Engineering Mechanics
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Module 03
Lecture No 25
Dry friction-I: Introduction with an example

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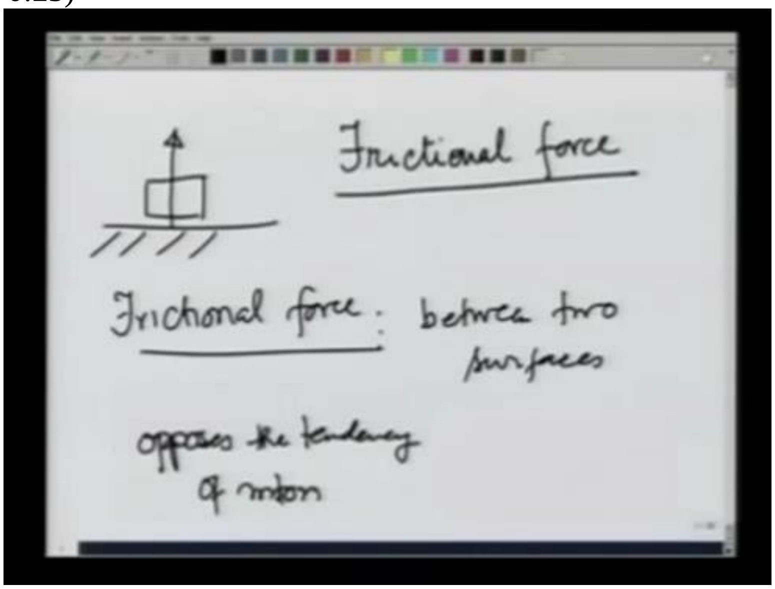

In our lectures on statics so far, you would notice that we have been taking the force on a body when it is on a surface mostly to be perpendicular to the surface. By doing so, we have been ignoring a very important force and the one that is all the time present known as the frictional force. And in this lecture, we pay attention to frictional force and discuss it in various different situations.

So to start with, frictional force could be between any two surfaces and it opposes the tendency of motion. So if a body tends to move in a particular direction, frictional force opposes it.

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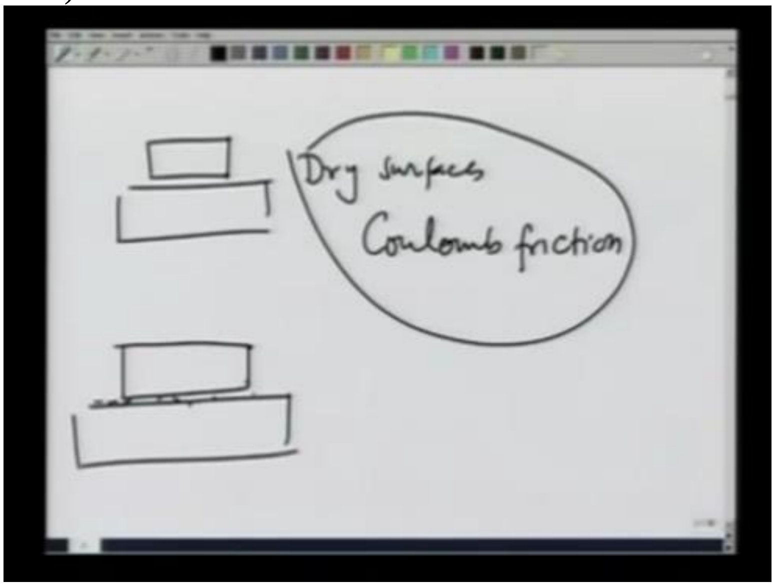

Now the frictional force could be between two dry surfaces and this is known as Coulomb friction. It could also between 2 surfaces that have a thin layer of liquid in between. There is a sort of wet friction. Friction is also present when something moves in a liquid known as the viscous force. In this lecture, we will be concerned primarily with: fiction or friction between two dry surfaces. To understand friction, let us perform a small experiment.

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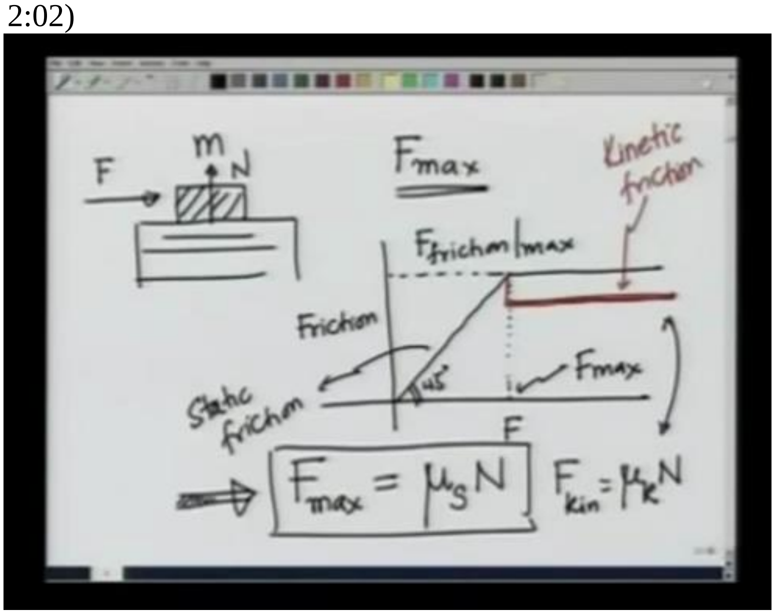

Let us take a block of mass M on a table or on a rough surface and push it by a force F. You will notice that when force is small, the block does not move. It is not move because there is a frictional force opposing it. You keep increasing the force, the frictional force keeps on increasing and it keeps the block in X position until you hit or you reach an F Max. When you reach F Max, the block starts moving.

So if I were to plot frictional force vs F, you would notice that friction adjusts itself with F up to a certain point which I will call F Max. This would be 45 degrees so that the frictional force is exactly equal to the force applied. As you go beyond F Max, the frictional force does not increase. It sort of saturates. So what you would notice after this is that frictional force would tend to saturate at that possible maximum value.

So this is the maximum possible value of frictional force and as you go beyond F Max, the body would tend to move because now I am applying a force which cannot be balanced. In practice, the line does not go like this. But when the body starts moving, the frictional force reduces a bit. So actually frictional force looks something like this. When it starts moving, when you go beyond F Max, frictional force goes below the maximum value of friction.

So we call these 2 regions, this is called kinetic friction and as long as the body is not moving, that is known as static friction. So notice that frictional force when you are applying a force on the body, is not constant. As you keep changing the force, the frictional force keeps changing until you reach a maximum force beyond which it cannot increase and then the fictional force works at its maximum.

What Coulomb observed is that this maximum value of the frictional force is equal to a constant which I will call myu S for static times N the normal reaction on the body. This is the maximum possible value of static friction and once the body starts moving, the kinetic friction F kinetic is equal to myu K, myu kinetic times N and you can make out from this figure that myu kinetic is slightly lower than myu static.

Although we write this formula F Max equal to myu S N I again emphasise that this is the maximum possible value of friction. It is not that if I apply any force on a body, the friction value would be myu S N. It will be less than myu S N. As I keep on getting the force, the frictional force will also keep increasing until it reaches its maximum value which is myu S times N.

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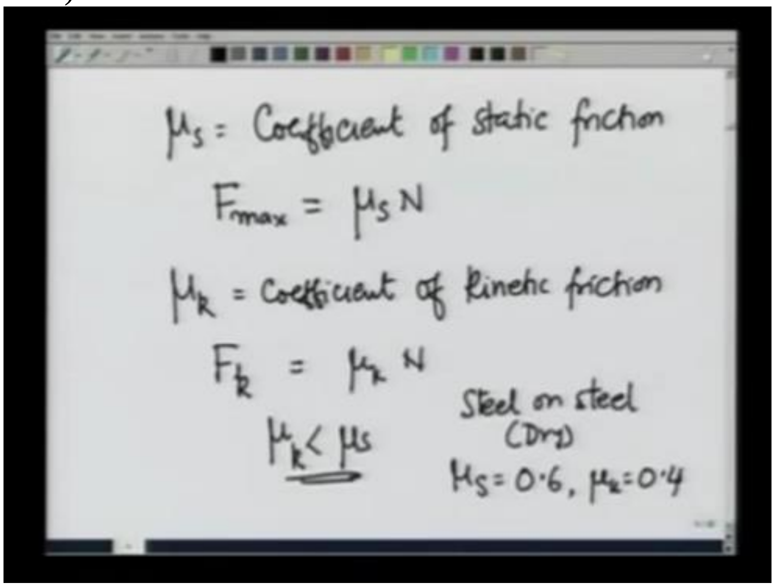

myu S is known as the coefficient of static friction and F Max, maximum value of friction or maximum force up to which the body will not move is myu S times the normal reaction on the body and myu K which is coefficient of kinetic friction so when the body is moving, the frictional force I will call it F kinetic is equal to myu K times N and it is observed that myu K is less than myu S. For example, for a steel moving on a steel, the steel on steel, obviously I am talking about dry friction, myu S is about 0.6 and myu K is 0.4.

One thing about friction is that it is independent of the area of contact. So having understood what frictional force is, how it comes about, now let us solve one problem with it so that we get a better feeling about it.

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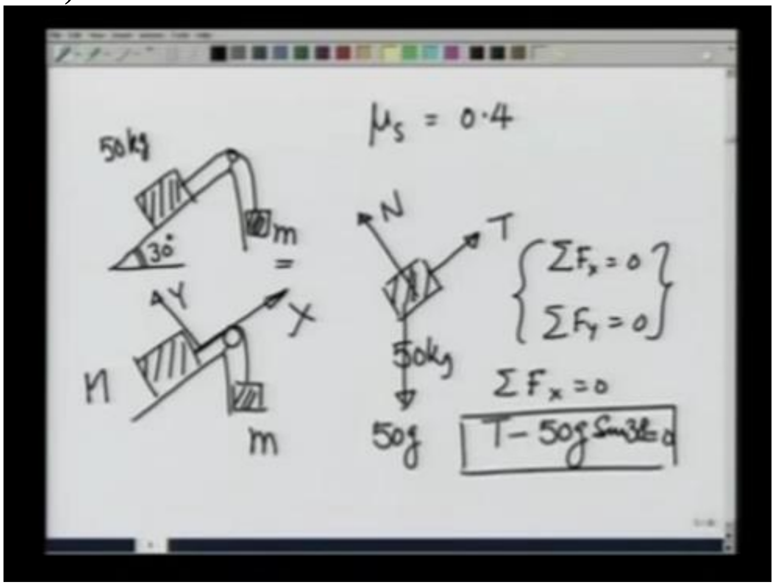

The problem I am going to solve is if I have a block of say 50 kg on a ramp with this angle being 30 degrees and let us say I tie it with a pulley and have another mass here small M, I want to know for what range of mass M would this block of 50 kg remain static on the ramp given that myu S that is the coefficient of static friction between the 2 bodies is 0.4. Why should small M have a range?

To start with, it has a range because there is frictional force acting on 50 kg block. Suppose there was no friction. Let us say if there was no friction, what would happen? If there was no friction, only one particular value of M would balance the mass M. How so? Let us understand that. If I make a free body diagram of the 50 kg mass, it would have the tension T pulling it up, force 50g pulling it down and the normal reaction from the surface.

If it is not to move, then all the forces, summation Fx, summation FY, all the forces should satisfy these 2 equations. Let us for convenience take our x-axis to be in this direction and y-axis to be in this direction. Then summation Fx equal to 0 gives me T - 50g sine of 30 is equal to 0. We can go over to the next page.

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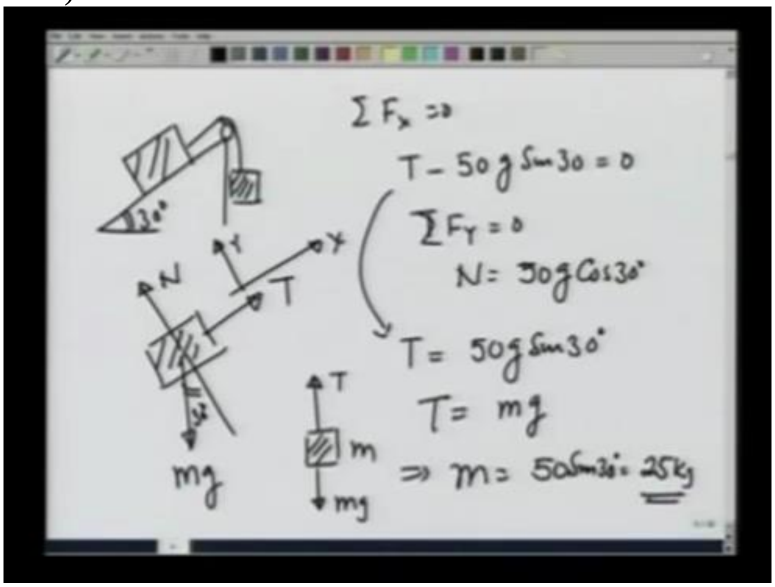

What we are doing is having a mass 50 kg on a ramp of 30 degrees, when I make the free body diagram of this big mass, it has the tension T, normal reaction N and MG pulling it down. This angle is going to be 30 degree. So summation Fx equal to 0 if I take this to be the x-axis and this to be the y-axis gives me T - 50g sine of 30 equal to 0 and summation FY equal to 0 gives me N equals 50g cosine of 30 degrees. This gives me that the tension T should be equal to 50g sine of 30 degrees.

If I look at the free body diagram of this small mass M, it has a tension T pulling it up and MG pulling it down. So T equals MG. And that gives me M is equal to 50 sine 30 or 25 kg. Just 25 kg would balance this mass on the ramp. What happens when we introduce friction? Friction provides an additional force that oppose the motion.

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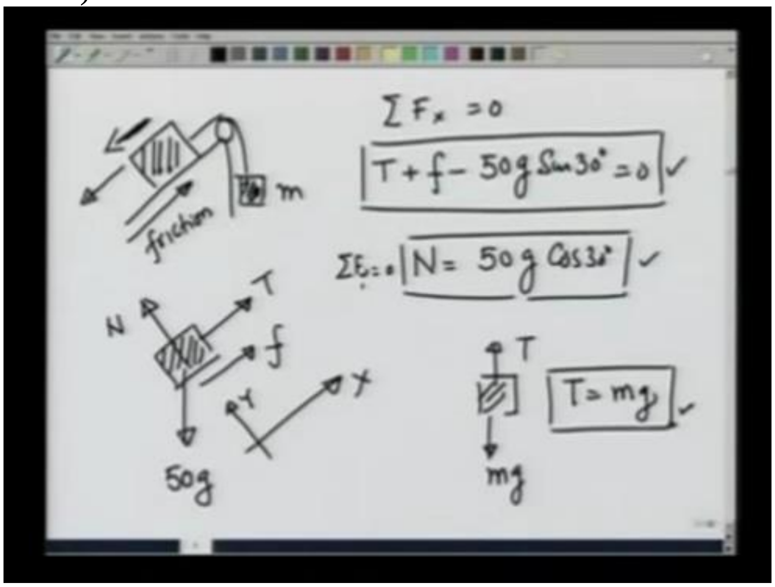

And therefore I can have a range of small mass M that would balance the bigger mass. Let us say 1st that I reduce this mass below 25 kg, then the bigger mass would have a tendency to move down this way. If this has a tendency to move down this way, there will be a frictional force opposing it. And this frictional force that oppose it makes it possible to have mass M much less than 25 kg and still have this block in equilibrium.

Let us see how. So this mass of 50g now if I make a free body diagram, has a frictional force let me call it small F acting this way of the plane. There is 50g pulling it down. There is a normal reaction N and a tension T pulling it up. Again taking this to be the X direction and taking direction perpendicular to the plane to be Y direction, summation Fx equal to 0 gives me T + F - 50g sine of 30 degrees is equal to 0. That is one equation.

And the other equation that I have is N is equal to 50g cosine of 30 degrees. That comes from summation FY is equal to 0. If I look at the equilibrium of small mass M, this has only 2 forces, T and MG pulling it down. So T must be equal to MG for equilibrium. So under frictional force, then this bigger mass has a tendency to move down, the equations for equilibrium are going to beat this.

Notice, I have written this to be small F, the frictional force to be small F, not myu N because myu S times N is the maximum possible value of friction. If 50g sine 30 - T is such that it is below the maximum possible value of friction, the block will remain in equilibrium and F would be equal to that value which is less than myu N. So let us look at the equations once more.

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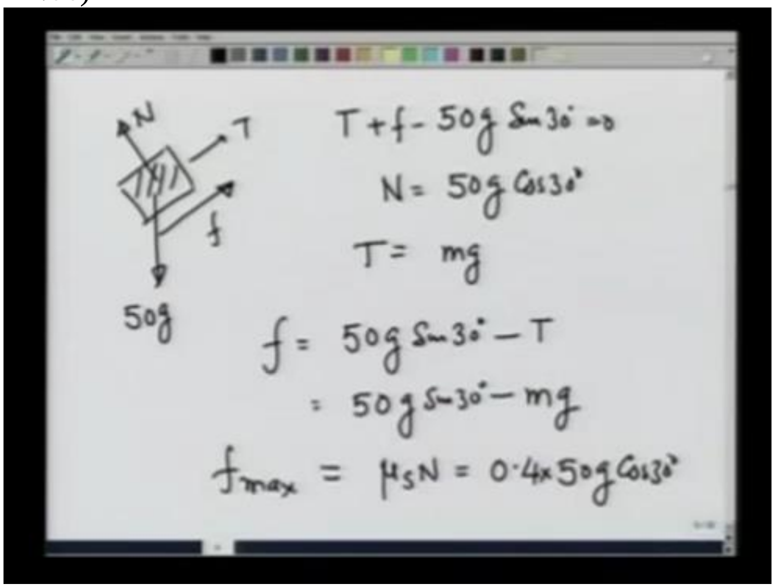

So I have this bigger mass which has a frictional force acting in this way, T acting this way, normal reaction acting this way and 50g pulling it down. And the equations that I wrote were T + F - 50g sine of 30 equal to 0. N equals 50g cosine of 30 degrees and T equals MG. Putting it all together gives me F is equal to 50 G sine 30 degrees - T or 50 G sign 30 degrees - MG.

Now the maximum possible value that F could have is myu S times N which is equal to 0.4 50 times 50G cosine of 30 degrees. That is the maximum possible value of the frictional force.

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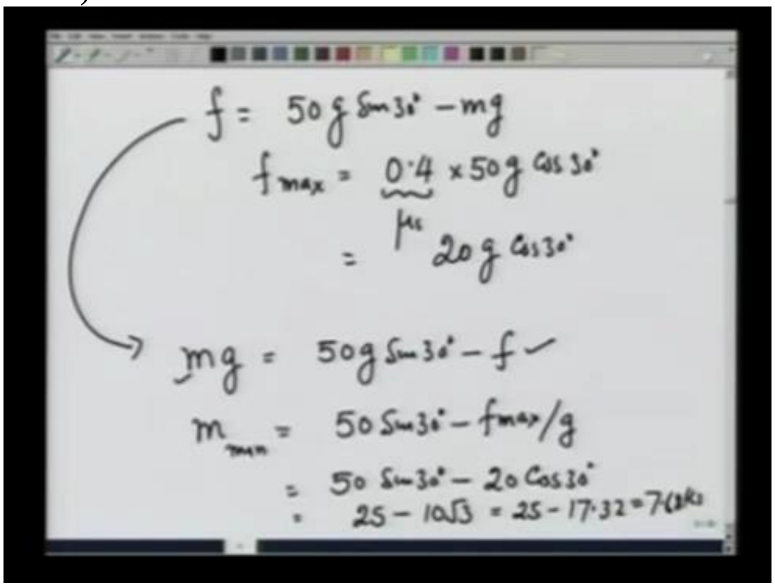

So we have equation F equals 50 G sine of 30 degrees - MG and F max possible is equal to 0.4 which is myu S times 50 G cosine of 30 degrees which is equal to 20G cosine of 30 degrees. Therefore, if I convert this equation to get M, I get MG equals 50 G sine of 30 degrees - F. When F is maximum, M is the minimum possible value of small M that gives me equilibrium.

So M minimum that will give me equilibrium would be equal to 50 I am dividing by G on both sides, sine 30 - F max divided by G which is nothing but 50 sine 30 - F max is 20 cosine 30 degrees G divide by G. So G cancels out. By putting in the numbers, you get this equal to 25 - 10 root 3 which is 25 - 17.32 or 7.68 kg. So you see because of the friction I could have a much smaller value of small M, 7.68 kg and still the whole system would be in equilibrium. This is when the bigger block has a tendency to move down.

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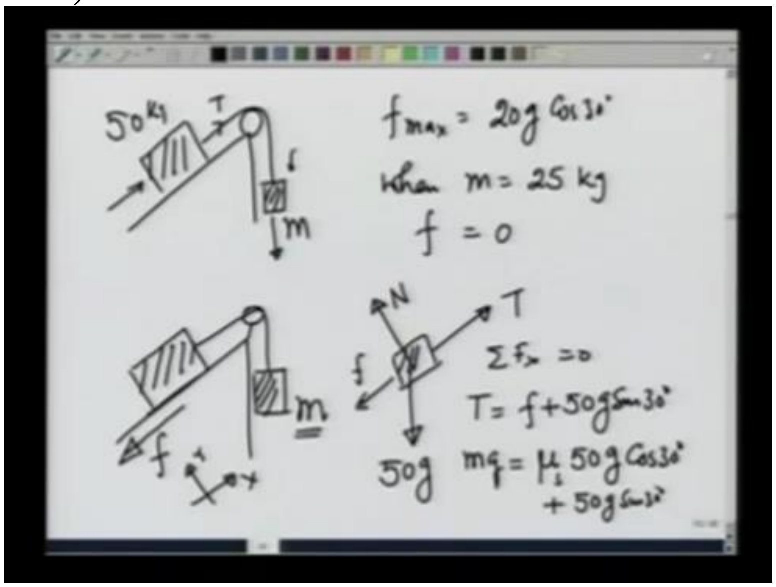
e 

So on this ramp, this is mass M, this is 50 kg. If I have 7.68 kg, M equal to 7.68 kg, this block is in equilibrium although it has a tendency to move down. What happens when I start increasing the mass on it? So I go beyond 7.68, maybe I make it 8, 9 kgs, 10 kgs, 11 kgs and so on. When I increase the mass, I would increase the tension. If I increase the tension, I would not require that large a frictional force to keep the big block in equilibrium.

So frictional force will go below its maximum possible value which was 20 G cosine 30 degree. So F maximum we had calculated was 20 times G cosine of 30 degree. If I increase this mass, it will start going down. In fact, when M equals 25 kg and we had calculated earlier that for M equals 25 kg, I do not really need any friction to have equilibrium. F, frictional force would be 0.

What if I go beyond 25 kg? If I go beyond 25 kg, this mass would now start pulling the bigger mass up and the direction of frictional force would change. So the other limit of this whole scenario is going to be, have this pulley, that if I increase this mass M, it tends to pull 50 kg mass up and therefore if I go beyond 25 kgs, this mass has a tendency to move up. And therefore, the frictional force on this would be in the opposite direction.

The question we ask now is up to what value of M can I go so that the system remains in equilibrium. Again, if I make a free body diagram of the bigger mass, it has 50 G pulling it down, normal reaction N, T pulling it up and the frictional force is in the opposite direction. So this is X, this is Y. Summation Fx equal to 0 now gives me T equals F + 50G sine of 30 degrees. T is nothing but MG as we have calculated earlier is equal to F.

F is nothing but my times 50G cosine of 30 degrees and + I have 50 G sine of 30 degrees.

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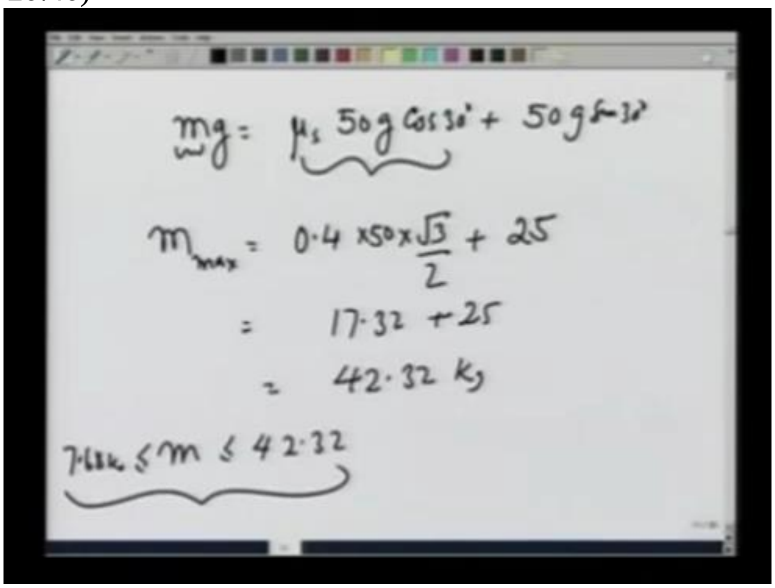

And therefore if I put in the numbers, numbers and getting as MG is equal to myu S 50 G cosine of 30 degrees + 50 G sine of 30 degrees. This is the maximum possible friction. So this gives me maximum possible mass M that will keep the whole thing in still in equilibrium. So M max is going to be myu S which is 0.4 times 50 cosine 30 which is square root of 3 over 2 + 25 which gives me 17.32 + 25 or 42.32 kgs. So for M less than 42.32 kgs and greater than 7.68 kgs, in this range the system would remain in equilibrium.