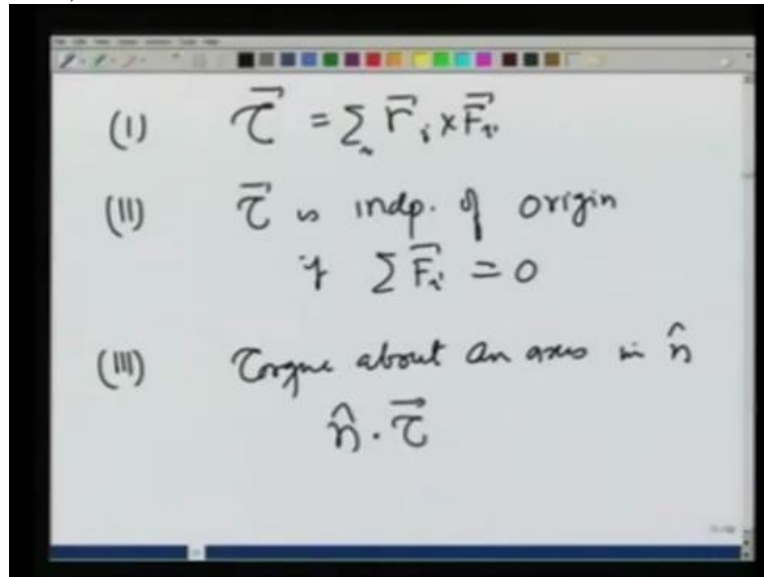


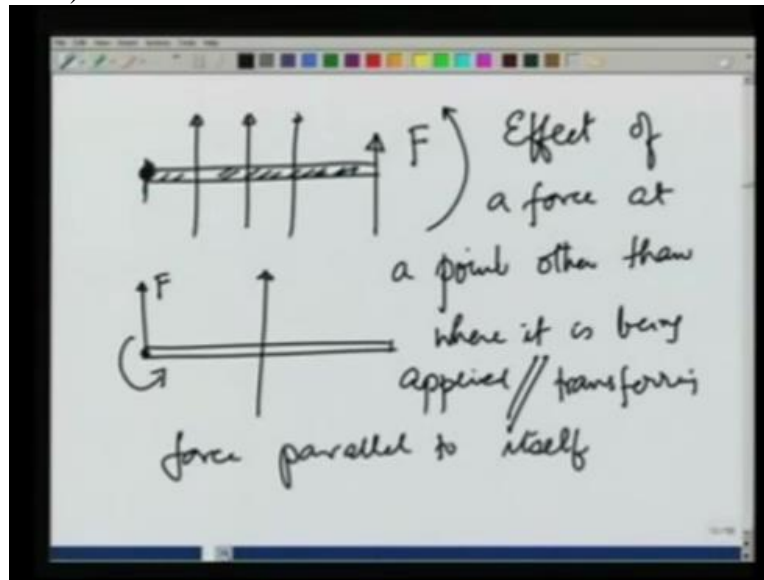
Engineering Mechanics
Professor Manoj K Harbola
Department of Physics
Indian Institute of Technology Kanpur
Module 1
Lecture No 13
Finding a force and a couple equivalent
to an applied force

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Next what we will do is using the couple moment and force we will like to transfer a force parallel to itself and see the effect of a force how is it seen at some other point.

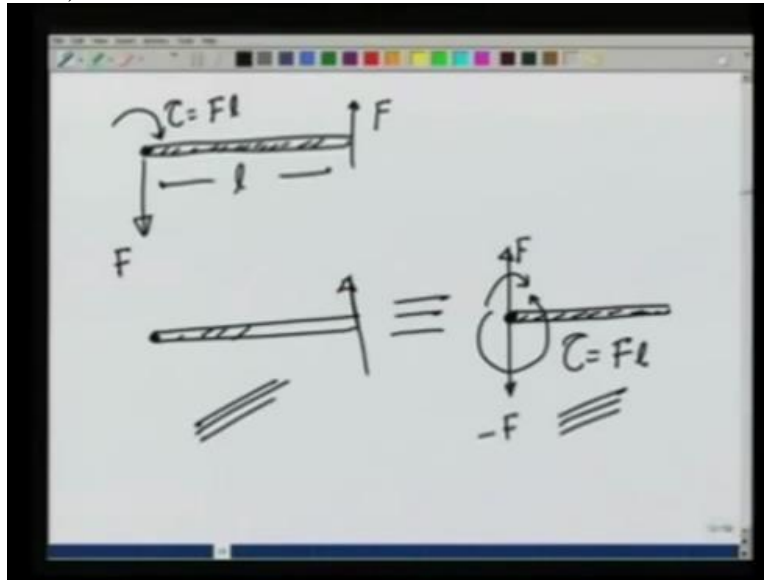
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For that, let me just motivate you by taking a rod and apply a force here. If this rod is fixed here, what you will see is that this force will number 1 push the rod in this direction and will also tend to rotate it like this. What I want to find is, what is I think of this therefore as a force pushing the rod in this direction what else can it do? I will replace this whole thing by a force at this point.

And a moment that is rotating the rod. That is the total effect of the force. So what we are going to learn is the effect of a force at a point other than where it is being applied equivalently in some books you will see this is called transferring force parallel to itself. And why this is important is at times you really want to see the action of the force by describing its what all can it do? Can it rotate things? Can it transport? Can it move things, accelerate things and so on?

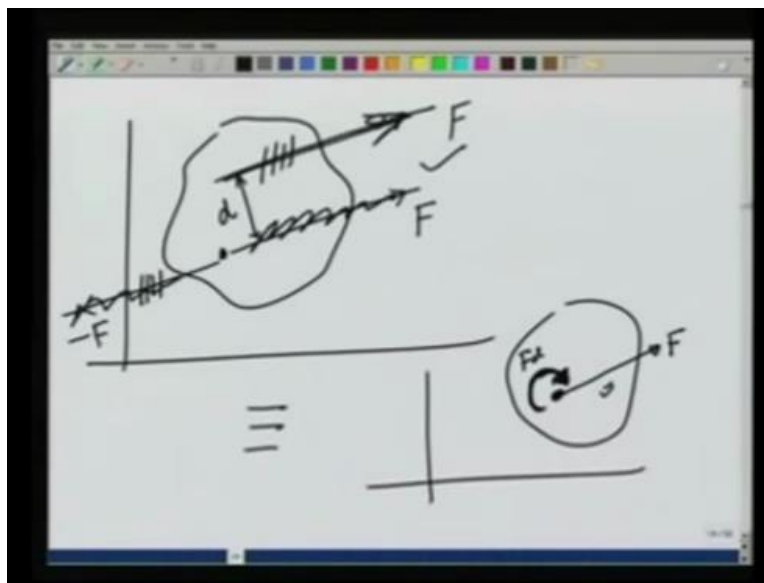
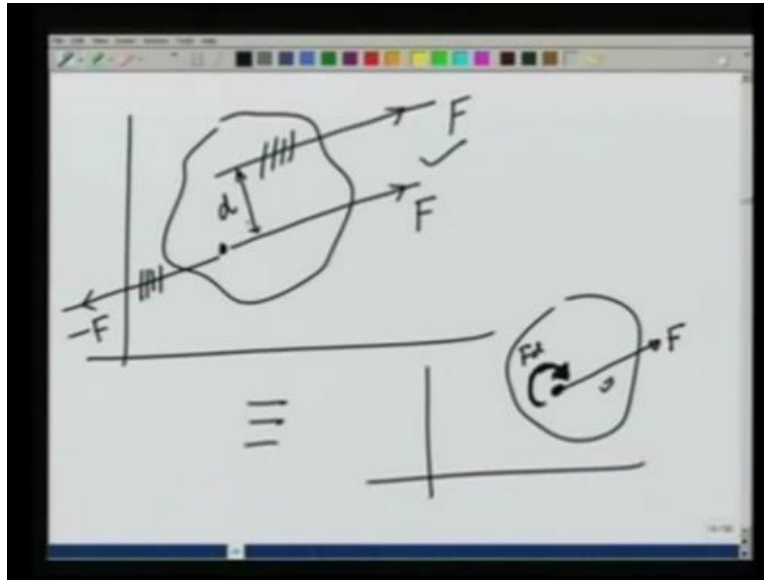
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So just in the example that I took, deep I take a rod and if I apply a force here, S, let us say the length of the rod is L and it is held at this point. And suppose the rod is in equilibrium. Then you would say that I need to apply an equal and opposite force F in this direction and because these 2 forces produce a couple, I need to apply a torque in this direction which is of the amount F times L to keep the rod in equilibrium.

In other words, what I can say is that this force is absolutely equivalent to a force F at the pivot point and a torque like this of the amount T_{ao} equals FL and a force F so that if I want to keep this rod in equilibrium, I need to apply at this point a force $-F$ and a torque like this. So this is absolute this is equivalent to this as far as equilibrium or statics of this rod is concerned. How do I understand this?

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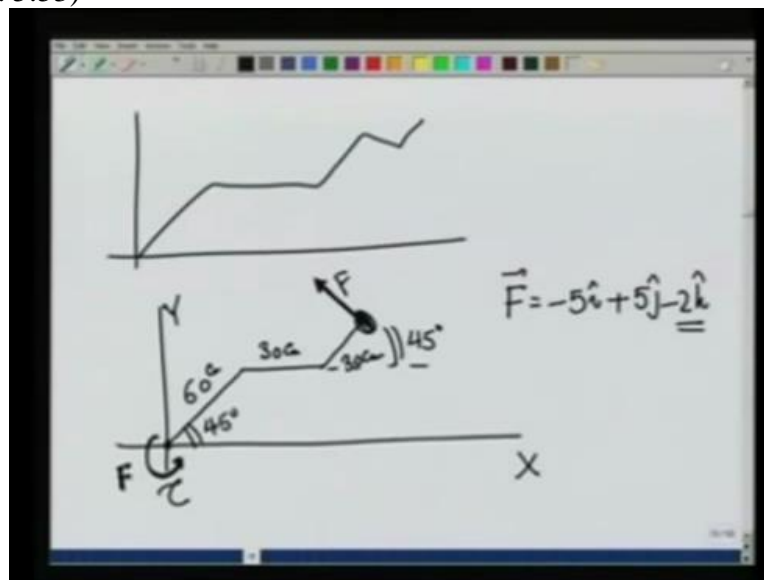
So let me take a system and let us say it is free to rotate about this point. It is held here and I apply a force like this F . Let me now add a 0 force. A very carefully chosen 0 force onto this rod. I will apply a force in this direction and in this direction of the same amount as F which is a 0 force. F in one direction and equal and opposite force at the same point. So I have really done nothing to this.

But now you notice that the original force and $-F$ give a couple which is equal to the perpendicular distance between the forces times the force itself. So I can say that this whole

thing is equivalent to applying a torque at this point of the amount FD and applying a force F in this direction. What I have done is transferred the force parallel to itself to this point and consequently have generated to keep the entire dynamics or statics the same I had to apply an additional torque here.

The effect of these, this torque and this force, these 2 elements is exactly the same as they effect of that single force, original force applied here without these on the body. So this is called two equivalent system. It is easier at times to think in terms of couples and forces and therefore we do this.

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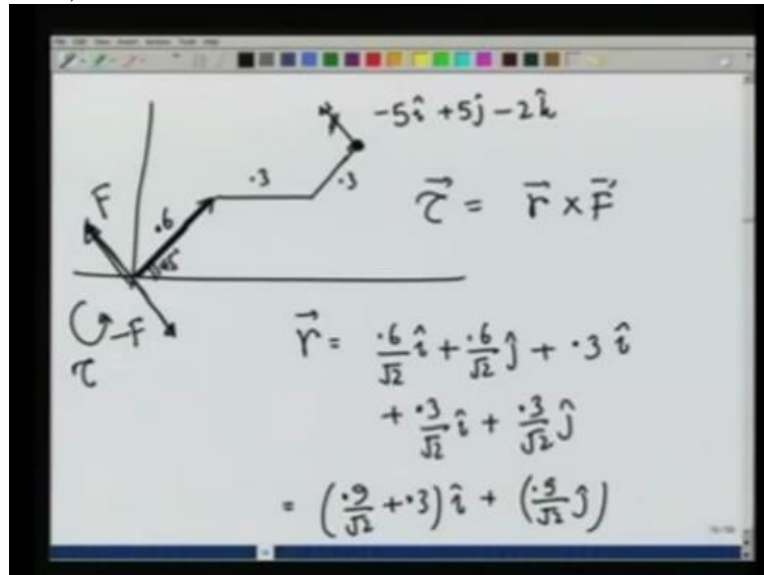


Let us take an example. You must have seen in buses, the handle to change the gear is of a funny shape. So let us say if I take a handle in the XY brain of this shape, the gear handle and this is the head where the driver applies the force and changes gear. So let us say he applies certain forces and I want to know what is the equivalent systems? How much moment does he generate at this point and what force is transferred here? And that is what we will do next.

So let us take the length of this part to be 60 cm. This part to be 30 cm and this part also to be 30 cm. Let this angle be 45 degrees, let this angle also be 45 degrees. And now we say that the driver applies a force F which is equal to $-5I + 5J - 2K$. Something like this but coming out of the board. It has us a Z component also.

Or going into the board because the Z component is negative. A force like this to change the gear. What is the equivalent torque and force at this point? So we are in a way transferring the force parallel to itself here and consequently, we will indicate this force by an equivalent moment, torque and a force F.

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So let me make the picture again. Here is the handle of the gear, this is the gear head. This is 60 cm or 0.6 meter, 45 degrees, this is 0.3 meter and this is also 0.3 meter. And the force out here, it is $-5\hat{i} + 5\hat{j} - 2\hat{k}$. The torque of the force is going to be $\vec{r} \times \vec{F}$. Equivalently I can think of an equal and opposite force being applied here and a force equal to the original force being applied at the bottom.

I have done nothing but added a 0 force onto the system. Now this $-F$ and the original force therefore create a couple here it is given by τ and the force is the original force here. Let us therefore calculate this couple. Either I can calculate the perpendicular distance between the forces or I can straightaway use the formula τ equals $\vec{r} \times \vec{F}$. C what \vec{r} is going to be in this case.

It is going to be equal to $0.6/\sqrt{2}\hat{i} + 0.6/\sqrt{2}\hat{j}$. That is the vector from here, the 1st part of the gear handle, this vector. $+ 0.3\hat{i} + 0.3/\sqrt{2}\hat{i} + 0.3/\sqrt{2}\hat{j}$. And therefore the total vector is $0.9/\sqrt{2} + 0.3\hat{i} + 0.9/\sqrt{2}\hat{j}$.

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$$\vec{r} = \left[\left(\frac{0.9}{\sqrt{2}} + 0.3 \right) \hat{i} + \frac{0.9}{\sqrt{2}} \hat{j} \right] \times$$
$$\left[-5\hat{i} + 5\hat{j} - 2\hat{k} \right]$$

And therefore, the torque is going to be equal to 0.9 over root $2 + 0.3 I + 0.9$ over root $2J$ Cross the force is $- 5I + 5J - 2K$. I leave this as an exercise for you to calculate the torque. But once you calculate this torque what you would find is this gear handle on which this force was being applied, this force system on this is equivalent to, on the same gear handle as if we are applying a torque here of the amount calculated here and a force, the original force which is $- 5I + 5J - 2K$.

So this sort of concludes an introduction to moments, couples and finding equivalent force and couple systems given a force about a particular point.