Engineering Mechanics
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Module 02
Lecture No 18
Forces in different geometric configurations

We considered a ball and socket joint.

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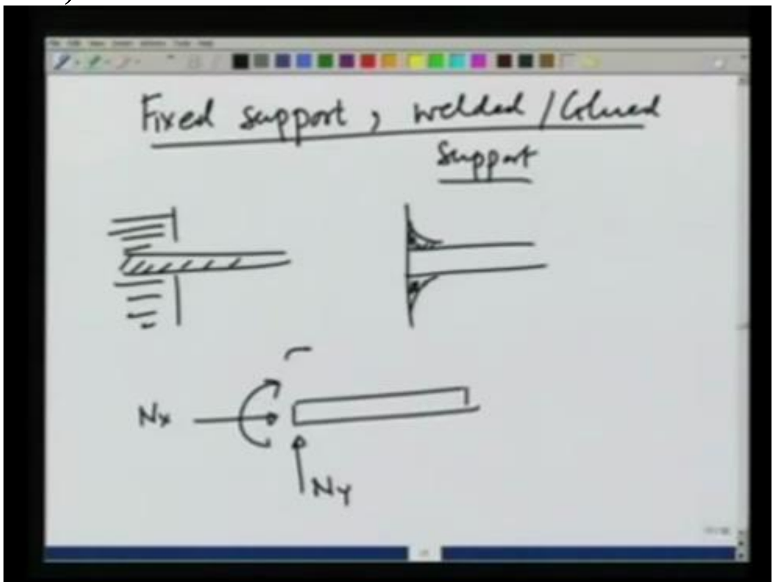

Recall now that in 2 dimensions, there is also a fixed support or a welded or glued support. Let me just remind you what it was. It was a support that was either built in a wall or it was something that was welded or glued here. And these supports could support or provide a force in X direction, in Y direction as well as a torque Tao. The three-dimensional generalisation of this is going to be again a beam either fixed in a wall or something which is glued or welded at its corners.

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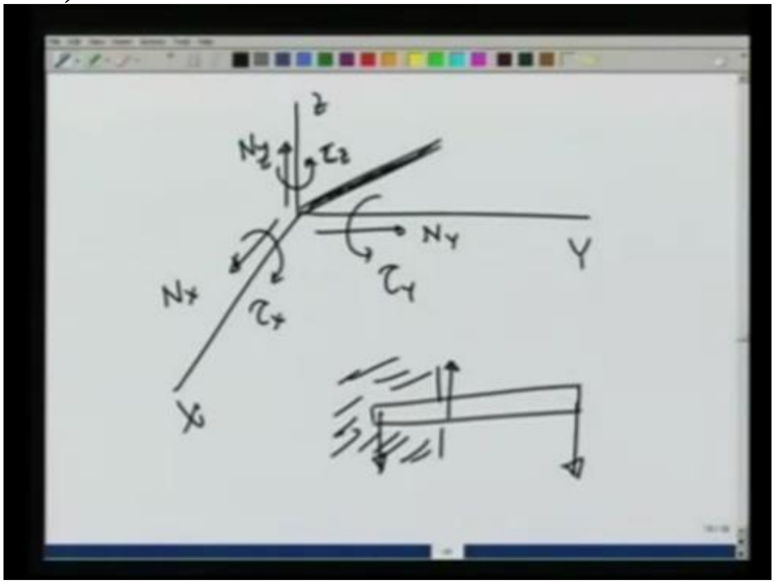

But now it can provide given X, Y and Z directions and suppose this is a fixed support, it can provide a force in X direction, it can provide a force in Y direction, and (inaudible 1:34) it can provide a force in Z direction. Since it is fixed in the wall, using arguments that we use in say 2 dimensions, it can also provide a torque about the Y direction and it can also provide a torque about the Z direction.

This would come out using arguments that we used earlier for the two-dimensional case. In that when we post a built-in support or fixed support in a wall like this, there were forces generated here. And these provided a torque and a couple and a force. You generalise it to 3 dimensions and you will get the answer that a fixed support in 3 dimensions can provide torques and forces along and about all the 3 axis.

Hopefully, by the analysis carried out so far, you would be able to now get an idea of what kind of force and what kind of torque can a new element, engineering element provide.

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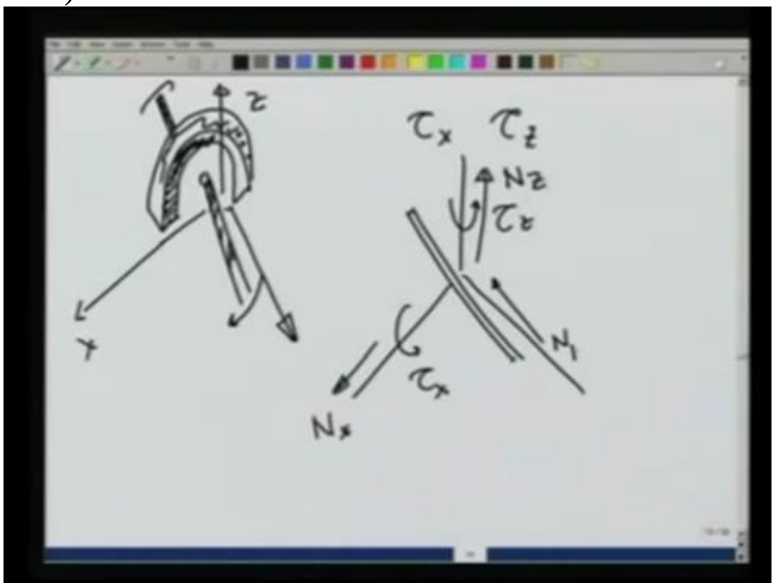

For example, suppose I were to give you a support like this with some thickness through which I put a shaft through a hole here. Now you will see, if I rotate the shaft like this, it cannot turn. If I rotate the shaft like this, it cannot turn. So if I take this to be X, this to be Y and this to be Z direction, let me be a little more careful, let me put Y perpendicular to the support like this. Then you can see that about the X axis, I cannot rotate it.

So the support gives a Tao X. About the Z axis I cannot rotate it. So it gives me Tao Z. However, I can rotate it about the Y axis. And therefore, this kind of support would give me on the shaft, a Tao X, a Tao Z but no Tao Y. On the other hand, I cannot push it along the X direction. And therefore, it gives me a normal reaction along the X axis. I cannot push it along the Y direction if the shaft is fixed.

So it gives me a, suppose I could sit along the Y direction, then there is no force in this direction. But I cannot push it along the Z direction. So it gives me a force in the Z direction. In addition if it were also fixed along the Y axis, if the thing if the shaft was completely fixed, then it would also provide a force in Y direction. If it was not free to rotate, it will also provide a torque along the Y direction.

So you can do analysis like this and find out what all a engineering element, what all kind of forces and torques can it tolerate or provide for equilibrium. Having talked about elements, let me not talk about a few cases of different forces that due to geometry, their particular geometry, give, make certain conditions, equilibrium conditions irrelevant.

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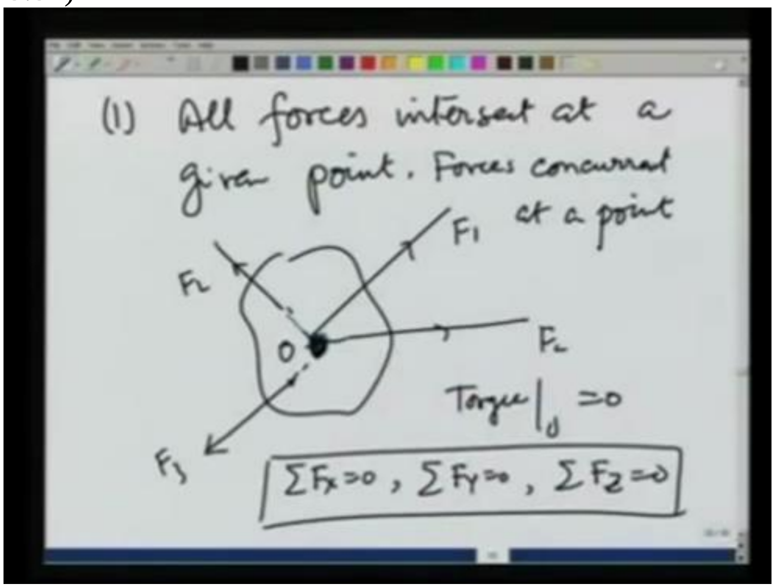

For example, let us take case 1. All forces intersect at a given point. So suppose there is a body and all the forces applied may not be applied at the same point. So F1, F2, F3, F4 but they all say intersect at a particular point. Let me show this point here, like this. They all intersect at this point. In that case, the condition that torques be 0 becomes irrelevant because if I take the torque about this point where they intersect, the torque about this point O would be 0 for all the forces.

And therefore the only equilibrium conditions I require are summation Fx is equal to 0, summation Fz is equal to 0 and summation FZ is equal to 0. The forces that intersect at a particular point are known as forces concurrent on a point. Forces concurrent on a point. So for forces concurrent at a point, the only equilibrium condition is summation Fx is equal to 0, summation FY is equal to 0 and summation FZ is equal to 0.

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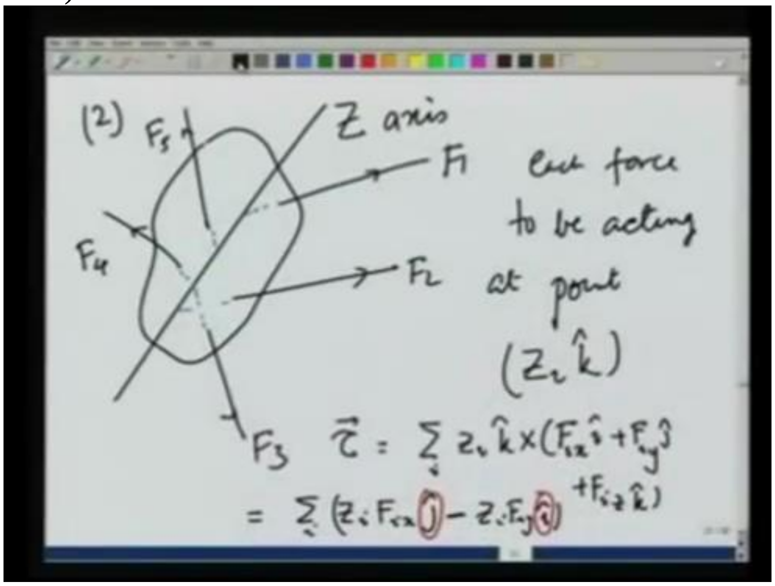

Let us take a look at another situation in which suppose the forces all intersect a particular line. No matter where they are applied but they all intersect a particular line and let us call that line the Z axis for convenience. That line can be arbitrarily chosen to be Z axis. So this is force F1, force F2, force F3, force F4, force F5 and they are all intersecting the Z axis or a given line at one point or the other.

Let us see what happens in this case. In such a case, by the transmissibility of force vector, I can take each force to be acting, force to be acting at a point say ZIK. That is at a distance ZI in the Z direction. And therefore, the torque by the forces is going to be summation IZIK Cross FI X component I + FI Y component in J direction + FI Z component in K direction. And if I evaluate this, since K Cross K is 0, you will see this comes out to be summation I ZI FIX.

K Cross I is J - ZIFIY. K Cross J is I with a - sign. So - sign will be taken care of. So you see the only component of torques that such forces that are all intersecting one particular line which we take to be the Z axis give torques in the direction only X and Y and therefore we need not worry about the condition that summation Tao Z is equal to 0.

Such forces are called concurrent on a line. So forces concurrent on a line give me torque Tao Z is equal to 0. So let us rewrite it.

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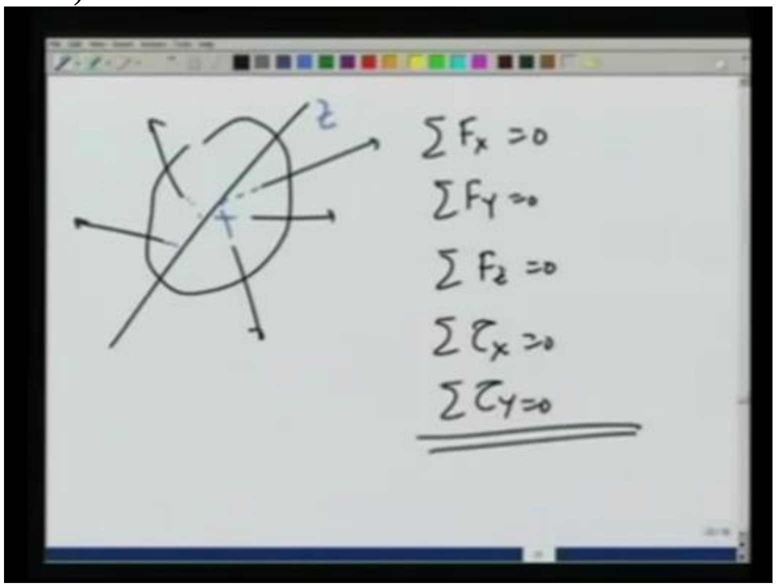
e 

So we have a situation where all the forces are concurrent on a line. That is they all intersect one particular line which we are taking to be the Z axis. In that case, of course I have to satisfy summation FX is equal to 0, summation FY is equal to 0, summation FZ is equal to 0 and summation Tao X is equal to 0, summation Tao Y is equal to 0.

There is no equation for Tao Z because that automatically is 0. So that is the another situation in which you realise suddenly that there is no component of Torque in the Z direction and therefore I need not worry about that equation.

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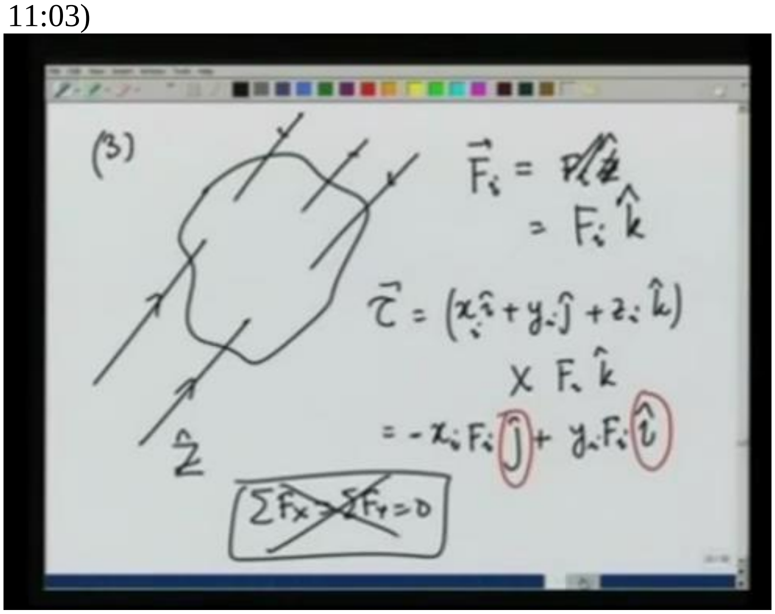

As a 3rd situation, let us take forces which are all parallel. Since they are all parallel, let us take that direction along which they are acting as the Z direction again. So that all forces FI vector can be written as FIZ tao. Let us write the unit vector that we have been using, FIK. And therefore if I were to calculate their torque about any given point, the Torque which is XII YIJ + ZIK Cross FIK would give me again XIFI. I Cross J is K with a - sign + YI FI. J Cross K is I.

So this also has components only in X and Y direction. So I need not worry about the Z component of the Torque. Since there are no components of the forces in X and Y direction in this case, therefore I have summation Fx is equal to summation FY is equal to 0 automatically satisfies. I need not consider it.

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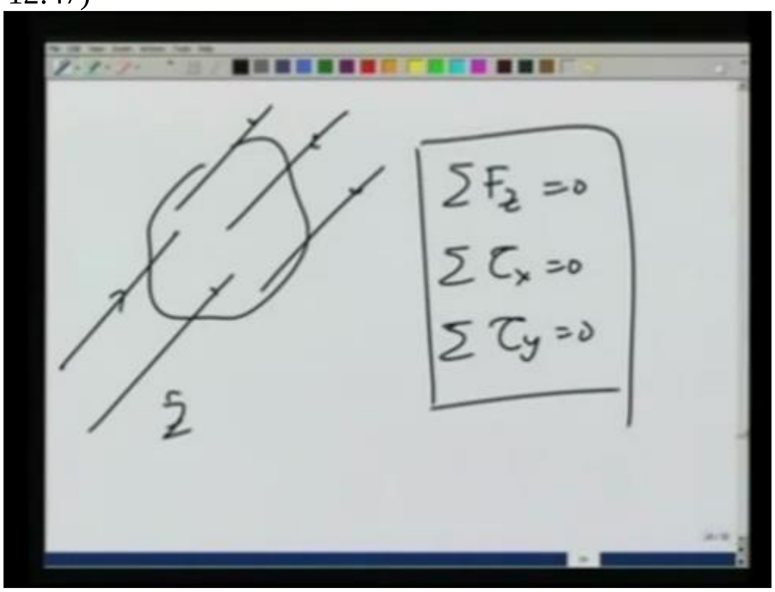

So in this case when all the forces applied are parallel and we take that parallel direction to be the Z direction. In that case, the equilibrium condition is going to be summation FZ is equal to 0, summation Tao X is equal to 0 and summation Tao Y is equal to 0. Only 3 conditions.

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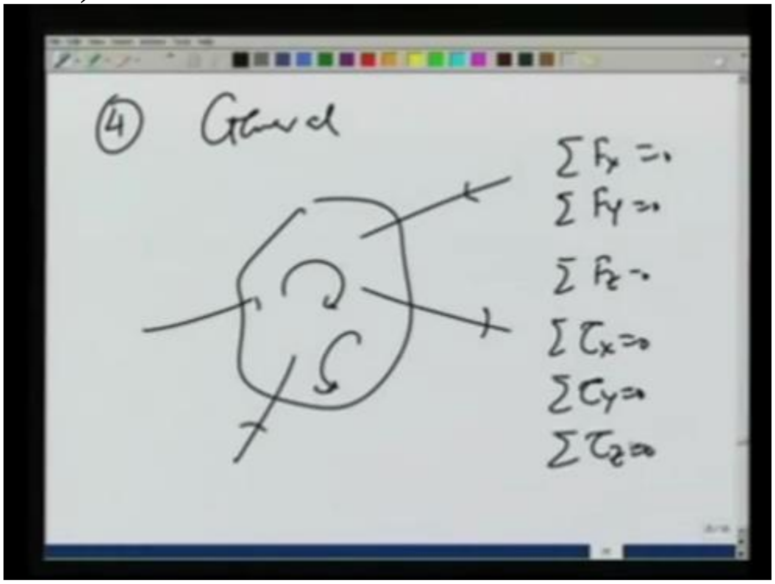

Of course, if none of these satisfies, in general we have the condition general, if there are all sorts of forces and all sorts of Torques applied, in that case of course we have summation FX is equal to 0, summation FY is equal to 0, summation FZ is equal to 0, summation Tao X is equal to 0, summation Tao Y is equal to 0 and submission Tao Z is equal to 0. That is the most general condition.

But what we covered in the earlier 3 cases as if the forces are concurrent or they are concurrent on a line or if they are parallel, some of these conditions are automatically satisfied and we need not worry about them.

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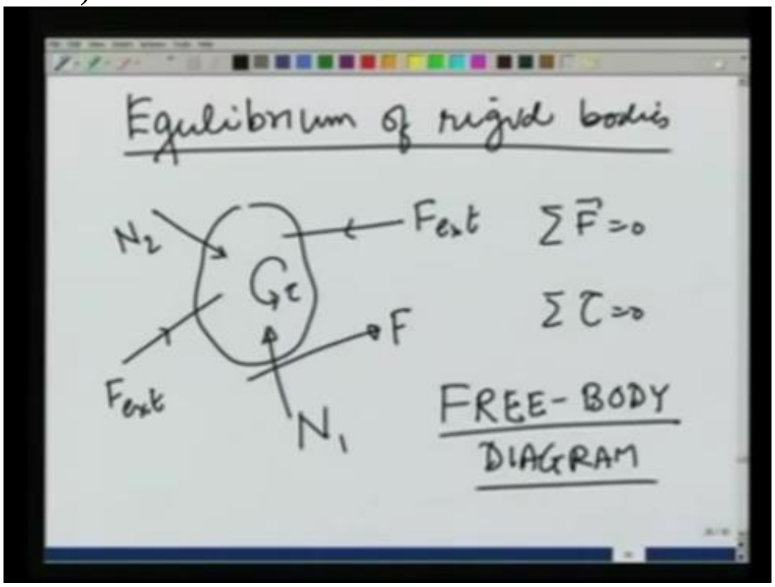

To review this equilibrium of rigid bodies and I emphasise they are rigid bodies because we have not allowed any deformation. What we have done so far is taken a particular body and saw what are the external forces applied on them and what are the forces generated is, normal reaction 2, 1, maybe a Torque Tao, maybe a force, frictional force F. What are these forces given by various elements by which it is held and then we did the equilibrium condition, summation F is equal to 0 and summation Tao is equal to 0.

Such a diagram where the elements which are holding the system, we isolate the system and replace those elements holding system by the normal reactions of those elements or the Torques provided by those elements is known as the body diagram. We have been using free body diagrams so far. I did not use this term. Now I am introducing it. So free body diagram is the one where I take the body, isolate it and replacing all the engineering elements that are holding it by their respective forces or Torques provided.

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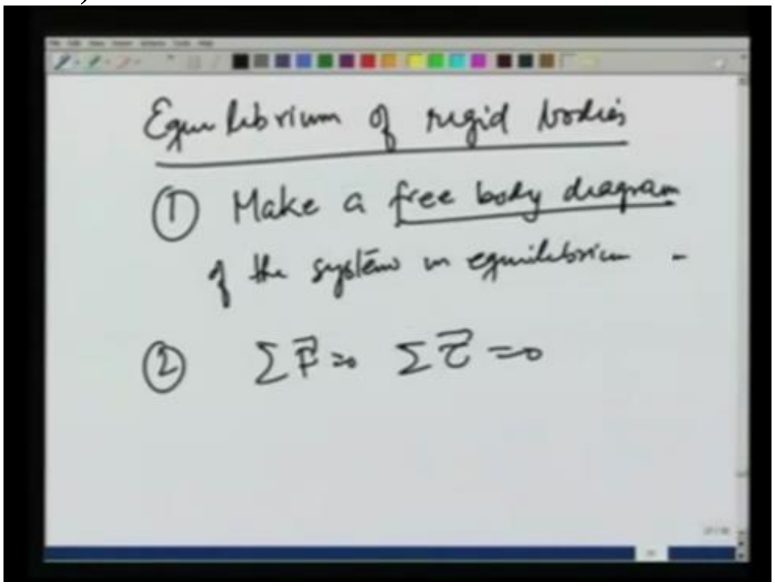

So in considering the equilibrium body, 1st thing we do is, step number 1, make a free body diagram of the system in equilibrium and 2, apply the condition summation F is equal to 0 and submission Tao is equal to 0 and solve it. So this is a brief introduction to equilibrium of rigid bodies. In the next lectures, coming lectures, we will be applying this condition to a very special engineering structure called trusses.