Engineering Mechanics
Professor Manoj K Harbola
Department of Physics
Indian Institute of Technology Kanpur
Module 2
Lecture No 17
Solved examples; equilibrium of bodies-II

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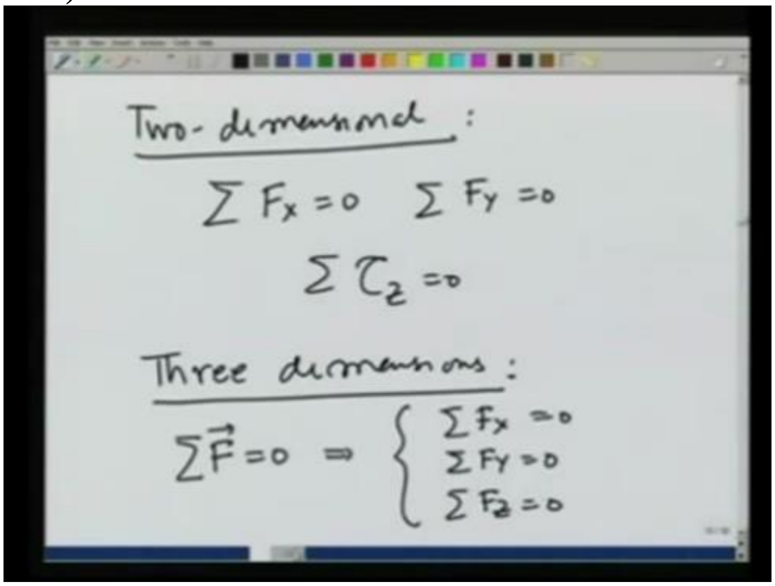

You may have noticed that so far we have been bring problems which are essentially two-dimensional in the sense that we have been looking at equilibrium conditions that Fx is equal to 0, summation FY is equal to 0 and summation Tao Z is equal to 0. So we have been talking about 2 force components X and Y and one component of the talk and that is in Z direction or the torque about Z axis. Now let us generalise this to 3 dimensions.

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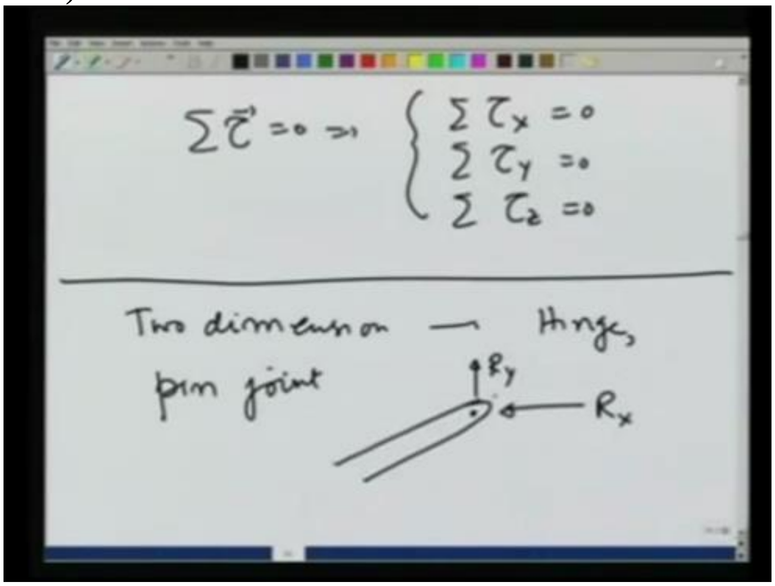

In 3 dimensions, obviously the conditions that summation FB is equal to 0 becomes summation Fx is equal to 0, summation FY is equal to 0 and summation FZ is equal to 0.

Similarly, the condition that summation Tao vector is 0 implies that summation of all the torques along the x-axis is 0. Summation Tao Y is 0 and summation Tao Z is 0. Similarly, now that we are talking about three-dimensional cases, the engineering elements also now we have to consider in view of three-dimensionality and see that they can apply forces in all 3 directions. Thus, remember in 2 dimensions, there was a hinge or a pin joint which was like this and it was capable of applying a force in X and Y direction.

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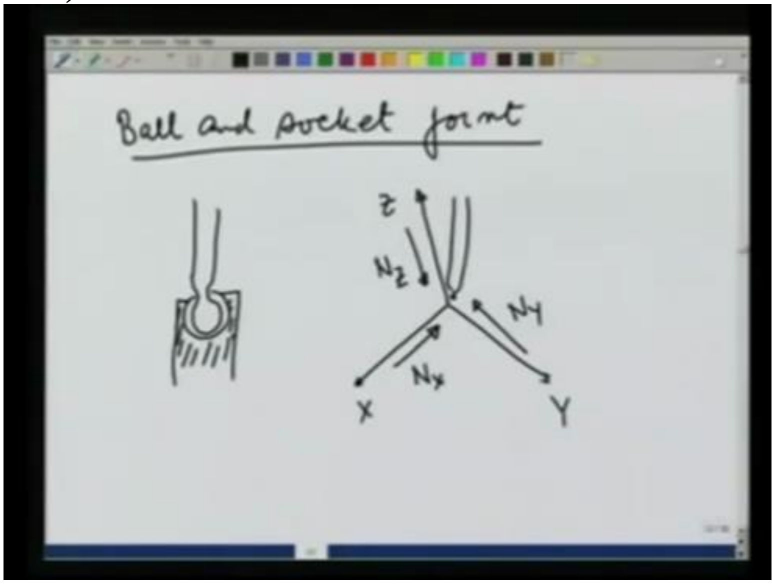

Sometimes they are given as reactionary force RX and RY. In 3 dimensions, a similar joint is called a ball and socket joint which is nothing but a socket in which a ball fits and it can rotate in any directions. So a ball and socket joint, let me just show it. It is like this. It is capable of giving force in X, Y and Z, all 3 directions. Let me make it like this. Let us say, this is the Z direction. On a socket joint, it can provide a force in X direction, it can provide a force in Y direction and it can provide a force in Z direction.

This is the three-dimensional counterpart of the pin joint or hinge joint of two dimensions. R solve problem using this concept.

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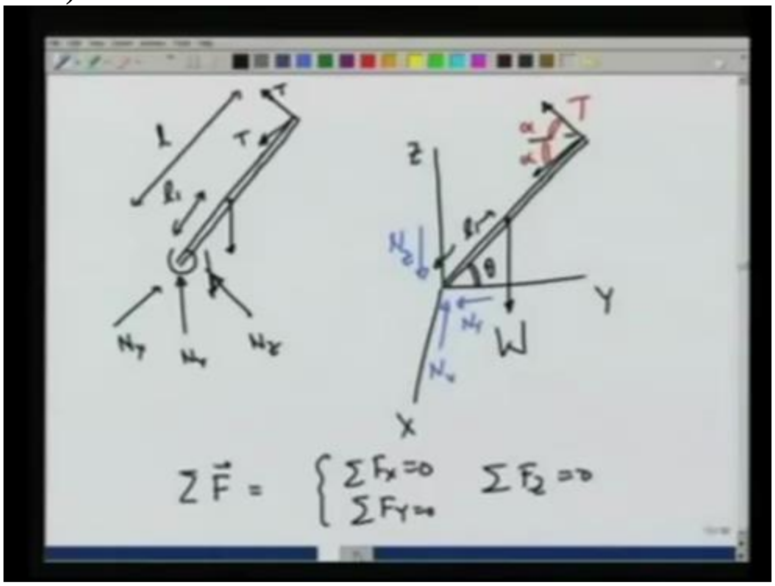

Suppose a person wants to lift a very heavy load and what he does is digs a whole in the ground and puts a pole in it, a long pole, hangs the weight somewhere in the middle C at a distance of L1. Let the length of the pole be L. And then he applies forces horizontally by 2 ropes in 2 directions like this. What is the tension in these ropes? That is we want to know. And what are the reactions here in X, Y and Z directions? In X, and Y and Z.

So let us 1st chose our axis X, Y and Z. We choose our axis in such a manner that the pole over the Y axis and in the YZ plane. X, Y and Z. Let the pole be in the YZ plane over the y-axis making an angle theta from the y-axis. The load is at a distance L1 and let the weight be W. The ropes are in the XY plane and let them make, each of them and angle of alpha from the Y axis. They are in the XY plane.

We want to find out the tension in the rope and the forces NX, the force NY and the force NZ at this point. I am making all the forces towards the origin because it does not matter. So let us now consider the equations for equilibrium. Summation over F gives summation Fx is equal to 0, summation FY is equal to 0 and summation FZ is equal to 0. Let us consider forces in all 3 directions.

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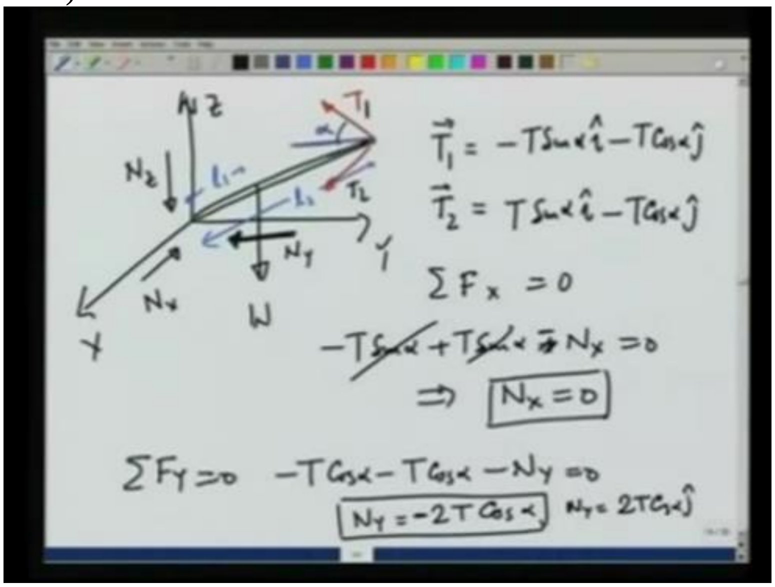

So let me make the picture again X, Y, Z. This is how the pole looks making it slightly tilted. There is a weight W acting downwards. This length is L1. This entire length is L2. This is normal reaction Nx, Ny and Nz and there are these 2 tensions in XY plane making angle alpha from here. So if I were to write the force, let me call this tension 1, let me call this tension 2. T1 vector is nothing but - T sine alpha I - T cosine of alpha J because the tension T1 has component in negative X direction and negative Y direction.

Similarly tension T2 is equal to T sine alpha I - T cosine alpha J. So these are the 2 forces from arising from the tension. The weight acts in the vertical direction. So summation FX is equal to 0 give me - T sine alpha + T sine alpha + Nx is equal to 0 or this implies, this cancels that Nx, the normal reaction in X direction is 0. It does not really matter actually. This should have been negative here.

Similarly, summation FY is equal to 0 implies that - T cosine alpha - T cosine alpha - NY is equal to 0 or NY is equal to 2T cosine of alpha with a - sign. So NY has a magnitude of 2T cosine alpha and the negative sign in the front indicates that it is in the direction opposite to what has been shown in the figure here going towards the origin. So NY is actually 2T cosine alpha J.

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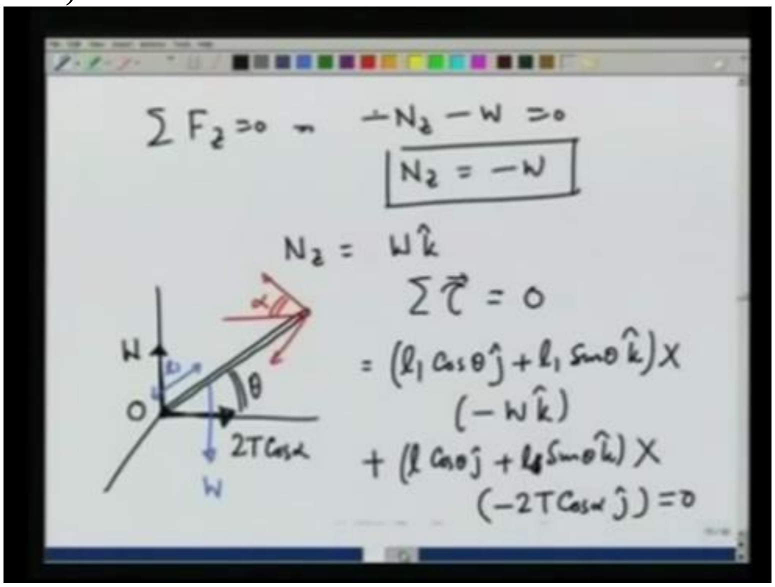

The 3rd condition, the summation FZ equal to 0 gives - Nz - W is equal to 0 or Nz is equal to - W. Again showing that Nz is equal to W and in the direction opposite to what has been shown. And therefore Nz is equal to W Times K. So what we have determined on this rod or on the pole that has been sort of put in a ball and a socket joint is that there is a force on this due to the ground, due to the hole on the ground which is equal to 2T cosine of alpha.

There is a normal reaction in the Z direction which is equal to W. There is a force W pulling it down. There is no force in the X direction. And then, there are these 2 tensions in the XY plane at, working at an angle alpha like this. This angle is given to be theta. Let us now take the condition, summation Tao is equal to 0. To calculate Tao, let us take the origin to be at the origin of the XYZ system O so that this comes out to be L1 cosine of theta J + L1 sine of theta K.

That is when L1 is this distance. So distance where W is acting is L1 cosine theta J + L1 sine theta K cross product of - WK. The other forces are tension. So that would be + L cosine of theta J + L1. Sorry L sine of theta K cross, since the 2 forces are acting at the same point, I can write the torque for their summation which would be equal to - 2T cosine of alpha J. And this should be equal to 0. This condition would determine the tension T.

So let us work this out. The 1st term is going to give me J Cross K which is I. L1 cosine theta times W.

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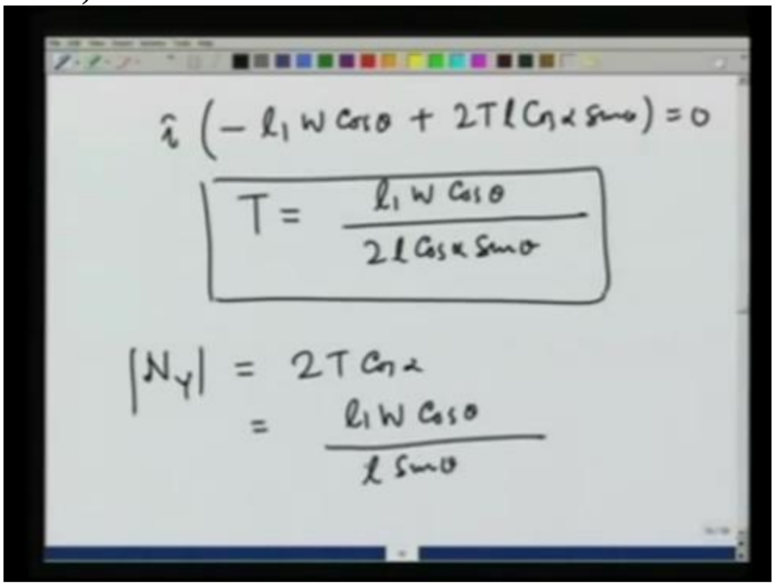

So 1st term gives me I - L 1 W cosine theta. The 2nd term, again K cross J is 0. So K Cross J, K Cross J is - I. So it gives me + 2TL cosine alpha sine theta. So this gives me + 2TL cosine alpha sine theta. This is also in I direction and this is equal to 0. And that gives me T is equal to L1 W cosine theta divided by 2L cosine alpha sine of theta. Once T is calculated, you can calculate Ny which is equal to 2T cosine of alpha in magnitude which will come out to be L1W cosine theta over L sine of theta. And that is your answer.