

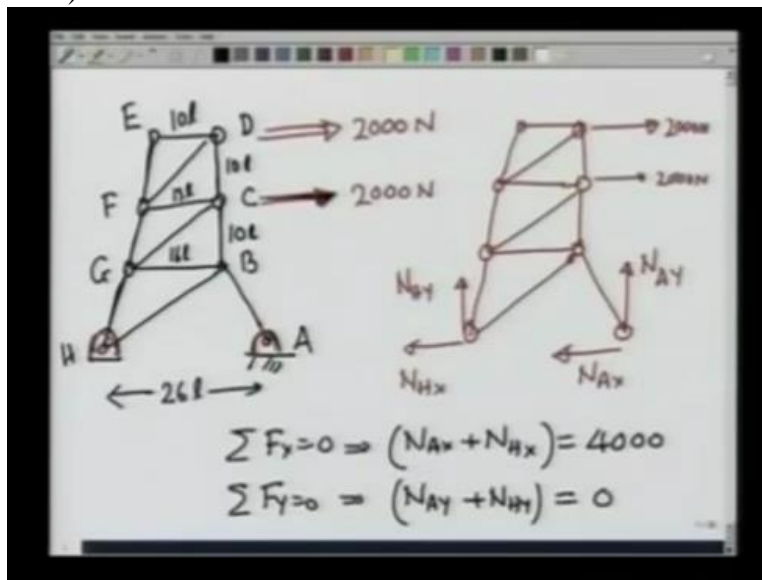
Engineering Mechanics
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Module 02
Lecture No 23

Plane trusses V-Solved examples calculating forces in a simple truss by method of joints

As a final example of the method of joints, I am going to solve a problem again based on a problem from the book of Merriam on statics.

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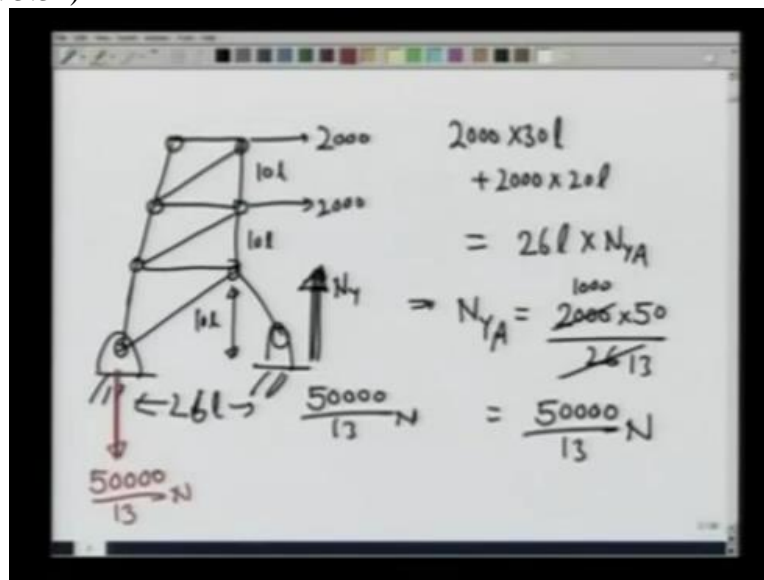
This is a truss like this where the lower point here is fixed. These are joined like this by rods and this point is also joined by rod and this point is then fixed here, point A. So this is a truss A, B, C, D, E, F, G, and H. Notice that these lines HG, GF and EF are slightly slanted, DC and BC are vertical. This is 10L in length. So is ED, 10L. This is 13 L, FC is 13 L, GB is 16 L and this length is 26L. The truss is loaded at points, see, let me make this in red at point C and at point B by 2000 newtons each.

And we want to calculate the forces on different members. Notice, since both points A and H are on fixed pins, this problem is statically indeterminate externally. Let us see how. So if I take this structure, then there is a vertical force here, let us call it N_{Ay}. And in anticipation, I am already

assuming, N_{AX} is going to be to the left. Similarly N_{HX} is going to the left and N_{HY} and there is this load, 2000 newtons, 2000 newtons.

Then the summation F_x equals 0 gives me $N_{AX} + N_{HX}$ equals 4000. Notice that I cannot determine from this equation, N_{AX} or N_{HX} separately. Summation F_y equals 0 gives me $N_{AY} + N_{HY}$ to be equal to 0. That means N_{AY} and N_{HY} are going to be opposite. The value of N_{AY} or N_{HY} can be obtained by applying the torque equation.

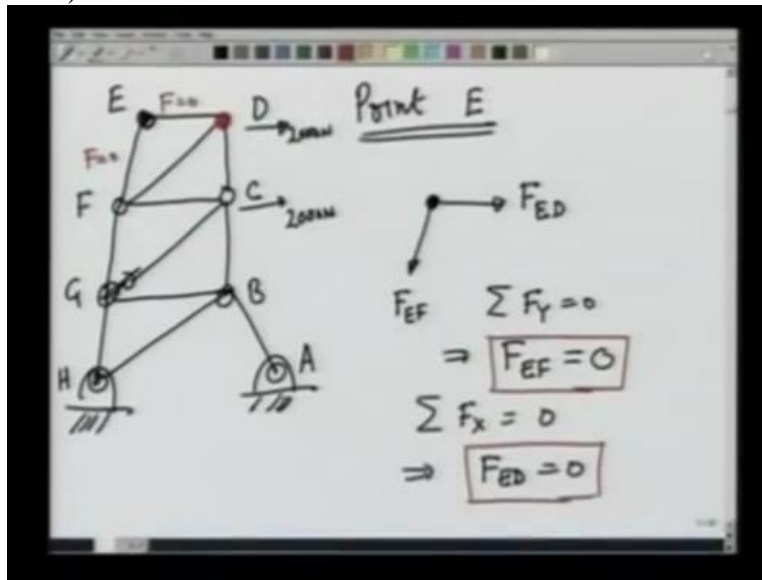
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And I am going to take the torque about point H. So if I apply torque about point H, this is 10 L, 10 L. This is also given to be 10 L. Then by torque equation I get 2000 times 30 L + 2000 times 20 L should be equal to this is N_Y , 20 6L times N_Y at A. And this gives me N_{YA} at point A equals 2000 times 50 divided by 26. That is 1000 13 equals 50,000 over 13 newtons in the direction shown.

So this force is 50,000 over 13 newtons. And from the Y force balance equation, the force out here, vertical force is going to be in the opposite direction 50,000 over 13 newtons. This is from the equation written on the previous page. What we are now interested in is, can we determine the forces on different members of the truss? And this is what we are going to do next.

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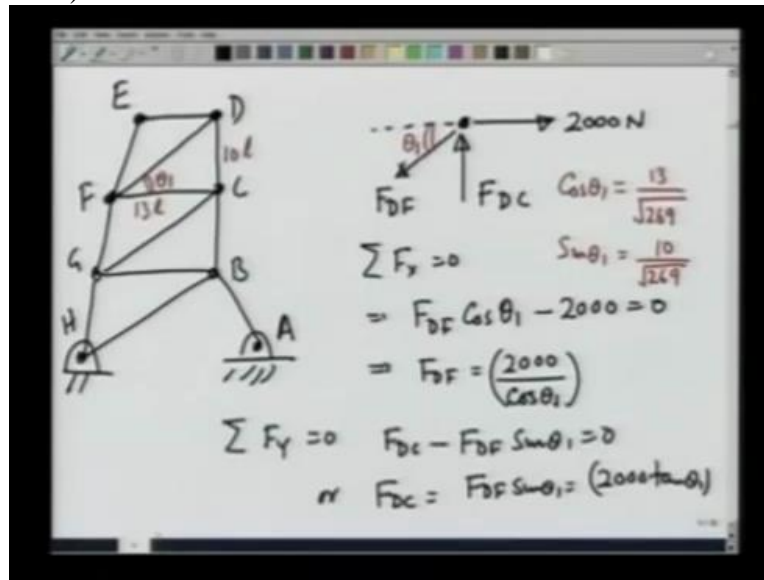


So let us look at this truss which is like this. I am sorry this is, I took this point. This is vertical. This is pin. Now this is A, B, C, D, E, F, G, H. Apparently an indeterminate problem but let us see if we can solve for forces in different members. To start with, let us look at point E. And I am choosing point E because from there we can solve, start solving things in a very very simple manner. At point E, there are only two forces.

This is of course loaded with 2000 newtons each. There is a force, let us call it F_{ED} and there is a force F_{EF} . Right away if I do summation F_y equals 0, this gives me that F_{EF} sine of whatever that angle is, should be 0 and therefore F_{EF} is 0. If F_{EF} is 0, then summation F_x equals 0 again implies that F_{ED} is 0. So starting from point E, I get 2 answers right away that the forces in the members F_{EF} and F_{ED} are 0. This force is 0, this force is 0.

Now if I go to point D here, there are only 2 unknown forces here, F_{DC} and F_{FD} . And therefore I can determine both from the 2 force balance equations. Let us do that.

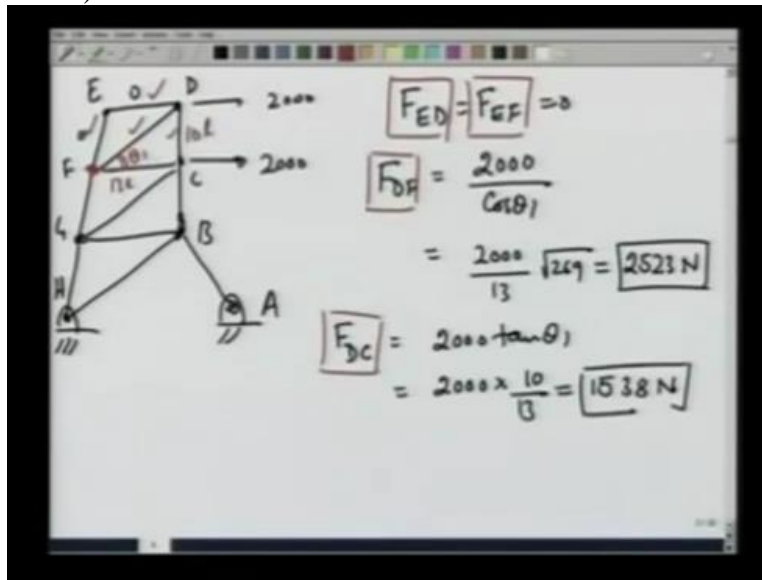
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So if I go to point D, let me again make the structure. Point D, A, B, C, D, E, F, G, H. At point D, FED is 0 and there is only one load, 2000 newtons. There would be a force FDF and are vertical force. Again in anticipation, I am making it like this, FDC. There are 2 unknowns and both of these, I can determine. Let this angle be theta and that will be the same angle as this. This is 10 L, this is 13 L. Let me in fact call it theta1 because I am going to require other angles later.

By summation Fx equals 0 I get FDF cosine of theta1 - 2000 equals 0 and that gives me FDF equals 2000 over cosine of theta1. Notice that cosine of theta1 from here is going to be 13 over square root of 269 and sine of theta1 is going to be 10 over square root of 269. Once I get FDF, I can also calculate FDC from the condition that summation FY is equal to 0 and that gives me FDC - FDF sine theta1 equals 0 or FDC equals FDF sine theta1 which is 2000 tangent of theta1. Let us calculate these numbers.

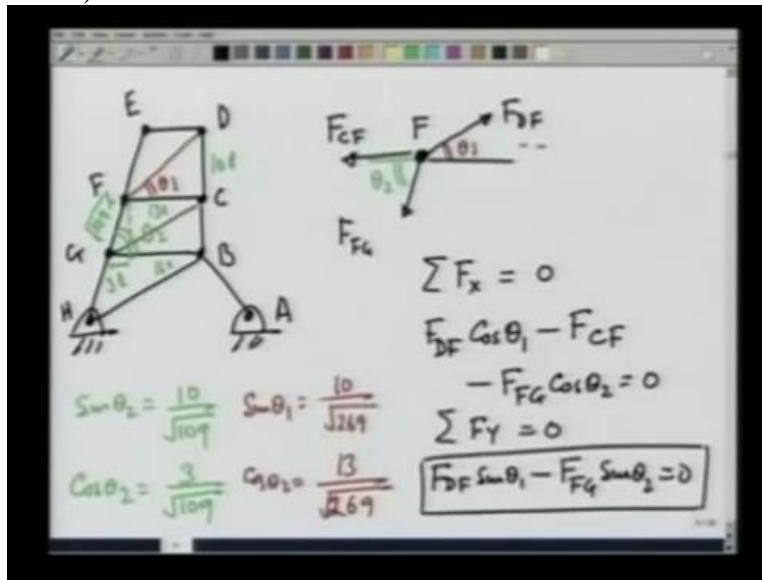
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So the numbers that we get are let me make the picture again. 2000, 2000, 0, 0, A, B, C, D, E, F, G, H. So what we of trend is that FED equals FDF equals 0. We obtain FDF equals 2000 over cosine of theta. Let me remind you, this is theta, this is 10 L, this is 13 L. So which is equal to 2000 over 13 square root of 269 and FDC, DC comes out to be 2000 tangent of theta which is 2000 times 10 over 13 which comes out to be 1538 newtons.

Similarly, FDF comes out to be 2523 newtons. So we have found the for forces. F let me box them, FED, FEF, FDF, FDC. We have found this, we have found this, we have found this, we have found this. Now we can move on to point F because on point F now I know 2 forces FFD and FEF. And 2 unknown forces are going to be FFC and FFG. So with 2 equations, I can find these forces also. Let us do that next.

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So to calculate the forces on F, let us balance the forces on F. Let me make the truss again. A, B, C, D, E, F, G, H and I have already shown you how these are the other lines. This is angle theta1. If I look at point F, it has a force EF which is 0. It has force , do I call it FD or DF? Let me see. I call it DF. FDF acting at an angle theta1 then this has a force and in anticipation I am going to make the direction down FFG and let us call this angle at which it is acting, theta2.

And therefore that angle is going to be this angle theta2. And we will calculate the sine and cosine soon. Then in anticipation again, let me just anticipate that the force FCF is going to be towards the left. So this point F is in equilibrium under these 3 forces, FDF, FCF and FFG. To calculate sine and cosine of theta2, let me just drop a perpendicular from here. Recall that this is 10 L, this line is 13 L, this is 16 L. So therefore this portion is going to be 3L and this length FC is going to be 109L.

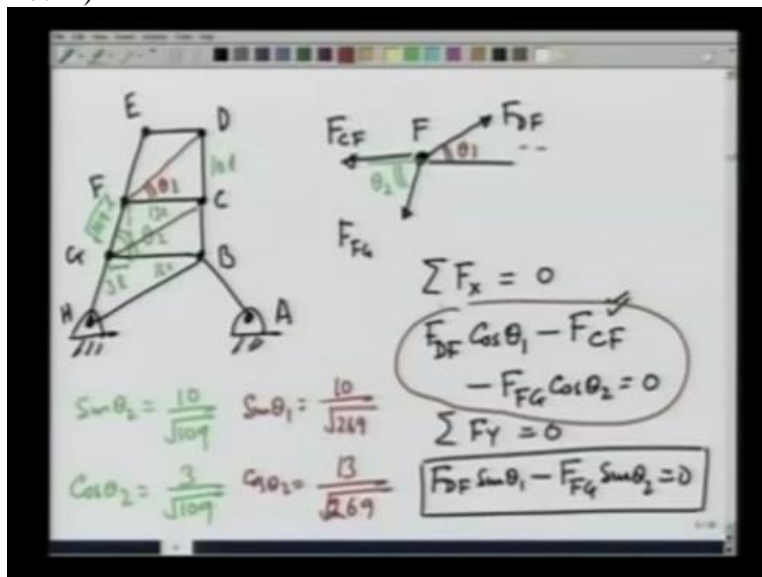
And therefore I can write, sine of theta2 is going to be 10 over square root of 109, cosine of theta2 is going to be 3 over square root of 109 and we have already seen that sine of theta1 is 10 over 269 and cosine of theta1 is equal to 13 over square root of 269. Now, summation FX at F equal to 0 gives me FDF cosine theta1 - FCF - FFG cosine of theta2 equals 0. Similarly, summation FY equals 0 gives me FDF sine of theta1 - FFG sine of theta2 equals 0.

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$$F_{DF} = \frac{2000}{\cos \theta_1}$$
$$F_{DF} \sin \theta_1 = 2000 \tan \theta_1$$
$$F_{FG} = \frac{F_{DF} \sin \theta_1}{\sin \theta_2} = \frac{2000 \times \frac{10}{13} \times \frac{\sqrt{109}}{10}}{10}$$
$$= 1606 \text{ N}$$

From the 2nd equation, I can find FFG right away. Recall that FDF was 2000 over cosine of theta1 and therefore FDF sine theta1 is 2000 tangent of theta1 and FFG is going to be FDF sine theta1 over sine theta2. That we saw from this balance equation which is going to be 2000 times 10 over 13 times square root of 109 divided by 10. And that comes out to be 1606 newtons. So you have found FFG also.

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Handwritten mathematical derivation on a whiteboard:

$$F_{DF} = \frac{2000}{\cos \theta_1}$$

$$F_{DF} \sin \theta_1 = 2000 \tan \theta_1$$

$$F_{FG} = \frac{F_{DF} \sin \theta_1}{\sin \theta_2} = \frac{2000 \times \frac{10}{13} \times \frac{\sqrt{109}}{10}}{\frac{3}{\sqrt{109}}} = 1606 \text{ N}$$

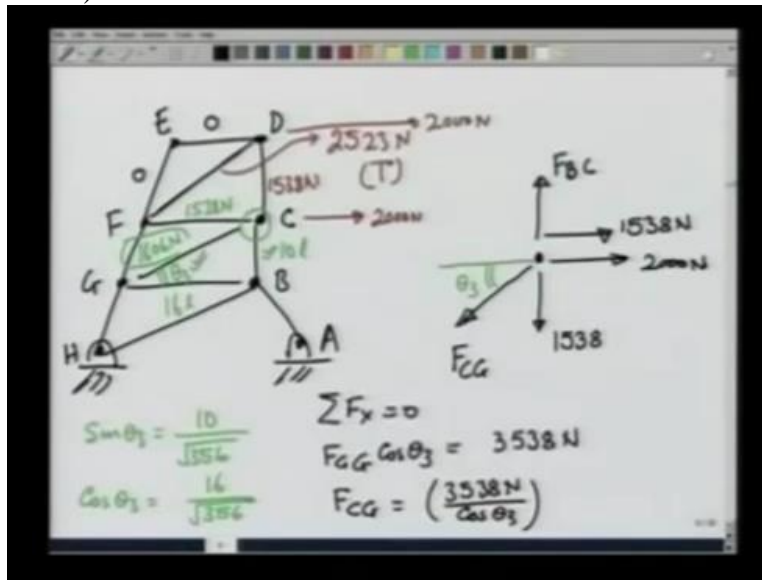
$$F_{CF} = +F_{DF} \cos \theta_1 - F_{FG} \cos \theta_2$$

$$= 2000 - 1606 \times \frac{3}{\sqrt{109}} = 1538 \text{ N}$$

Having found FFG, we can now calculate FCF also because FCF then is going to be FCF from the previous equation if you look at this equation is FDF cosine theta 1 with a - sign + FFG cosine of theta 2. We plug in the numbers. FDF cosine theta1 we already know is 2000. So this is, did we get the sign right? Let us check that. We have + and -.

So this is + and this one is - so that the direction that we have anticipated is already correct. If we plug in the numbers, this comes out to be 2000 - FFG is 1606 times cosine of theta2 which is 3 over root 109. And if you calculate this, it comes out to be 1538 newtons. So let us see what all forces have be found in the truss?

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This is a truss. We have found this force to be 0, this force to be 0. We have found this force. Let me put it in red. We have found this force, let me just write this 1st A, B, C, D, E, F, G, H. So we have found this force FFD to be 2523 newtons. This is of course loaded with 2000 newtons this way and 2000 newtons this way. And this force is tensile that pulls D in. Similarly, now we have found FDC, this force which is 1538 newtons.

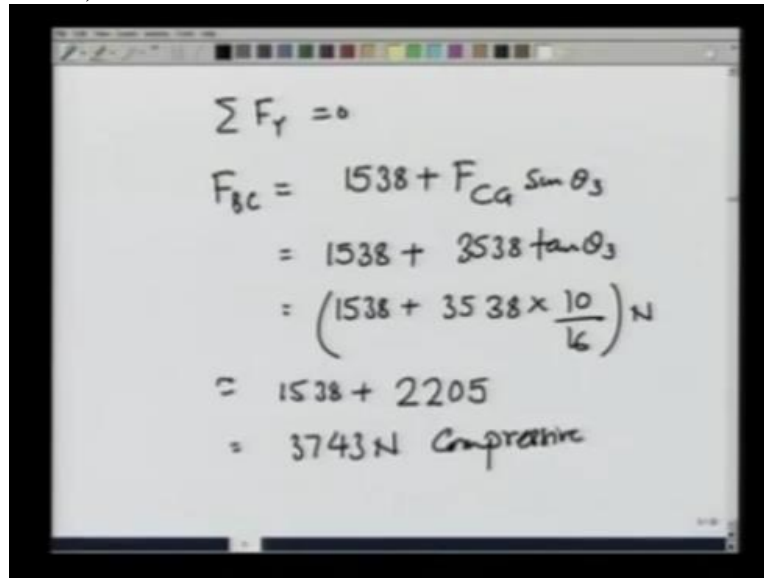
We have found FCF which is also 1538 newtons and we have found FFG which is 1606 newtons. Similarly now I can go on. Now you see, if I look at point C, they are the 2 forces 1538 and 1538 newtons are known here and therefore I can calculate FCG and FCB by balancing forces on point C. So let us balance the forces on point C. On point C, there is a lot of 2000 newtons acting this way.

FFC is a compressive force because it was pushing point F outwards. So here also, it pushes point C this way 1538 newtons. Similarly, FCD which is a force which was pushing point D up is going to push this point C down with 1538 newtons. Then in anticipation, we are going to have FCG acting in this way and FBC acting up. The 2 unknown sources here are of course FBC and FCG.

Let us look at these lines. This length is 10 L, this is 16 L, so if I look at this angle and let me call it theta3 I am going to have FCG CG acting at an angle theta3 with sine of theta3 equals 10 over square root of 356 and cosine of theta3 equals 16 over square root of 356. Now we are ready to

calculate F_{BC} and F_{GC} . So let us do summation F_x equals 0 which gives me F_{GC} cosine θ_3 equals 3538 newtons. And therefore, F_{GC} is equal to 3538 newtons divided by cosine of θ_3 . I can plug in the numbers and get my answer.

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The image shows a whiteboard with handwritten mathematical steps. The first line is $\sum F_y = 0$. The second line is $F_{BC} = 1538 + F_{CG} \sin \theta_3$. The third line is $= 1538 + 3538 \tan \theta_3$. The fourth line is $= \left(1538 + 3538 \times \frac{10}{16} \right) N$. The fifth line is $= 1538 + 2205$. The sixth line is $= 3743 N$ Compressive.

Once I have known that, I can also calculate F_{BC} by saying that summation F_y equals 0 and that in this case gives me F_{BC} equals $1538 + F_{GC}$ sine θ_3 which is $1538 + 3538$ tangent of θ_3 which is nothing but $1538 + 3538$ times $\frac{10}{16}$ newtons. This is approximately $1538 + 2205$ which is 3743 newtons and it is pushing point B up and therefore this is compressive. So we have calculated forces up to GC and BC. If I know other angles, I can go further and calculate all the forces in all the members.