Engineering Mechanics
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Module 02
Lecture No 19
Plane trusses I: building a truss and condition for it to be statically determinate

We have been looking at equilibrium conditions under static positions of bodies.

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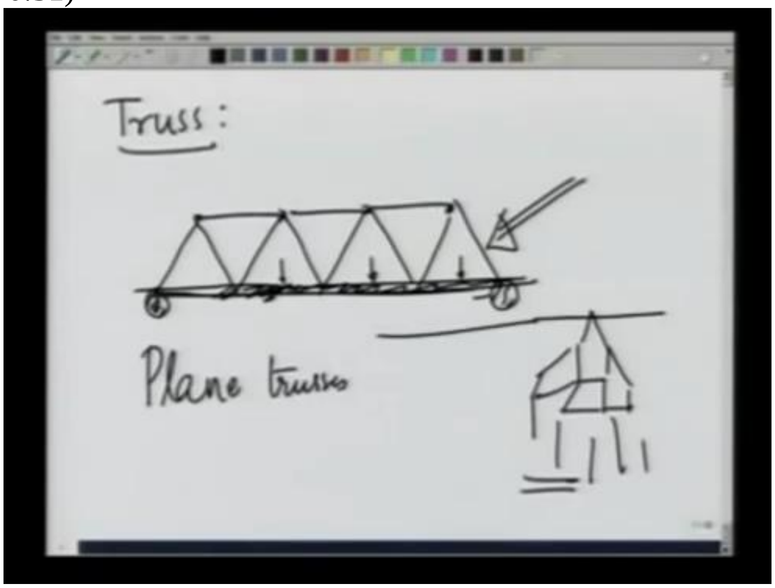

We now wish to apply this to a particular structure called truss. These are structures that are used to distribute loads and you have seen them when you travel. For example, across all this, you see a structure like this. And here is the road. This structure helps to distribute whatever load there is on the road and takes it to the sides here. This is known as a truss.

And in this lecture we wish to analyse the trusses using equilibrium forces that we have learnt earlier. Another example of truss that we see around is the pillars that carry electric wires. They have this kind of structure and the wires are going like this. These are also trusses. In this case, the structure is essentially a two-dimensional structure. So these are known as plane trusses and these are three-dimensional trusses.

In this lecture, we will be focusing on plane trusses. Before we start on trusses, I would like to remind you of a certain things using the force and torque balance condition that we have learnt earlier. So let us see what happens when there are only 2 or 3 forces acting on a system.

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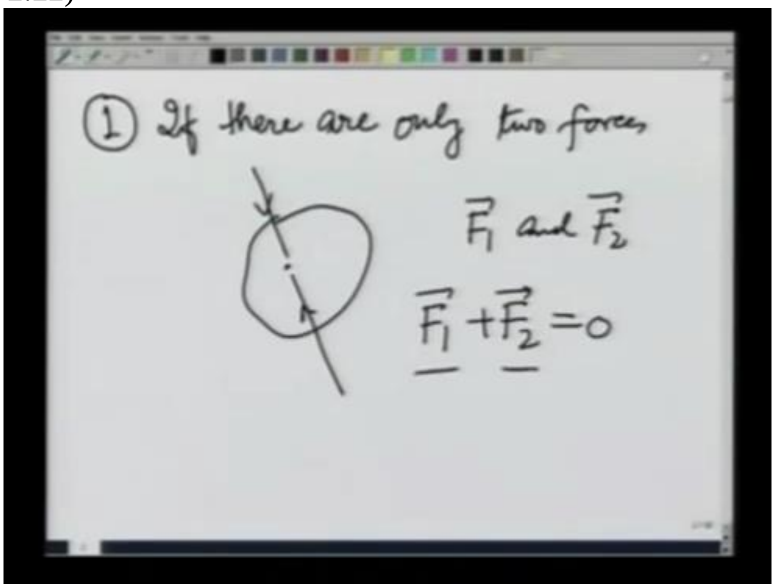

Number 1, I am going to look at if there are only 2 forces. If a body is in equilibrium under 2 forces F1 and F2, then for body to be in equilibrium, the forces must be co-linear and opposite so that F1 + F2 is equal to 0. If they are co-linear than a tiny point, the Torque also vanishes if they are equal. So this is the condition for equilibrium.

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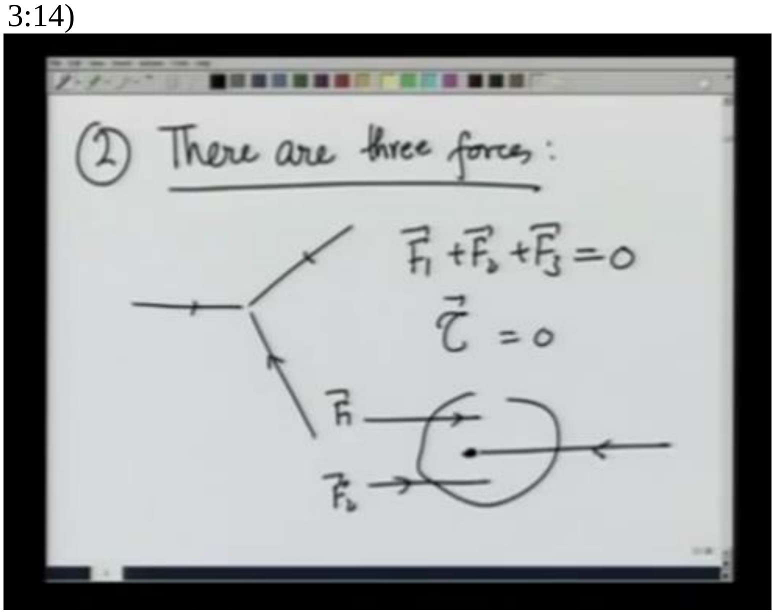

Next, if there are three forces, in that case the forces should either meet at a particular point so that F1 + F2 + F3 not only is 0, the Torque due to 3 forces is also 0. Another way they can provide equilibrium to the body is if they are parallel. So 2 forces can act in this direction and that 3rd force would act opposite. In that case, about this point for example, the torques would be in opposite directions due to F1 and F2 and they will balance out. This is something we are going to use later.

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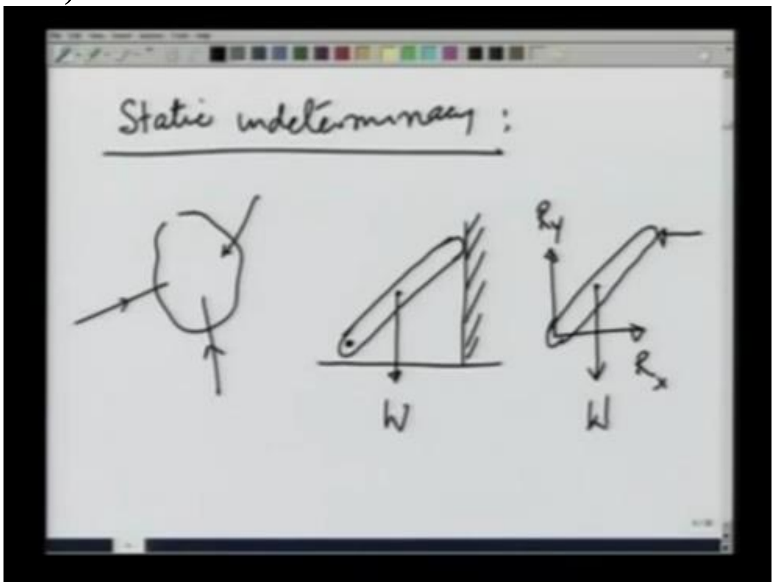

Next, a very important thing. Let me talk about a static indeterminacy. If there is a structure or a body and it is in equilibrium under certain forces and constraints which are holding it together then if the number of constraints are more than minimum required for it to be in equilibrium, than the system is known as statically indeterminate. If the number of forces, number of constraints are just the number that is required to keep it in equilibrium then the system is statically determinate.

In the case of statically indeterminate systems, the extra constraints that we put which may be removed without discovering the equilibrium is known as the degree of indeterminacy. This is all illustrated very well with an example. Let us take a rod which has a pin joint at the lower end and let us say it has a weight, W. For it to be in equilibrium under this weight, it will have to be supported on this side by say a wall. And that is enough to ensure equilibrium.

Because in this case if we look at the free body diagram of the rod, we have a force acting down which is its weight, there is a reaction in the Y direction due to the plane. One in X direction due to the plane and there is normal reaction on the wall. These forces are sufficient to bring this rod in equilibrium. Let us see how.

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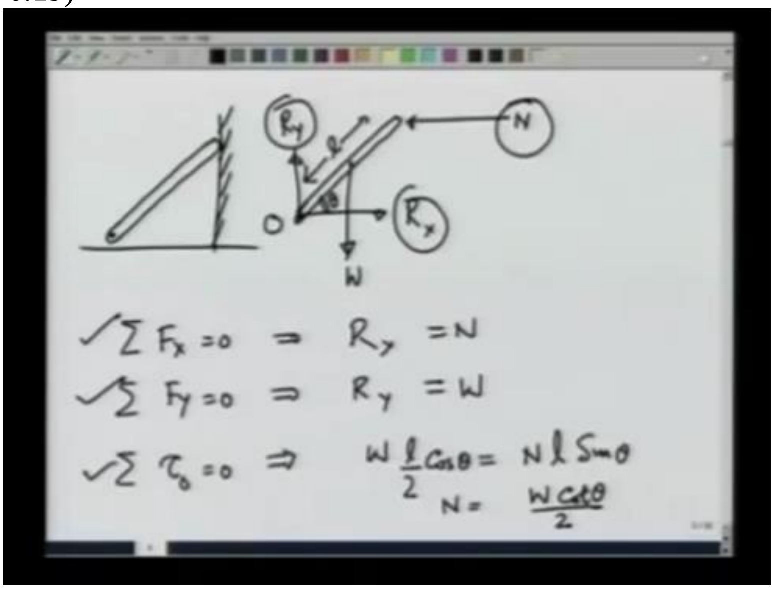

So I have this rod, a pin joint here and a wall here. This is a force RY, RX, normal reaction due to the wall and its weight. The equilibrium condition, summation FX equal to 0 gives RX equals N. Summation FY equal to 0 gives RY equals W. Now, about this point, the lower point let us say this pin middle joint is O, the Torque also must be balanced. If this angle is theta, the length is L, then we have summation torque about O is equal to 0 gives W times L over 2 cosine theta is equal to N times L sine theta or N equals W Cotangent theta divided by 2.

Notice that in this problem, I have 3 unknowns , N, RX and RY and precisely 3 questions, 1, 2 and 3. So I can determine each and every unknown exactly. This problem is statically determinate.

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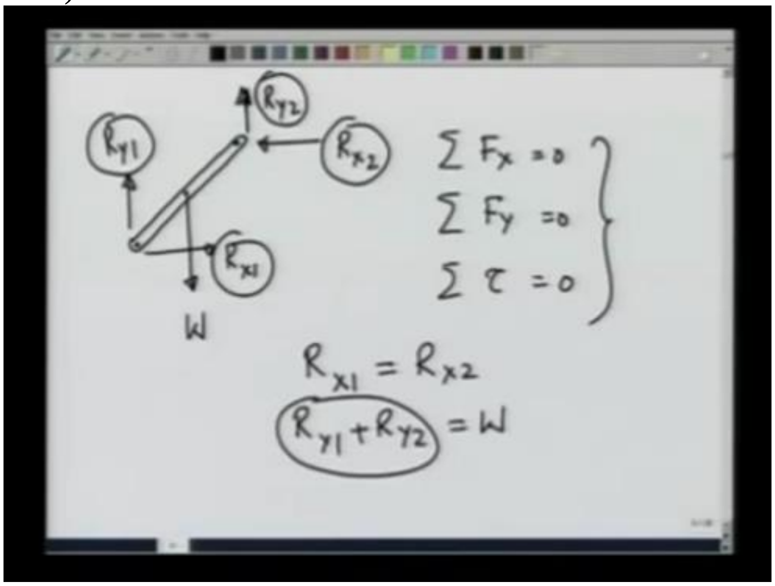

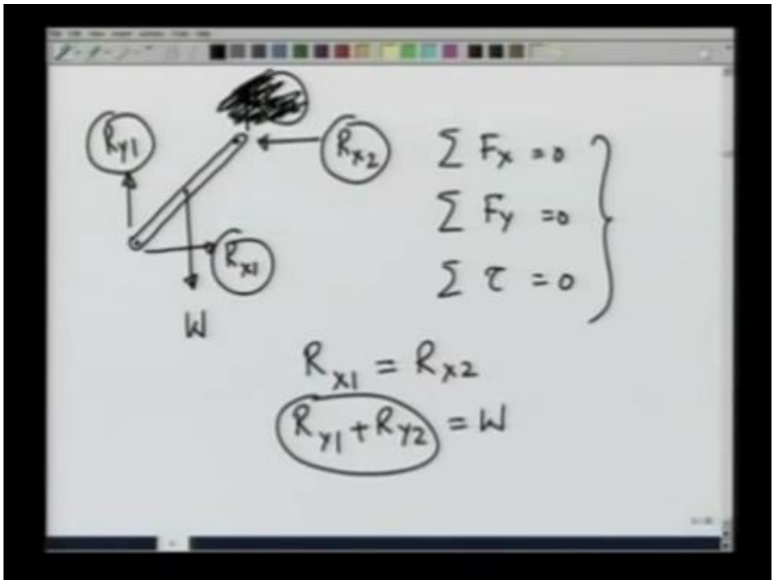

Let us look at indeterminate, corresponding indeterminate problem. If I take the same rod and put a pin joint here and also a pin joint here, in that case, this is the weight W. I will have RY1 and RX1 on the lower pin joint. RX2 and RY1 on the upper pin joint. In that case, you see that the number of unknowns are 4, RX2, RY2, RY1, and RX1. And number of equations are still 3. Summation Fx equals 0, summation FY equal to 0 and summation Tao about any point is 0.

So I am putting an extra constraints here by providing a vertical force in the upper plane also. The conditions I get are that RX1 is equal to RX2 according to the way we have shown directions. RY1 + RY2 equals the weight and 3rd condition is the Torque condition. You see, all I can determine through this is RY1 + RY2, not RY1 and RY2 individually.

And this is the degree of indeterminacy. So this is a statically indeterminate problem and degree of indeterminacy is 1 because I can remove this constraint of providing a vertical force and still keep the system in equilibrium.

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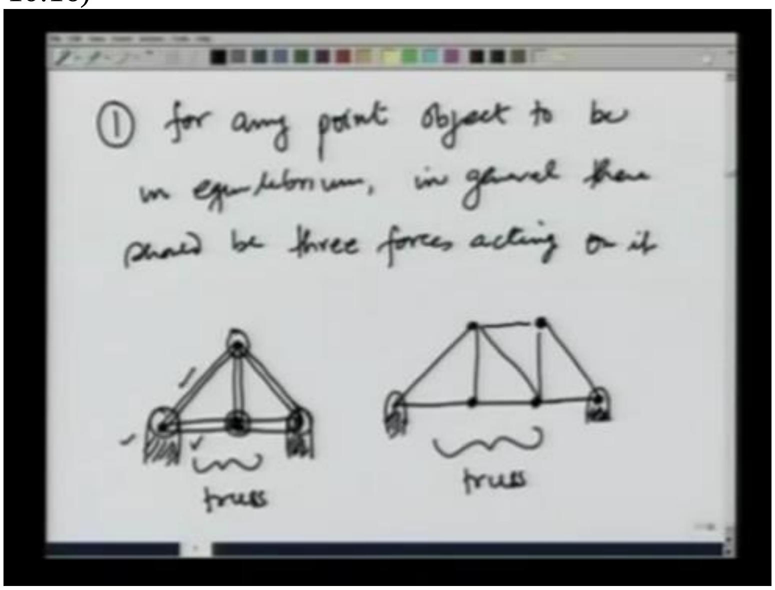

Now to motivate the trusses, I note 2 points. Number 1, for any point object to be in equilibrium, in general that should be 3 forces acting on it, minimum of 3 forces. And therefore, if I want to make a structure where each joint is in equilibrium, I should provide 3 forces there. There could be one external and 2 by something else or 3 members applying forces. For example, let us take a structure where I take a rod and make a pin joint at one end of it.

And suppose I want to load it at a particular point. Let me 1st make a structure of 3, taking 3 rods and provide a pin joint at the circles. Now you see, these pin joints has 3 forces acting on it, one by this member, one by this member and one external forces. So this can be in equilibrium. However, this joint and this joint are not acted upon by 3 forces. And therefore to make the structure, to make these points come under equilibrium, I have to provide more forces.

I could do that by providing one more rod here which provides 3 forces to this point and then make one rod here so that this point is also under 3 forces and make a joint here. However, the moment I do that, this joint is only under 2 forces. I have only 2 choices. Either provide a full support here or another choice and I will make it in a simple manner now. One joint here, one joint here, one joint here.

Here I earlier provided a fixed joint but now I will put another rod here, another member and make a fixed joint here. However, the moment I do that, this joint will not be in equilibrium. Because this is acted upon only by 2 forces, one by this fixed joint and other by this rod. So I have to provide one more member to make, to bring this point into equilibrium. The moment I do that, I have one free end of this member here which also has to be brought to into equilibrium.

I could provide one rod this way, one rod this way and his entire structure would be in equili would be capable of remaining in equilibrium. This is a truss, a plane truss. So is this. So you see if I keep making structures like these, I would provide at each point enough forces to keep that point in equilibrium. If I load this structure somewhere, I could provide, I could load it with some great here, I could load it here.

Each point is now, can be brought into equilibrium by forces generated in these members. These members would provide just the sufficient forces to bring this entire structure into equilibrium and in the process transfer this load onto the fixed supports. This is the job of a truss.

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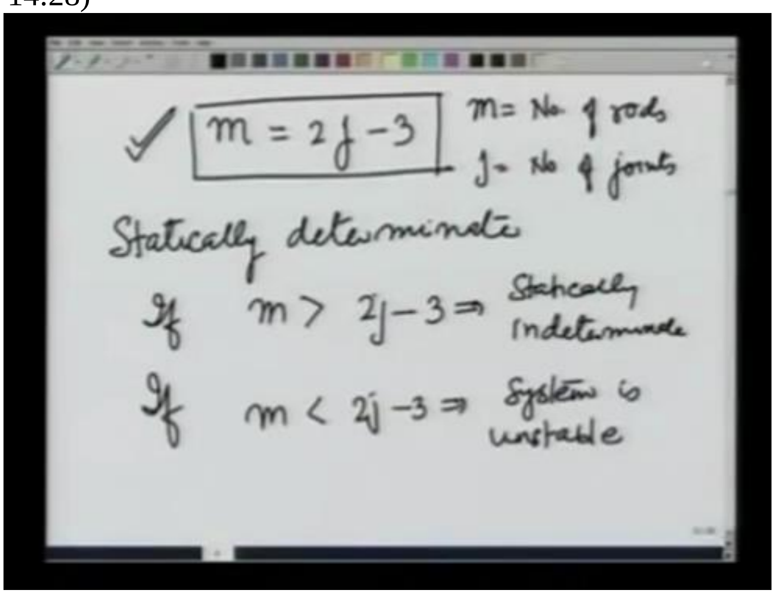

If you analyse it carefully, you will see that the number of members that I need to do this is equal to let me write the number of members equal to M is equal to 2J - 3. In this case, I provide just enough constraints in the system to make the system statically determinate. If M is larger than 2J - 3 where J is the number of joints and M is the number of members, let me write it here, M is equal to number of these rods let us say and J is number of joints, then the system becomes statically indeterminate.

I have provided more constraints than are needed. If on the other hand, so this is statically indeterminate. On the other hand if M is less than 2J - 3 then the system is unstable. I have not provided enough members or rods to provide enough forces to keep the system in equilibrium. And therefore, the system may collapse.

So to make it statically just right number to make a statically determinate, I need M members in a truss with M being 2J - 3 there J is the number of joints. It is understand how this condition comes about.

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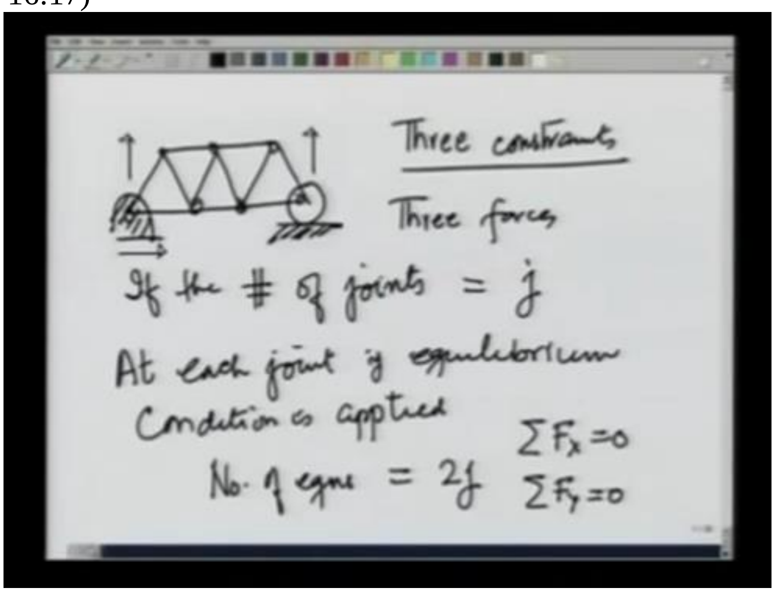

So if I take a truss which is externally statically determinate, that means it has 3 constraints from outside. 3 constraints could be in the form of 3 forces, 2 forces, 1 Torque or whatever. So let us take 3 forces. And then if the number of joints is equal to J, then at each joint if equilibrium condition is applied then we have number of equations equals 2J because at each joint since it is a point, there will be only two equations, summation Fx equals 0 and summation FY equals 0.

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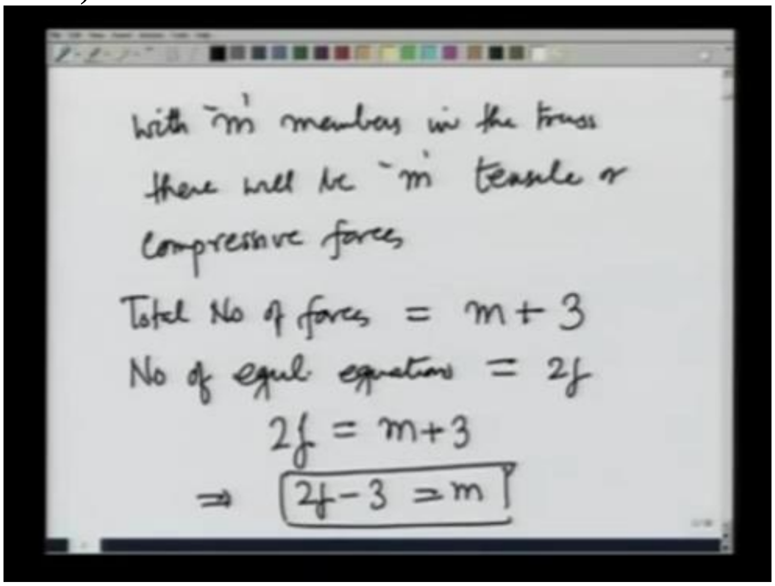

And as we saw earlier, with M members in the truss, there will be M tensile or compressive forces. So total number of forces is equal to M which are arising from the members of the truss + 3 external forces. And number of equations, equilibrium equations counting 2 for each joint is equal to 2J. If I should be able to solve for all the forces then the number of equations should be good to the number of unknowns with gives me 2J - 3 equals M and that is the equation for statically determinate trusses if the trusses are externally statically determinate.

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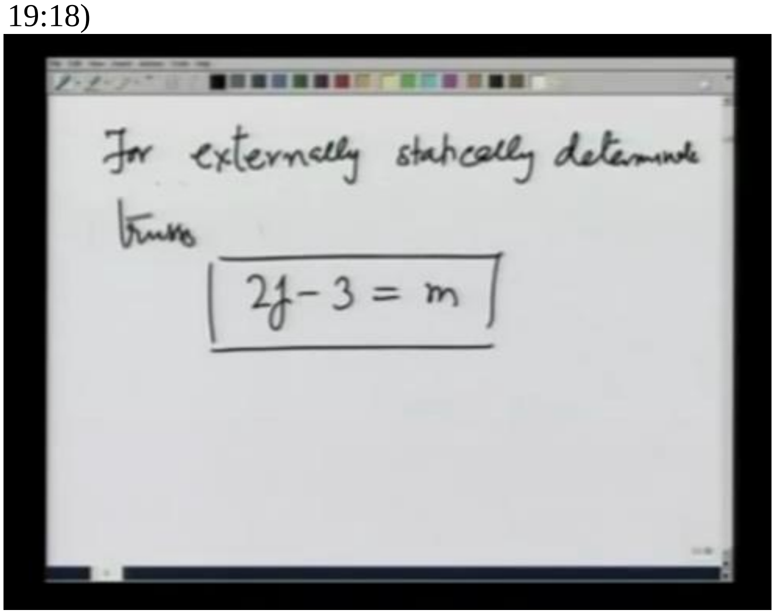

So for externally statically determinate trusses, 2J - 3 equals M and that is how we get this equation. Let us see the structures that I made earlier whether they satisfy this or not.

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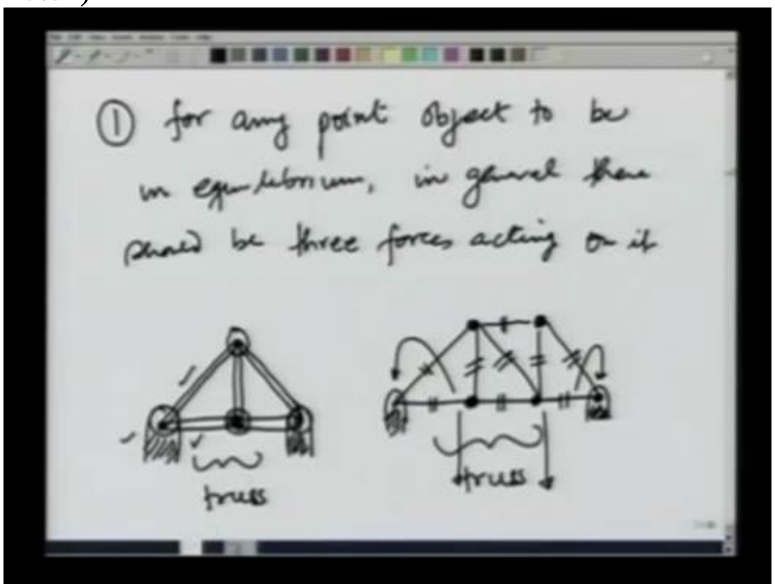

So let us see the 1st structure that I made was this one and 2nd one was this one. Let us see that.

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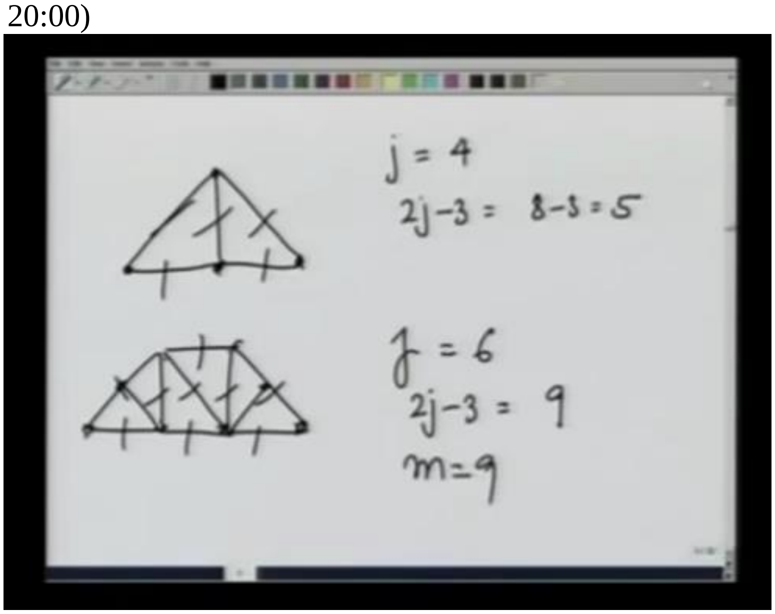

So the 1st to the time it was like this. Number of joints is equal to 1, 2, 3, 4. So 2J - 3 is equal to 8 - 3 which is equal to 5. And I have precisely 1, 2, 3, 4, 5 members. This is a statically determinate structure. The other structure that I made was like this. The number of joints here is equal to 1, 2, 3, 4, 5, 6. And therefore 2J - 3 is equal to 9. And I have 1, 2, 3, 4, 5, 6, 7, 8, 9 members. So this is also a statically determinate structure.

If I add more rods here, maybe like this, maybe like this, although I am increasing the number of joints so at some point it may become a system which is statically indeterminate. On the other hand if I have less members than 9, then it will be a structure which may collapse. It will not transfer forces in a easy manner or a consistent manner to the sides.

You notice that both the ends are put on a fixed point. Actually this makes the system indeterminate because the number of external forces is now 4. We could make it determinate by putting the right-hand side of the system on a wheel or on roller.