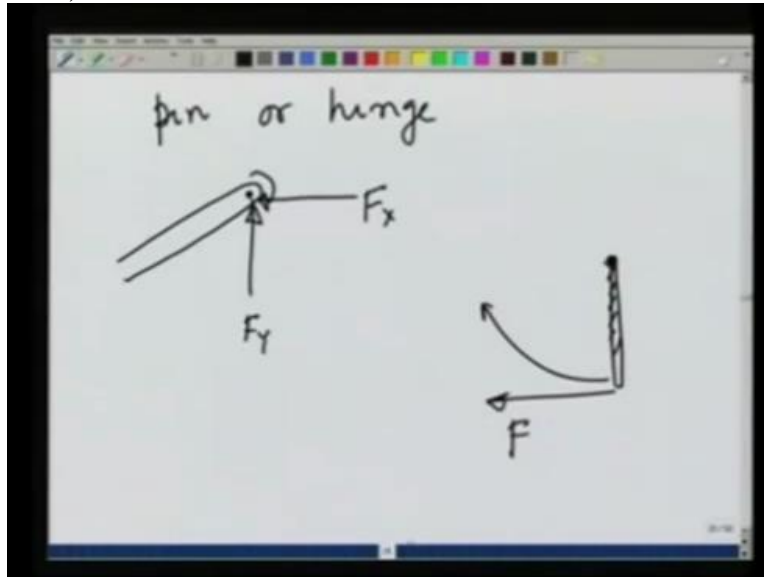


**Engineering Mechanics**  
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**Module 2**  
**Lecture No 15**

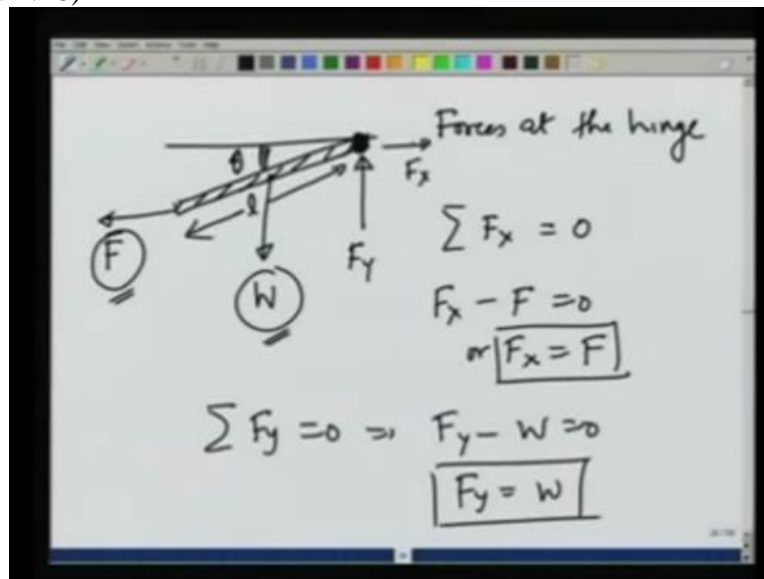
**Differential elements and associated forces and torques - II**

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Next, we consider an element which is a pin or a hinge. The same thing. That is if I have an element, a rod or a plank and it is free to rotate about this point. It is hinged here. In that case, the hinge cannot apply any torque but applies a vertical force and a horizontal force,  $F_y$  and  $F_x$ . These are the only 2 forces that it can apply. Nothing else. So let us see an example of this. You have all travelled in trains and sometimes when the bus is in vertical position, you apply a force  $F$  like this in order to pull it up. And this is hinged at this point.

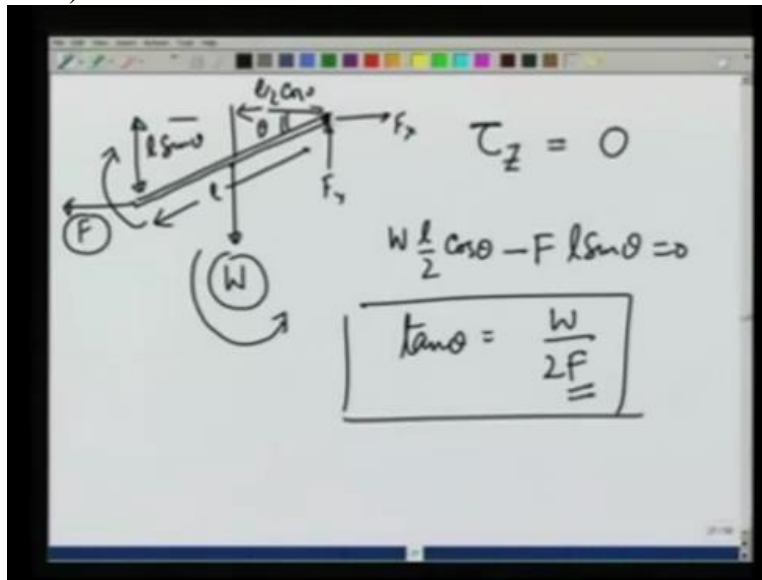
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So the problem we want to solve is, suppose you pull this bus out and you are applying a constant horizontal force  $F$  like this. The weight of the bus is  $W$  and it is hinged here. I want to know the forces at the hinge. As I said just now the hinge is capable of applying a vertical force and a horizontal force.  $F_y$  the vertical force and  $F_x$  the horizontal force. And summation  $F_x$  is equal to 0 gives me the horizontal force and that gives me  $F_x - F$  is equal to 0 or  $F_x$  equals  $F$ .

Similarly summation  $F_y$  is equal to 0 gives me  $F_y - W$  is equal to 0 or  $F_y$  is equal to  $W$ . Next, we want to calculate, given this force  $F$  and given this weight  $W$ , what is the angle  $\theta$  that the bar makes from the horizontal.

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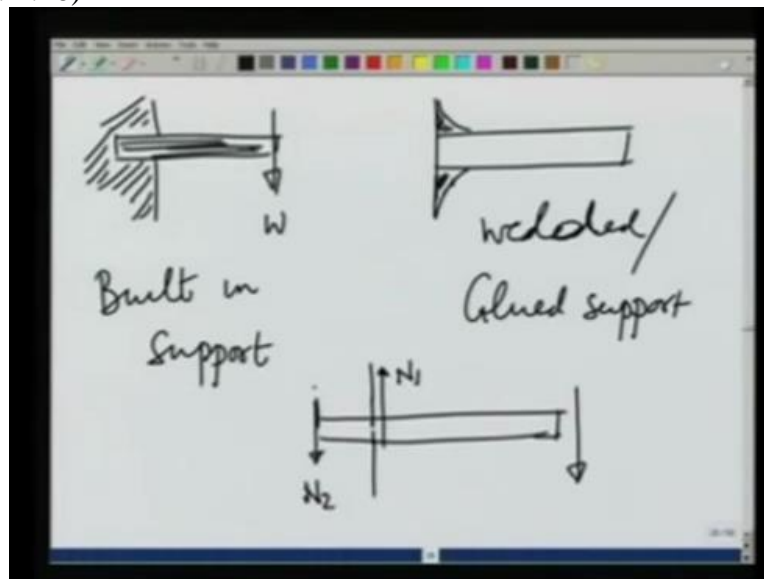


Let me make this picture again. So this is the bar. You have pulled it out with force  $F$ . This is being pulled down by its own weight,  $W$ . You are applying a force.  $F_y$  and  $F_x$  are being applied by the hinge. The length of the rod is  $L$ . There are torques. When I take torque about this point being generated by the weight  $W$  and the force that I am pulling with, if I take torque about  $Z$  axis should be 0 then the weight is giving a torque in this direction, counterclockwise is going to be positive.

And  $F$  is giving a torque in this direction which is negative. This gives or let us calculate the distances 1<sup>st</sup>. This distance is going to be  $L$  over 2. This angle is theta. Cosine of theta and this distance going to be  $L$  sine of theta. And therefore when I put  $\tau_z$  is equal to 0, I get  $WL$  over 2 cosine of theta -  $FL$  sine of theta is equal to 0 and that gives tangent theta is equal to  $W$  over  $2F$ .

You notice, larger the force that you apply, smaller the angle and that you experience when you do something like this. You pull the bar out. It goes higher and higher and higher. So this is an example of solving a simple problem using hinge forces.

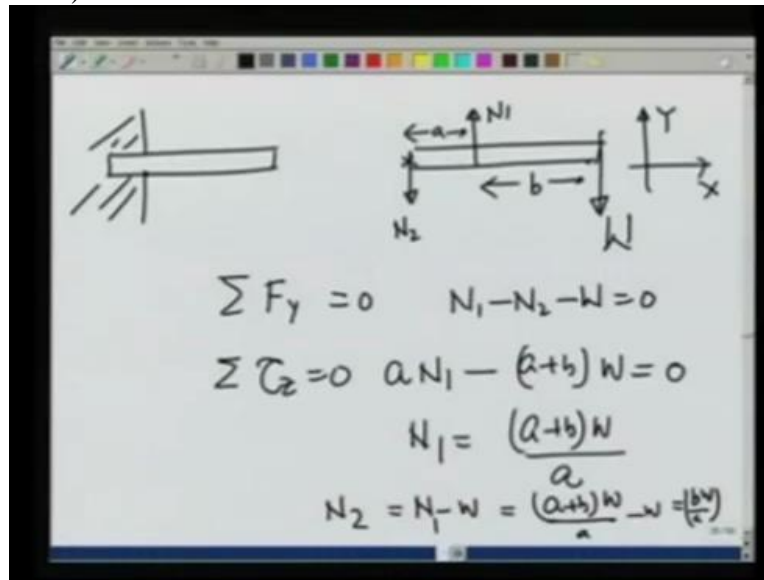
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Next, we look at built in supports. That is suppose there is a support or a beam which is inserted in a wall and is like this. And I apply the load here in any direction. Or a support which is glued to a wall. Suppose I glue it here. glue or weld. So welded or glued support and this is built in support. What all are these capable of? So to understand this, let us say, I take this built in support 1<sup>st</sup> and apply a load here.

Maybe hang something. Then you know, this point tends to go down and this point will tend to move up. As the support pushes the wall down, there is going to be a force let us say  $N_1$ , the normal reaction of the wall on the support. And similarly, this end is being pushed up. So this would experience a force  $N_2$ . Let us go to the next page and see.

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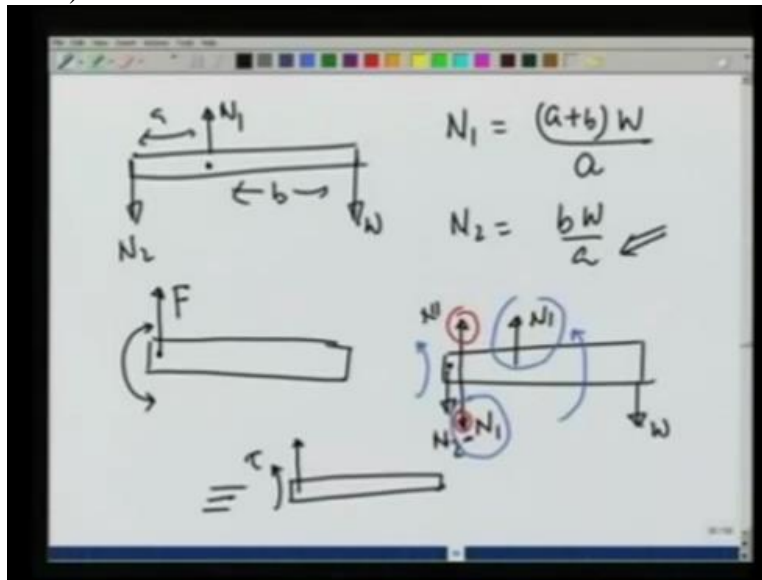


So what we did? We are looking at this built-in support and when I apply a force in this direction, let us say  $W$ , at this point there is a normal reaction  $N_1$  and at this point there is a normal reaction going in this direction  $N_2$ . Let us say this distance is  $B$  and this distance is  $A$ . Then, since there is no force in  $X$  direction, I need not worry about it. I am taking  $X$  direction like this and  $Y$  direction like this.

Summation over  $F_Y$  is equal to 0 gives me  $N_1 - N_2 - W$  is equal to 0. And summation  $Tao Z$  about the  $Z$  axis is equal to 0. Let us say I take it about this point. Although once the forces are 0, it does not really matter. It gives me  $A$  times  $N_1 - A + B$  times  $W$  is equal to 0 or  $N_1$  is equal to  $A + B$  times  $W$  over  $A$ .

And similarly  $N_2$  would then come out to be  $N_1 - W$  or this is equal to  $A + BW$  over  $A - W$  which is equal to  $BW$  over  $A$ .

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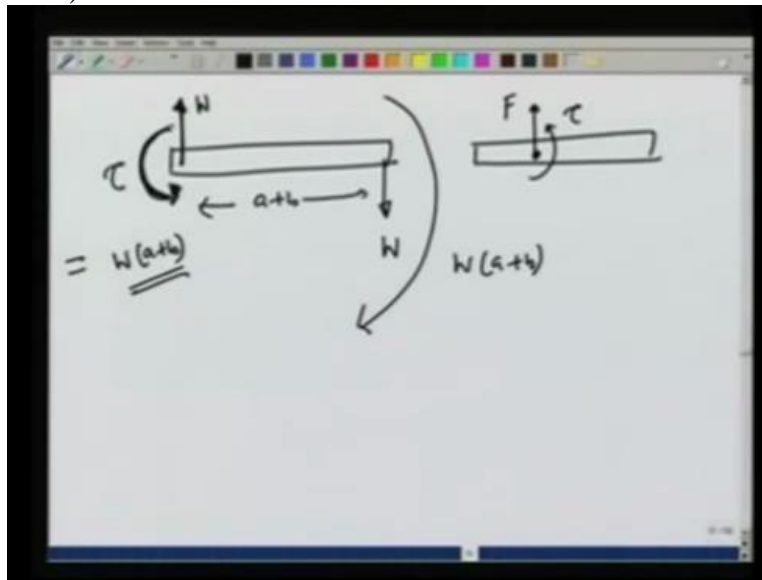


So what we find is for this built-in support,  $N_1$ ,  $N_2$ ,  $W$ . We have  $N_1$ , this is  $B$ , this is  $A$ , this is equal to  $A + B$  times  $W$  over  $A$ . And  $N_2$  is equal to  $BW$  over  $A$ . I could have gotten this answer directly if I took the torque about this point. But the point I am trying to make now is going to be slightly different. I could think of this whole thing this built-in support as if it is applying a torque at this point at the end in one direction or the other and a force  $F$ .

And that is how I want to represent a built-in support. Let us understand how this can happen. So once I have determined  $N_1$  and  $N_2$ , this is  $W$ , I can add a 0 force at this point and I choose this 0 force in a particular way. I add  $N_1$  and  $-N_1$  at this point. Now this  $-N_1$ , let me use or different colour.  $-N_1$  along with this  $N_1$  gives a couple, a couple to turn the whole thing counterclockwise.

So this gives a couple at this point in this direction. And this force  $N_1$  and  $N_2$  added together give me a force. So this whole thing is equivalent to a couple or moment torque here and a force  $N_1 - N_2$  here which obviously is going to be equal to  $W$ .

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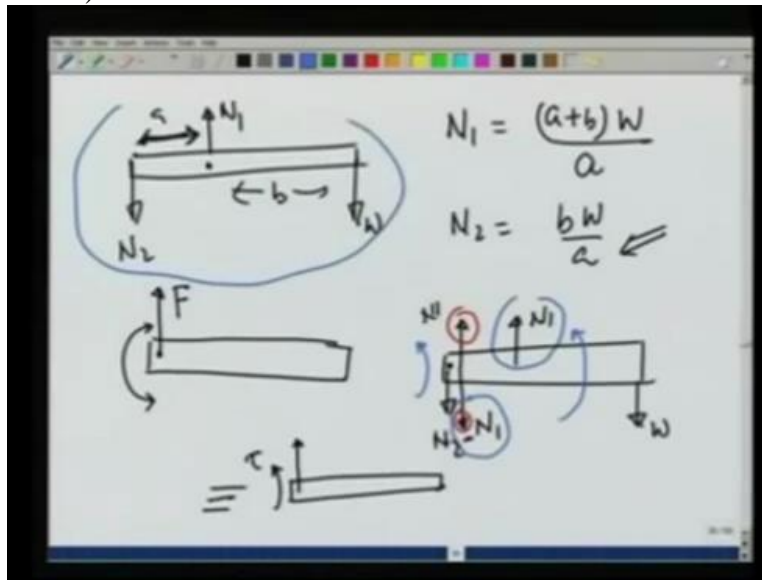


So what we say for a built-in support system is that this is capable of giving a torque and a force at this point. Of course I could have done the entire analysis about point B, this point near the wall where it enters the wall.

I would still say, by a similar analysis that this is capable of giving a torque. Its value would be different because now I would be doing things about this point and a force here. So I can think of is built in support system as providing a torque and a force. Of course the moment I apply a weight here, this force has to be equal to this and therefore, weight it has to be equal to  $W$ . The length is  $A + B$ .

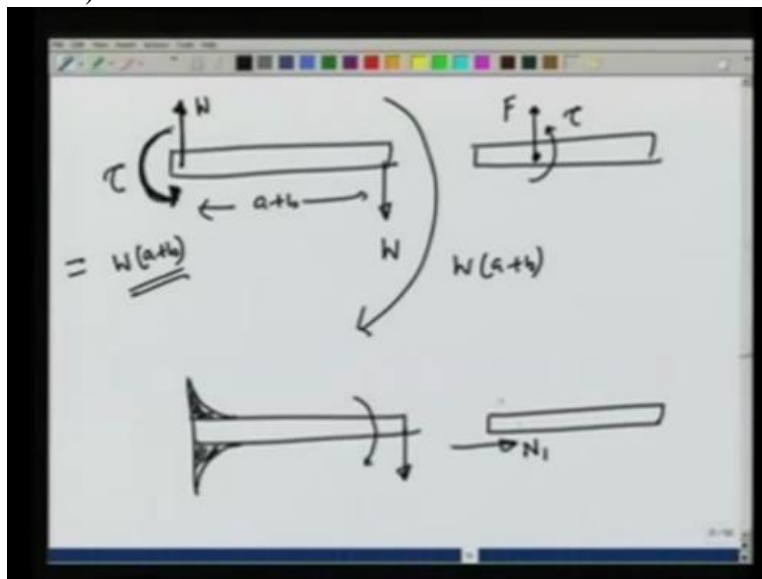
And therefore, this forms a couple which gives a torque in this direction as  $WA + B$ . And therefore the torque generated at this point to counter this torque is also going to be equal to  $W$  times  $A + B$ . So think of a built in support system as something that can generate a torque as well as a force. You notice that with  $A + B$  increasing, if  $A + B$  is large, this torque is going to be larger and larger and larger.

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It is actually better understood if we go back to the slide and see if  $A$  is large, if  $A$  is large, I am talking about this picture, if  $A$  is large, then  $N_1$  provides a larger and larger and larger torque. And therefore, with large  $A$  I need less  $N_1$  at this point. And therefore if you want to have a very very stable support, push it really deep into the wall.

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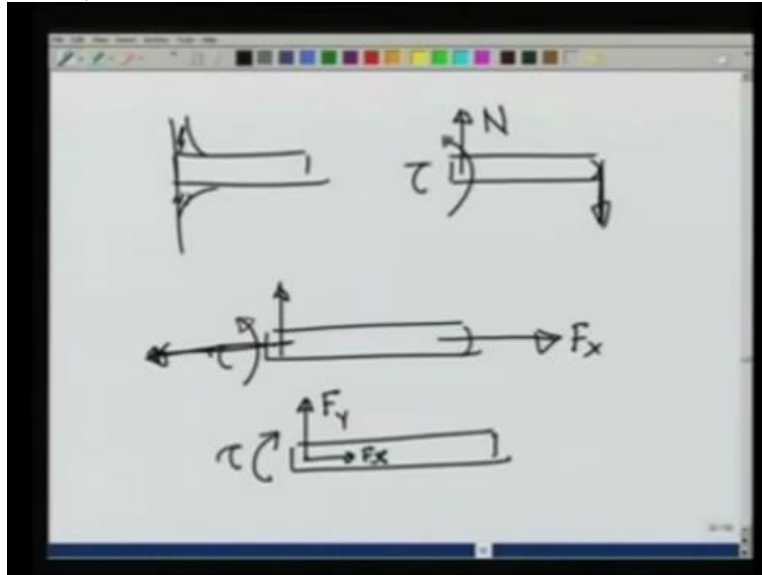


Let us see now a similar support which is not into the wall, which is not built in but is welded at the corners and I apply a load here. Let us see what will happen. As this load is applied, you will see that this has a tendency to turn like this and therefore there will be a force generated in this



direction, let us call this  $N_1$ . And there will be a force generated in this direction, let us call it  $N_2$ .

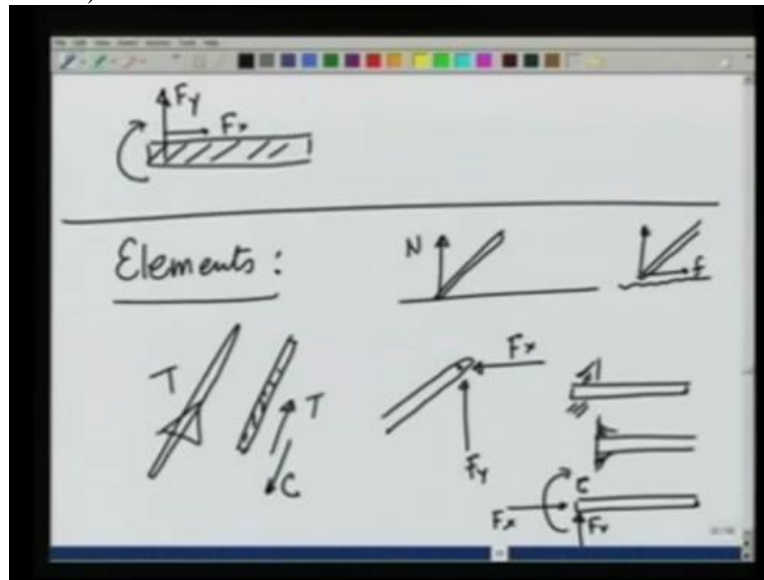
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Using the analysis similar to what I did just now for the built-in support system, I can again show this to be equivalent to giving a torque and innate force normal reaction to this load. So both are built-in support systems and a glued or welded system are capable of generating a counter torque and a force. And this is how I am going to represent them. We are not considered X component of the force so far.

You know obviously if I pull this thing by  $F_x$ , the built-in system or the glued system also generates a force in the opposite direction because the thing does not come out. And therefore, the complete picture, a built-in support system or a welded system, is capable of generating a torque, a vertical force and a horizontal force. And this is how we represent a built-in or glued system.

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I will do an example of this kind of element, that is a built-in support system which can provide a torque as well as a vertical force and horizontal force in the next lecture. But let us see what all did we learn today in this lecture. We 1<sup>st</sup> completed our analysis or discussion **or** of moments of force and then we looked at elements, various engineering elements and what kind of forces they can apply.

We looked at a string and saw that it can generate a tension. We looked at a rigid rod. It can generate a tension as well as compressive force. Then we looked at a smooth surface where we saw that it can generate only a normal force and a rough surface which I am going to show like this, which can generate a normal force as well as a frictional force along the surface.

Then we looked at in or hinge joints and this can create a force in the X direction as well as the Y direction. And finally we looked at built-in support systems or welded or glued support systems and saw that these are capable of generating a vertical, a horizontal force as well as a couple moment.