Engineering Mechanics
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Module 2
Lecture No 16
Solved examples; equilibrium of bodies-I

We have been looking at equilibrium of bodies and in the previous lecture, we looked at certain elements and what kind of forces do they apply.

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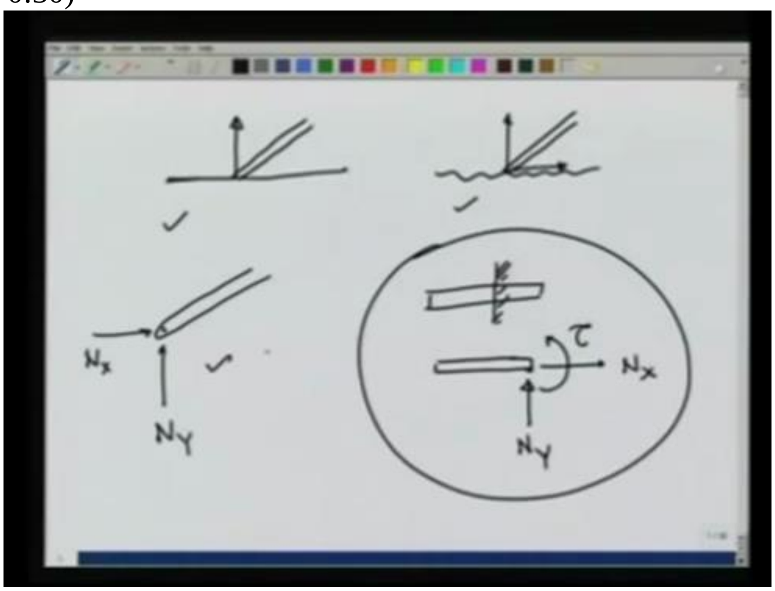

For example, we looked at smooth surface, rough surface and we saw that on a particular body, a smooth surface applies a normal force. On a particular body, rough surface applies a normal force as well as it is capable of applying a fictional force. Then we looked at a hinged joint and saw that this can apply force, both in the X and the Y direction.

And we looked at fixed or welded joints and we saw that is capable of applying a force in the Y, in the X direction is as well as it can create a torque. We have solved examples using this, this and this. Now let me solve an example using a fixed joint.

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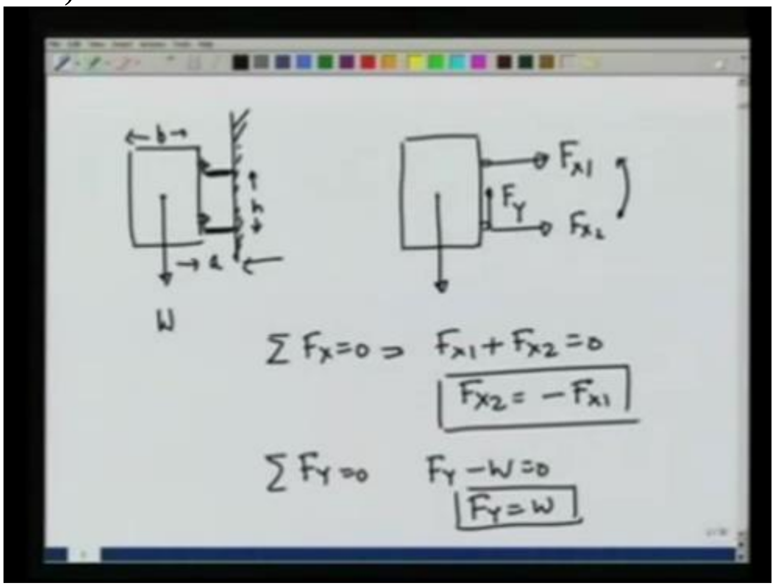

For this, I take a household example of a gate which is on 2 supports like this. Let the weight of this gate be W, let its width be b, let the distance between the supports be H and let this distance from the support wall to the point where the gate is supported to be A. We want to find the forces that are being applied by the wall on these supports assuming that the entire weight of the gate is borne by the lower support.

So let us see how the gate is being supported. If I look at the gate, it has a weight pulling it down. Then the entire weight is being supported by the lower support. So it has a force FY pulling it up. In addition, there will be forces in this direction. So let us call this FX 1 and FX 2. These are the only forces that are there on the gate. Then the equilibrium conditions, the summation FX is 0 gives me FX 1 + FX 2 is equal to 0 or FX 2 is equal to - FX 1.

Similarly the condition summation FY is equal to 0 gives me that FY - W is 0 or FY is equal to W. You may ask at this point, how come these 2 forces, FX 1 and FX 2 are generated? It is because FY and W give a couple in this direction. And to oppose this there must be horizontal forces generated at the joint to counterbalance this couple. So we have gotten these 2 answers. Let us now look at balancing the torque.

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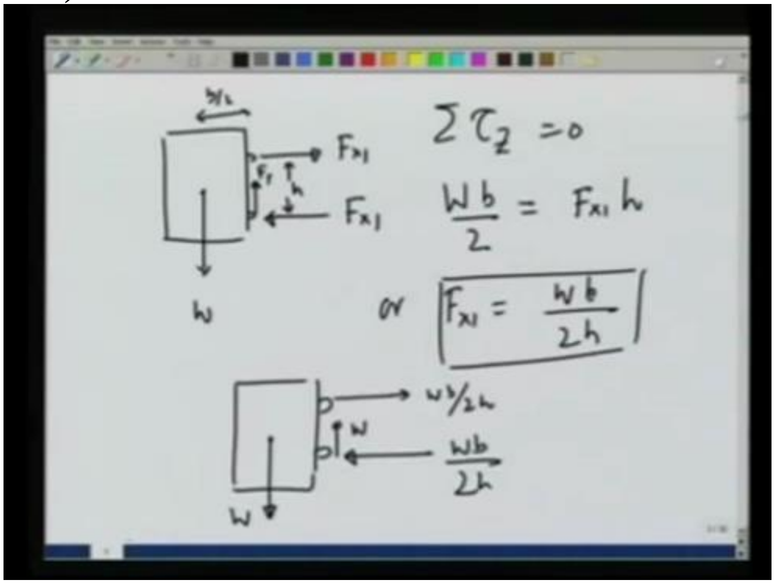

So I have this gate. There is a force Fx 1 and now we have already figured out that there is a force in the opposite direction of the same amount Fx 1. They are separated by a distance H. So summation Tao Z is equal to 0 gives me W times B over 2. That is the couple due to FY and W, should be equal to Fx 1 times H or Fx 1 is equal to WB over 2H.

So the forces acting on the gate are W, W, this way W times B over 2H and this way W times B over 2H. These are the net forces acting on the gate. How about the supports themselves?

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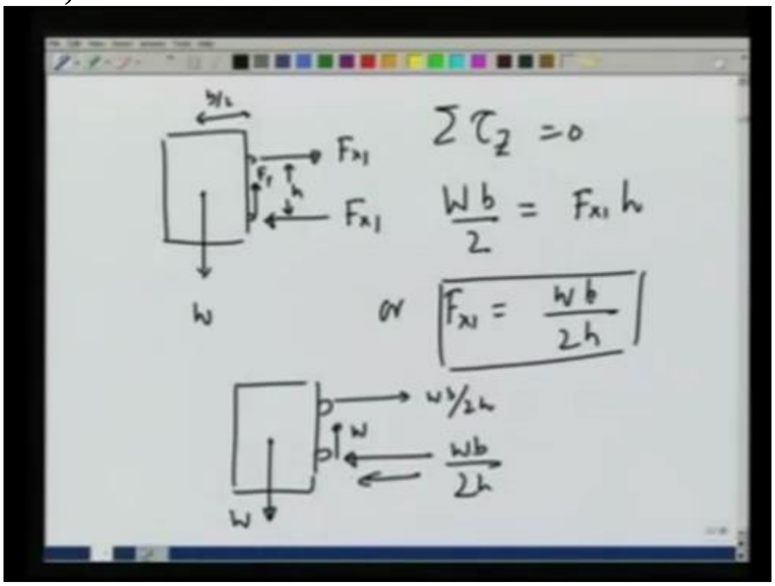

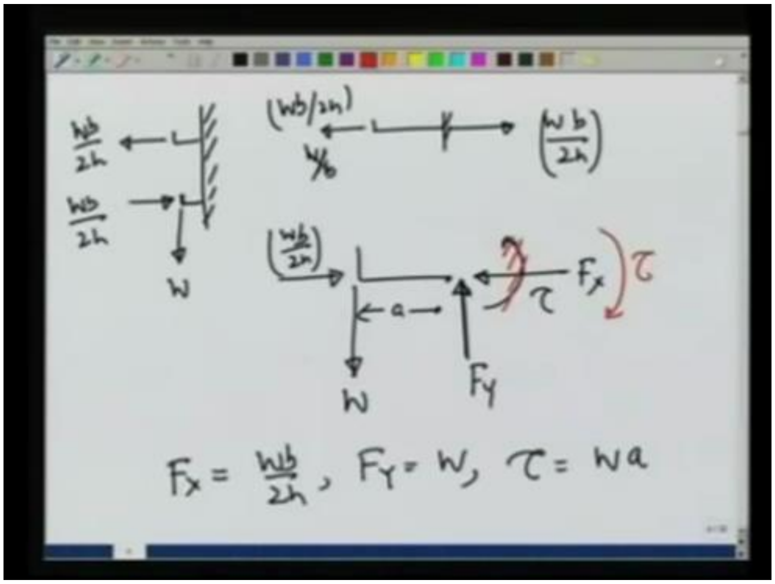

If I look at the support, this is the wall. This is one support, this is the other support. This support, the upper support is pulling the gate in by a force Fx 1 which is WB over 2H. And therefore gate must be pulling by Newton's 3rd law the support in this direction with WB over 2H. Similarly on the lower support, the gate is being let us look at the, gate is being pushed this way. So therefore there will be force pushing the support in with a force WB over 2H.

On the lower support, there are more forces. The lower support is providing a vertical force W to the gate and therefore there must be a force W pulling it down. Now we have to, ready to calculate the forces and moments generated on both the supports inside the wall. So for the upper support, the only force is this way, WB over 2. So the fixed support is capable of providing another force.

So WB over 2H. It will pull it in with the same force WB over 2H. For the lower support, situation is slightly more complicated. This distance is A. There is a force W down, there is a force WB over 2H this way. Therefore at the wall with respect to this point, there would be a force to balance the vertical force FY. To balance the horizontal force, there will be a force Fx and there will be a torque, Tao. Let us see how, what these values are.

So straightaway, Fx is going to come out to be WB over 2H, FY is going to come out to be W and the torque is going to come out to be W times A but not in this direction because this the same direction as W is providing the direction could be this way. This is the torque generated at the support. So this is one example of how a fixed support is able to support the applied forces from outside, both vertical and horizontal as well as the torque applied from outside.

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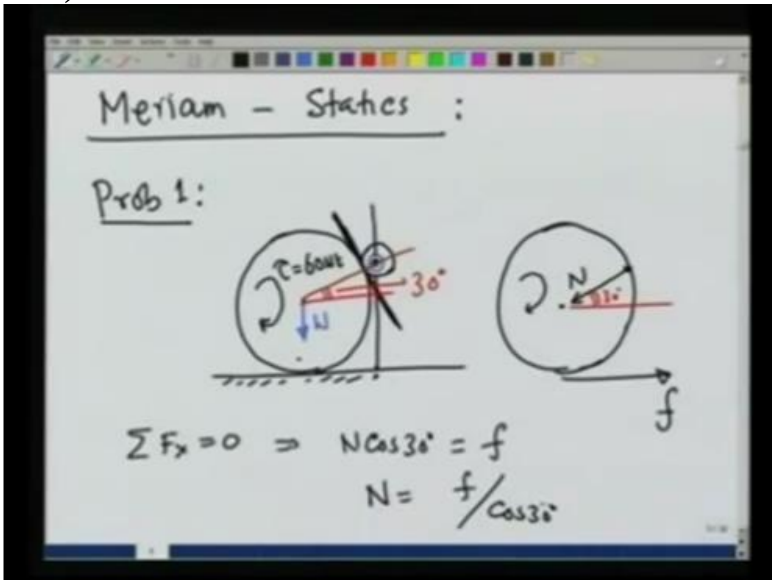

Having done all this, it may now solve 2 or 3 examples for you from the book of Merian. As the 1st problem, let me take the problem is like this. We have a wheel on a rough surface and it is touching a roller is free to move and it is fixed to a support like this. We apply a torque Tao is equal to 60 newtons metre on the wheel such that the wheel does not move.

We want to find what is the force applied by the wheel on this support. Let me show the support in blue here. We want to calculate the force on the support at this point. The weight of the wheel is sum W. You see, it does not really matter. Since the wheel does not move, therefore it is a static equilibrium problem. There is a torque like this.

Wheel would have a tendency to rotate in a clockwise direction and therefore there would be a frictional force generated like this. The other force would be provided where the roller is touching the force. What about the direction of this force? Since roller is free to move, there cannot be any force in the horizontal direction like this. If there were a force, this roller would start moving.

Therefore the only forces in the direction perpendicular to the surface and in this direction. We are given that this angle is 30 degrees. So here is this angle 30 degrees. If we now live the equilibrium conditions, summation Fx is equal to 0 gives us let this force be N. N cosine of 30 degrees is equal to F. And therefore the normal reaction is going to be F divided by cosine of 30 degrees.

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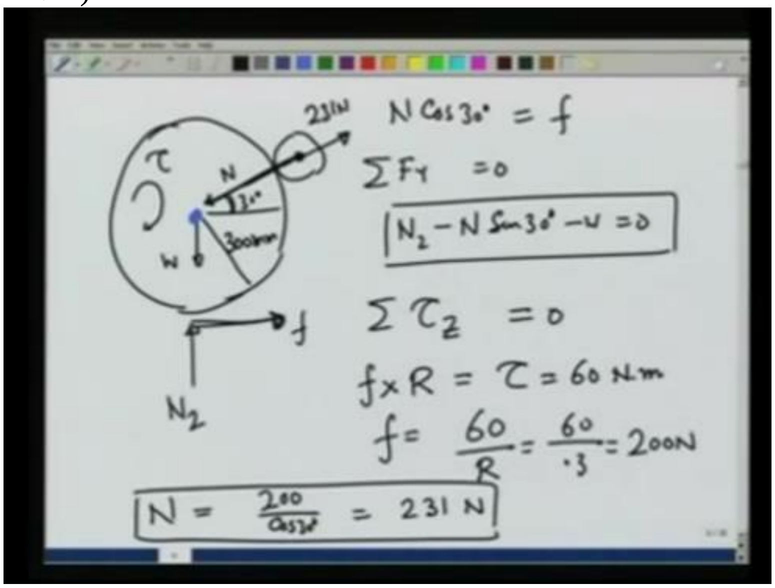

Similarly, let me make the picture again. Since the weight W, here is F, here is normal reaction N, here is the torque Tao. This angle is 30 degrees. So we found that N cosine of 30 degrees is equal to F. Summation FY is equal to 0 would give me if there is a normal reaction N2 here. N2 - N sine 30 degrees - W is equal to 0. That is the other condition.

And 3rd condition for the torque is summation Tao about Z is equal to 0. Let me take the torque about the centre of the wheel right here shown by blue. If we do that, the torque due to normal reaction N due to this force vanishes. That is the advantage of taking torque about this point. And therefore, I would have F times the radius of the wheel is equal to Tao applied which is 60 newtons metre.

And therefore F is equal to 60 over R. The radius R is given to be 300 millimetre. Here we will write this in metre. So this is going to be 60 over 0.3 is equal to 200 newtons. So we have found that the frictional force to support or the frictional force required so that the wheel does not move even after applying this torque is going to be 200 newtons.

And therefore the normal reaction N on the roller is going to be 200 divided by cosine of 30 degrees which you calculate, it comes out to be 231 newtons. So the force by the roller on the wheel is 231 newtons. And therefore on the roller, there will be forced by the wheel of the same amount, 231 newtons. And that is your answer. Let us solve one more problem.

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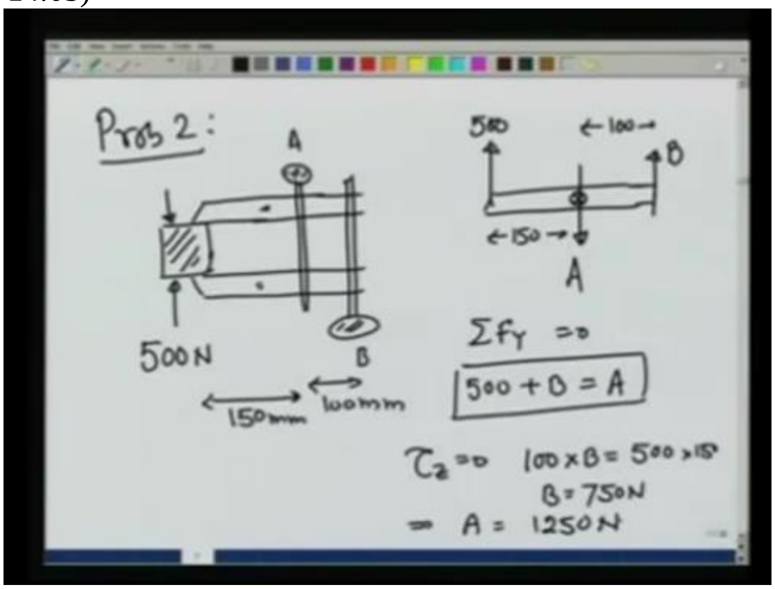

And this is, suppose we have a block of wood or steel and it is being clamped by tightening the screws on these 2 clamps. So here is one screw A, here is the other screw, B and they are being tightened in the opposite directions. They are tightened to the extent that the force on this block on both the directions, the compressive force of the amount 500 newtons. If this distance is 100 mm and this distance is 150 mm, we want to find the forces applied by the screws on these clamps assuming that the forces applied by the screws are in the along the screw direction.

So let us look at the upper clamp. On the upper clamp, there is going to be a force of 500 newtons in the direction opposite to the force it is applying on the block and the screw would apply a force like this. Let us call it A. And the other screw applies a force in the opposite direction, B. You can see right away, I made the forces acting in opposite directions because that is how I would balance the torque and the forces.

Since this is essentially a one-dimensional problem, I can straightaway write FY is equal to 0 and that gives me 500 + B is equal to A. This distance is 100 mm and this is 150 mm. Let me now balance talk about this point, the point in the middle where the A screw is. So Tao Z is equal to 0 gives me 100 times B is equal to 500 times 150. Or B equals 750 newtons. And this implies A equals 1250 newtons. That is the other problem.

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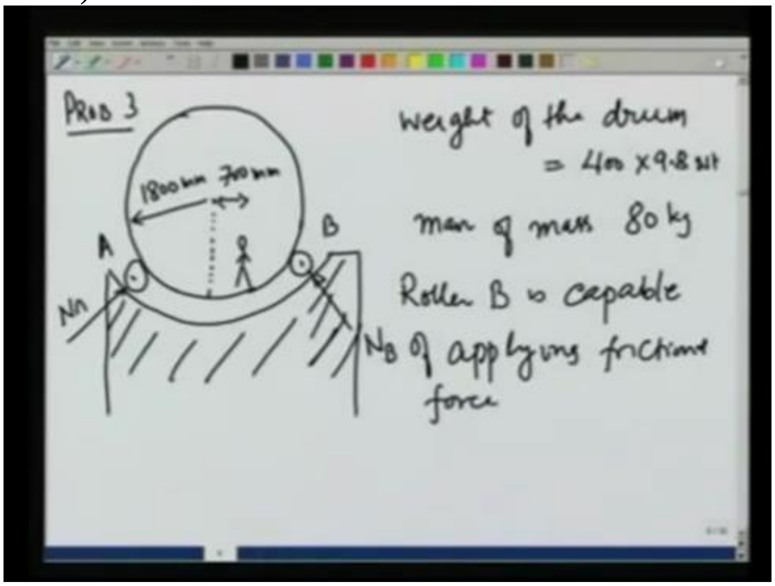

Let me solve one more problem. And that is a problem where we support a big drum, this problem 3, on a cylindrical rollers. The radius of the drum is given to be 1800 mm. Its weight is given to be 400 times 9.8 newtons. That is, it is a 400 kg drum.

A man of mass 80 kg starts walking on the drum and when he reaches here, which is 700 mm distance from the centre, the drum starts moving. Just begins to rotate on these rollers A and B. The B roller, these rollers are also cylindrical, is capable of applying a frictional force. Roller B is capable of…

That is why it is only after the man moves certain distance that the drum starts rolling. We want to find the reactions on A, NA and reaction on NB.

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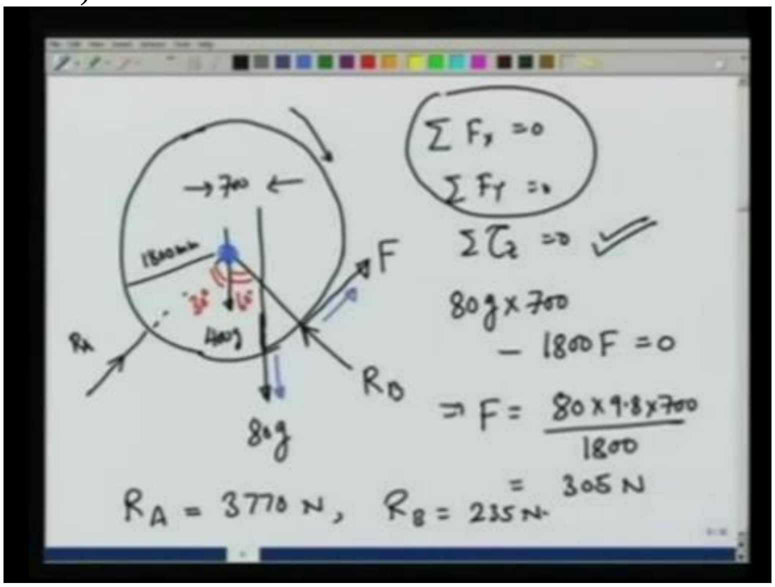

To solve the problem, let us make this drum and see what all forces are acting. There will be a force RA acting here. There will be a force RB acting here. The man is as this distance 700 mm. there will be a force of 80g acting this way and there is a force of 400g acting this way. The angles are given to be 30 degrees and 60 degrees here. In addition, since the drum has a tendency to rotate like this, there will be a frictional force.

Let us call it F at Roller B which is capable of applying a frictional force. Now we have taken all the possible forces that are there on the drum. And let us now apply our equilibrium conditions. So if we write summation Fx equal to 0, summation Fy equal to 0 and summation Tao Z is equal to 0, if we want to find F, that we can do right away. So let us 1st apply the torque equation to find F.

For this, let us take the centre of the drum as our point about which we take the torque. In that case, the only talk that will be coming into picture would be torque due to this weight and torque due to F. If we do that, we will find that torque is equal to 0 gives us 80g times 700 - 1800 which is the radius times F is equal to 0. Notice that I did not divide by 1000 which converts 700 and 1802 m because I am using the same units mm on both the forces.

And this gives me, force F is equal to 80 times 9.8 times 700 over 1800 which comes out to be 305 newtons. Having determined the frictional force F, it is easy to find RB and RB by taking the force conditions. I leave that exercise for you but give you the answer which comes out to be RA is equal to 3770 newtons and RB comes out to be equal to 235 newtons.