said earlier, there is another way of doing this problem and that would be that given this block and given this force F which could be 600 newtons or 500 newtons or 100 newtons, calculate the force required to keep the block in equilibrium and calculate the maximum possible frictional force.

If the force required to keep in equilibrium is more than the fictional force, maximum possible frictional force, the block would move. If it is less, the block will remain in equilibrium. In a way, that is what we did but you can proceed along the calculations in a slightly different way. Having done 2 simple examples of frictional forces, let us now move on and look at some other situations.