Engineering Mechanics
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Module 1
Lecture No 2
Addition and subtraction of vectors

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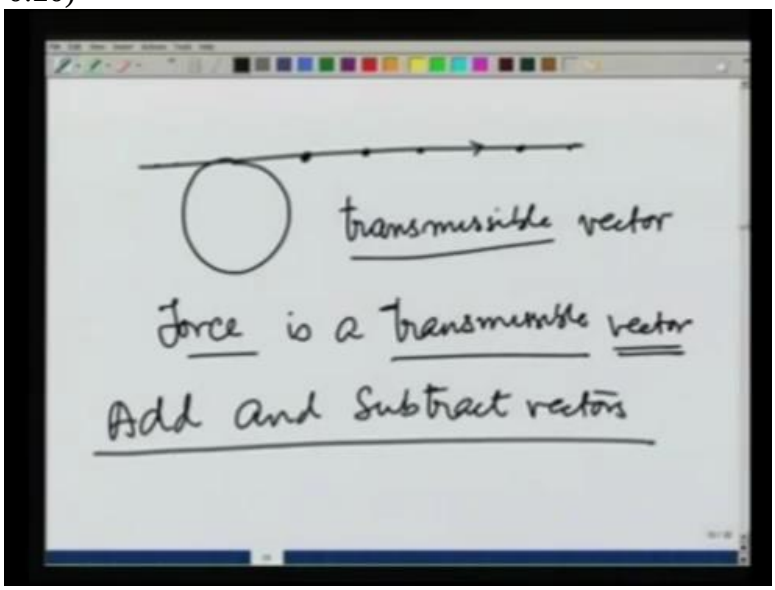

Having talked about equality of vectors and transmissibility of vectors, let us now see how do we add and subtract vectors using our graphical method. This is more or less a review because I am sure you have learned this in the 12^{th} grade.

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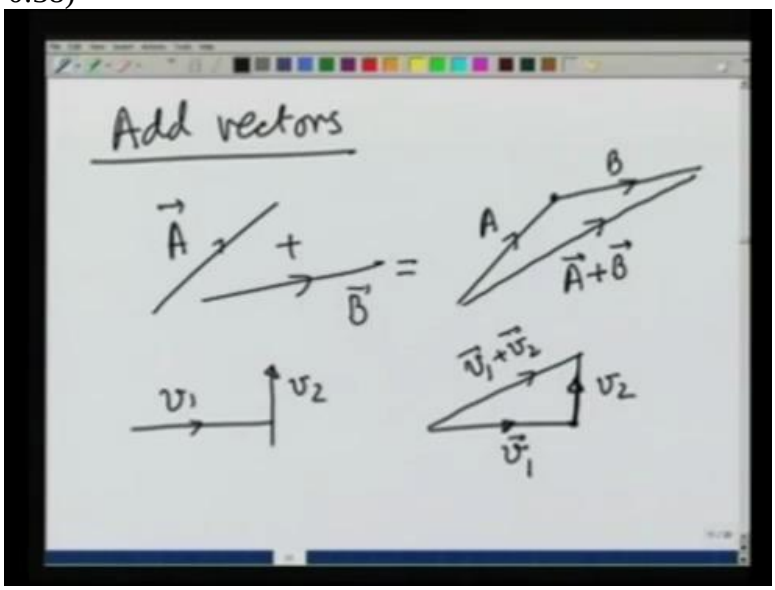

So if I want to add two vectors, A and B, what we do is, so we are doing A + B. What we do is, take A, on its head put the tail of B and then draw a vector starting from the tail of A to the head of B. And that gives you A + B. You may ask, why do I add this way? Does it make sense? It certainly does. Suppose there is a ball moving like this with velocity V1 and I hit it so that it acquires a velocity V2 in this direction.

Experience tells me that it will be moving in a direction like this which would be a sum of V1 + V2. You see, I have added V1 and V2 using the prescription I just gave you. I draw V1 and put the tail of V2 on the head of V1. Draw this vector and this gives me V1 + V2. So this is how we define the addition or we obtain the addition of vectors.

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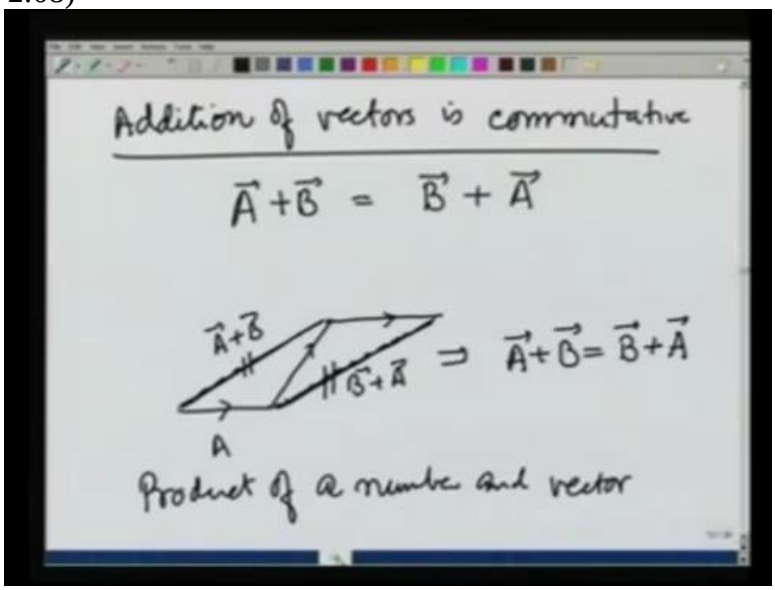

It is also easy to see that addition of vectors is commutative. That is, A + B is same as B + A. Let us see that. So let there be a vector A, let there be a vector B then this is A + B. On the other hand, if I draw the vector A with its tail at the head of B, then this would give me B + A. But this arm is parallel to this because this is the parallelogram. So these 2 vectors are parallel.

And because this is a parallelogram, this magnitude is same as this and this implies A + B is the same as B + A. So summation of 2 vectors is commutative. You see, graphical method is giving you a very visual way of looking at vectors. Next, we can use addition of vectors to define the product of a number and vector.

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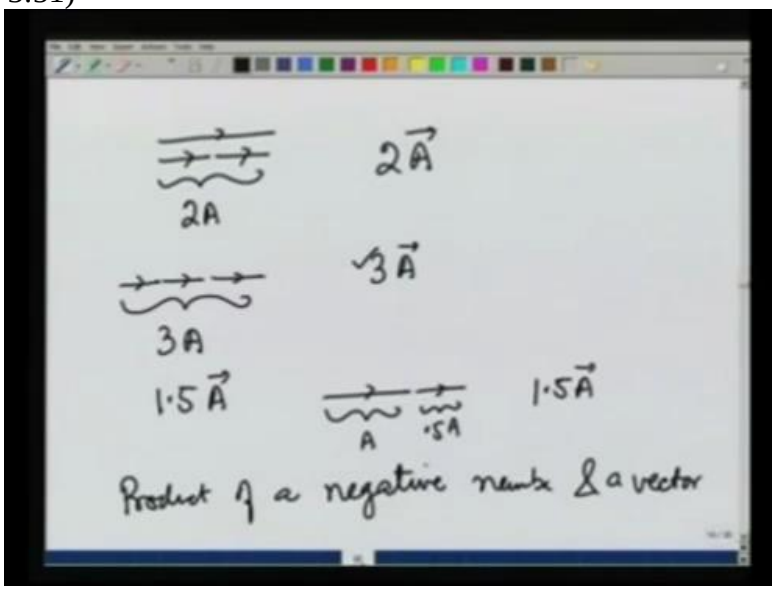

For example, if I bring A and add A to it, you can easily see that the left becomes double. Therefore its magnitude is 2A. And since it is in the same direction as A, I can write this as 2 times A vector. Similarly, if I add 3 vectors, the magnitude will become 3 times A. And it is the same direction as A. Therefore I can write, this is 3A.

So this defines multiplication of a number with the vector. It is nothing but adding that vector that many times. How about 1.5 A? 1.5 A would be A + half the magnitude but in the same direction. So this would be A. This would be 0.5 A and their addition gives me 1.5 A. Can I multiply a vector by negative number? The answer is yes.

So product of a negative number and a vector, recall from your school days, what does multiplying by a negative number mean?

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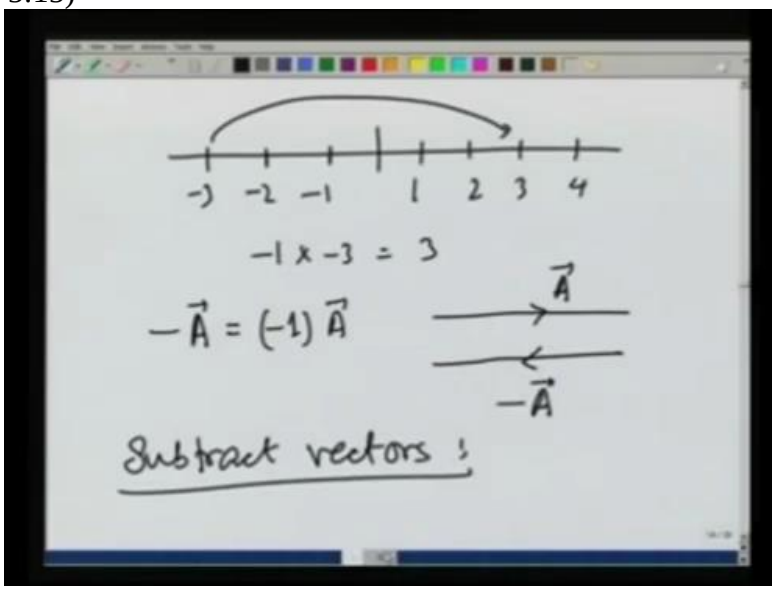

If I go to number line, this is 1, 2, 3, 4. - 1 is in the other direction, - 2 is also in the other direction, - 3 is in the other direction and so on. If I multiply - 3 by - 1, it becomes 3. So - 1 times - 3 becomes 3. What the multiplication by a negative number is doing therefore is changing the direction. Similarly if I take negative A, it will be - 1 times A. This would be nothing but change in the direction of A.

If A is this, - A would be a point in the other direction. That is the meaning of multiplying by a negative number. Of course, once you define multiplication by - 1, you can define multiplication by - any number. You just make it that many times more. Having obtained - A, it is now very straightforward to subtract vectors. So how do you do subtraction of vectors?

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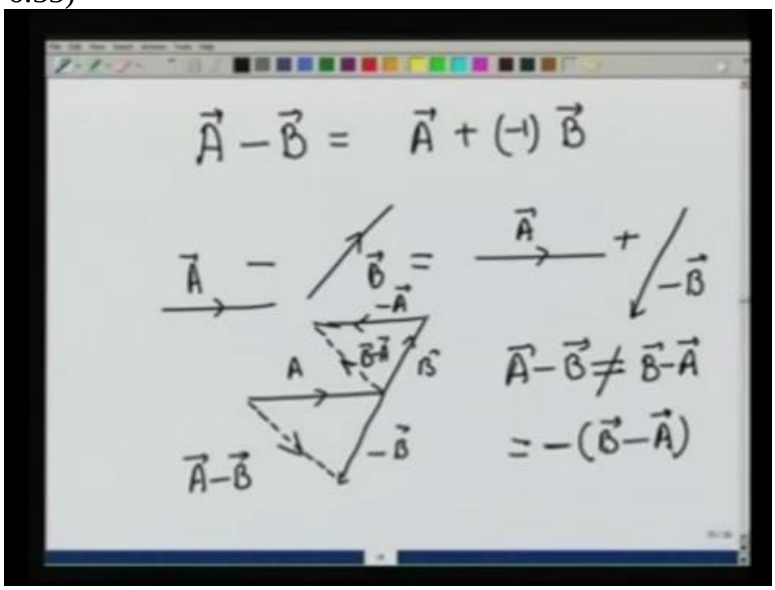

Suppose I have to subtract B from A. This would be same as A + - 1 times B. So what do we do? We take vector A and I want to subtract B from it. This would be the same as A + - 1 times B. So what do we do? We take vector A and I want to subtract B from it. This would be the same as taking A and adding - B to it. So A - B would be A, - B would be this.

This would give me A - B. And you can see right away that A - B is not same as B - A because B - A would be B - A. - A would be opposite to A. Therefore this would be like this. This is B - A. So you can see that A - B is not the same as B - A. But through graph, you can see very easily that A - B is equal to - of B - A.

So what we have done is addition and subtraction of 2 vectors. Similarly if you have to add 3 vectors, you can do A + B. Then add 3rd one to it. If you have to do 4 vectors, you can do A + B, C + D, add the 2 resultants again and so on. Let us do 2 examples showing the summation of 2 vectors and subtraction of 1 vector from the other.

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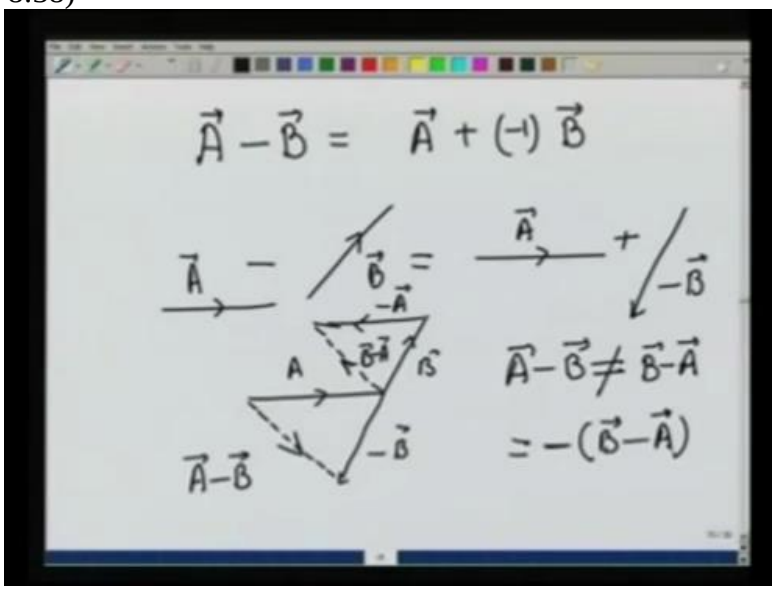

So example 1, suppose there is a person who walks 300 m to the east and then 400 m to the north. We want to ask, what is the total distance he covered? This is obviously 700 m and then we ask, what is the vector displacement which is the vector quantity of the person from the original position? To get that, we have to add the 2 displacements vectorally. So suppose the person started from here.

He moved 1, 2, and 300 m to the east and then 1, 2, 3, 400 m to the north. So this is his 1st displacement up to this point and then 2nd displacement from here to here. And therefore, the net displacement would be in this direction. This is 400, this is 300. Therefore, this would be 500. So net displacement of the person is 500 meter and in which direction?

At an angle Theta where Theta is going to be tangent inverse of 4 over 3 from the east towards north. So this is his net displacement. This is how you would find the addition of 2 vectors and which gives you the magnitude and its direction.

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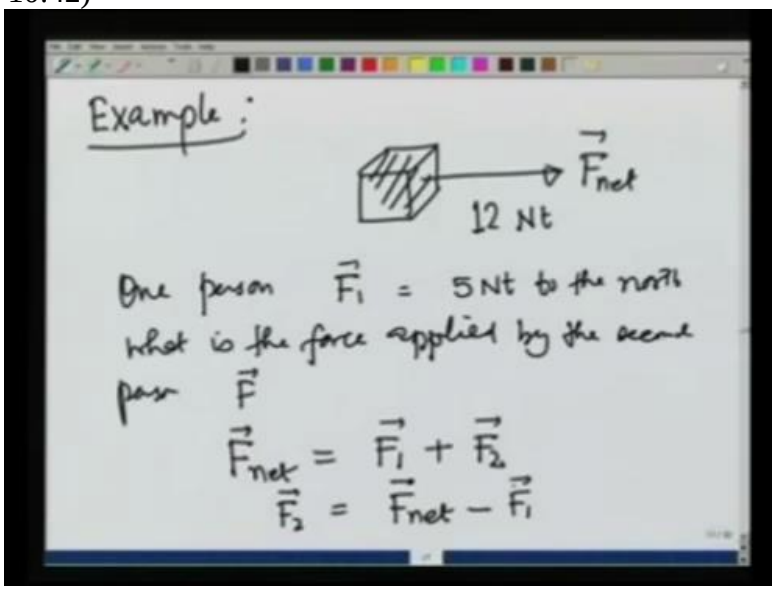

The 2nd example, let us take the case of a, 2 persons pushing a box so that the net force F net is let us say 12 newtons to the right or to the east. One person applies a force, F1 which is 5 newtons to the north. We ask, what is the force applied by the second person? Let it be F2. Then we know that F net is going to be equal to F1 + F2. And therefore, F2 is equal to F net - F1.

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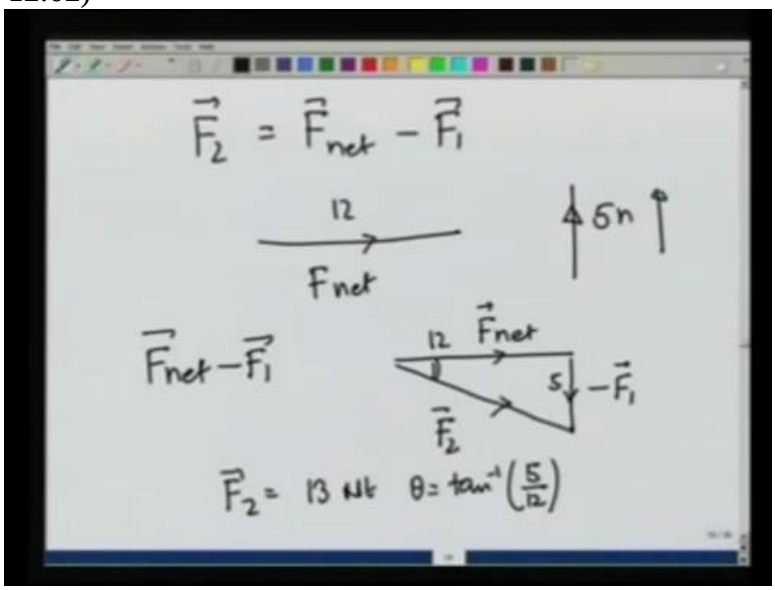

F2 is equal to F net - F1. We have been given that F net is 12 newtons to the east. F1, this is F net. F1 is 5 newtons to the north. And therefore, F net - F1 is going to be, this is F net - F1, we make going to the South and this would give me F2. You can see that F2 is going to be 13

newtons because this is 12, this is 5. At an angle Theta tangent inverse 5 over 12 towards South from the east.

So I have given you 2 examples. One in which we added 2 vectors and one in which we subtracted 1 vector from the other.