Engineering Mechanics
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Module 1
Lecture No 12
Calculating torques and couple moments-II

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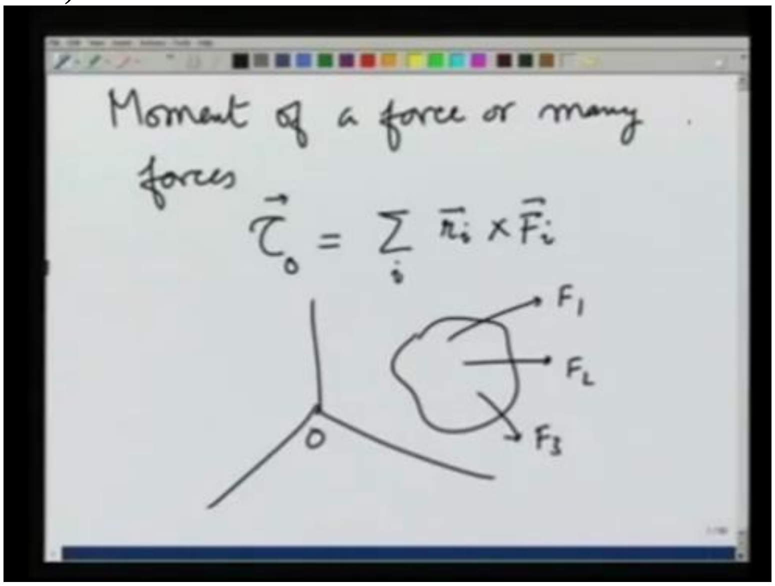

We have been learning about the moment of a force or many forces. What we have learnt so far is that the torque about origin O so that I will indicate by this is RI cross FI where we are applying these different forces F1, F2, F3 and so on about point O. We have also learnt that the torque is origin dependent.

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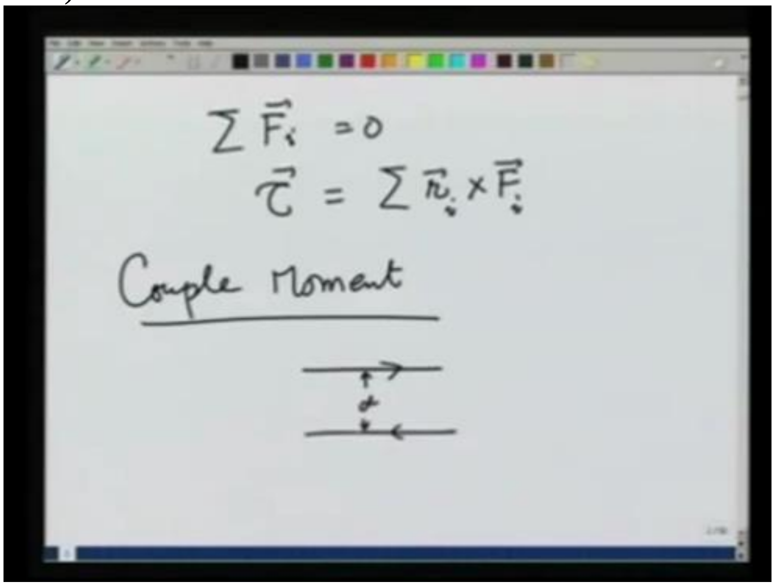

There is a very special case when the total force on the body is 0, then torque is independent of the origin. No matter about which point it is taken, it comes out to be independent. Then as a very special case of this, we define a couple moment which is nothing but two equal and opposite forces separate by a distance D. In this lecture, we study moment a little more. We will define the moment of a force or a torque about an axis.

Then I will give you an example to show you what you have been learning in 12th grade and our definition is the same. Then we will want to discuss the different elements in mechanical systems or civil engineering systems and what kind of forces do they apply with examples.

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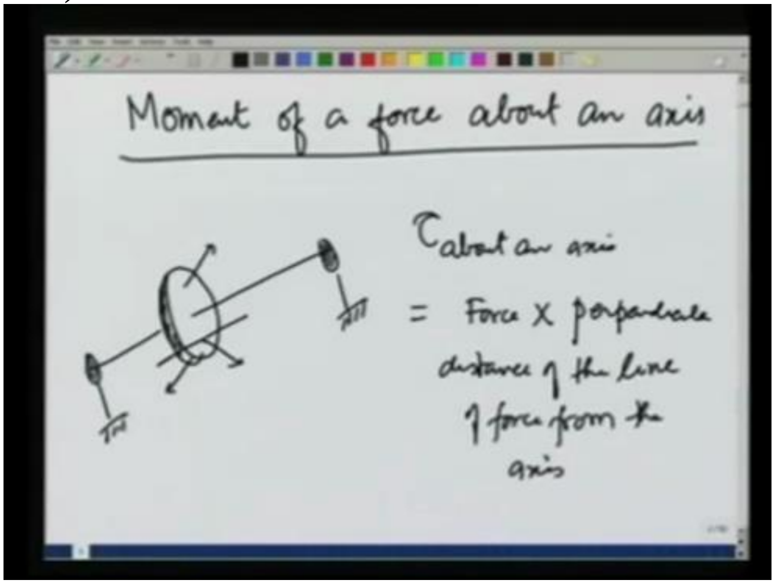

So, we are going to study in this case, the moment of a force about an axis. Why this is important is a lot of times, for example you have situations where something is actually made to rotate about an axis. This axis of a rotating disc. I may have fixed in ballbearings which are fixed in someplace. In that case, no matter what force you apply on the disc, the only component that is responsible for its rotation is that in the plane of the disc.

And then if you recall your 12th grade physics, you have been defining torque about an axis as force times the perpendicular distance of the line of force from the axis. Now we will get more sophisticated now that we have mastered the vector algebra and so on.

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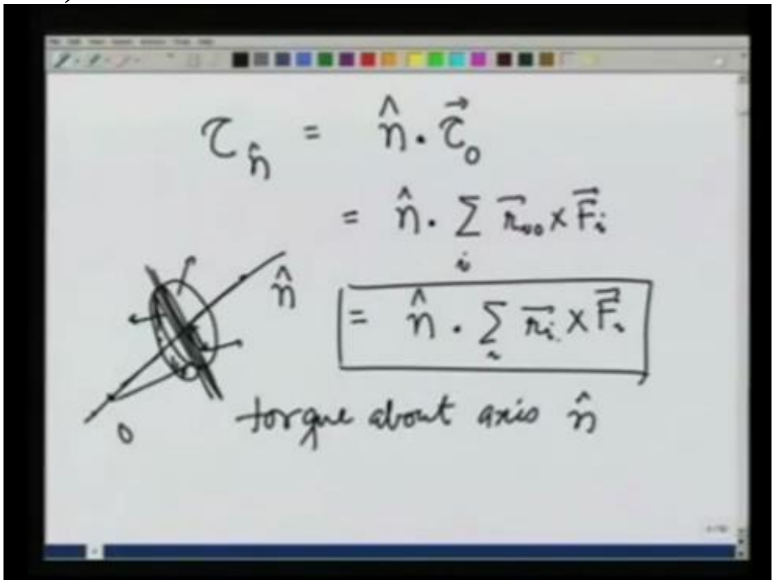

And define the torque about an axis in a slightly different manner which I will show through an example is equivalent to what you have been learning. So let us say that is a disc which is rotating about a fixed axis which is in the direction of a unit vector N and there are various forces working on it. Then the torque about the axis N is going to be equal to the dot product of N with respect to the total torque about said the origin O.

Although I am writing origin O, the torque about an axis is actually independent of where this origin is taken because it only depends on the perpendicular distance of the forces from the axis. So this I can write as N dot submission I RIO cross FI. And to emphasise that this is really independent of the origin, I will write it further as N dot summation IRI. I removed this index O cross FI.

This is the component of the total torque along the direction of the axis and this is what I will call the torque about an axis let us say N. If you really work it out, what is happening is since the axis is fixed, it is held by certain forces. No matter what forces you apply on the disc, the forces generated at the point where the axis is being held, are such that they will cancel certain the applied forces and certain components of them.

And then only force that is responsible for the rotation of the disc about this given axis is that which is in a plane perpendicular to this axis and more effective it is if it is farther away from the axis. This you know from intuition and from whatever you learnt in your 12th grade.

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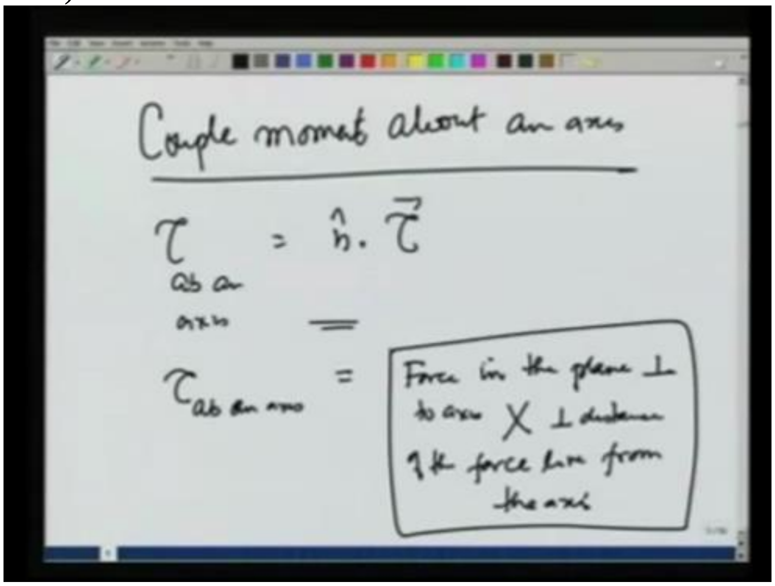

In a similar manner, the couple moment about an axis is the couple moment Tao dot N Tao about an axis. Exactly the same thing as as a torque because torque couple moment is nothing but is a very special torque. Let us now see if this fits well with our definition of torque about an axis is equal to force in the plane perpendicular to the axis times perpendicular distance of the force line from the axis. This is the definition that you have learnt in your 12^{th} grade.

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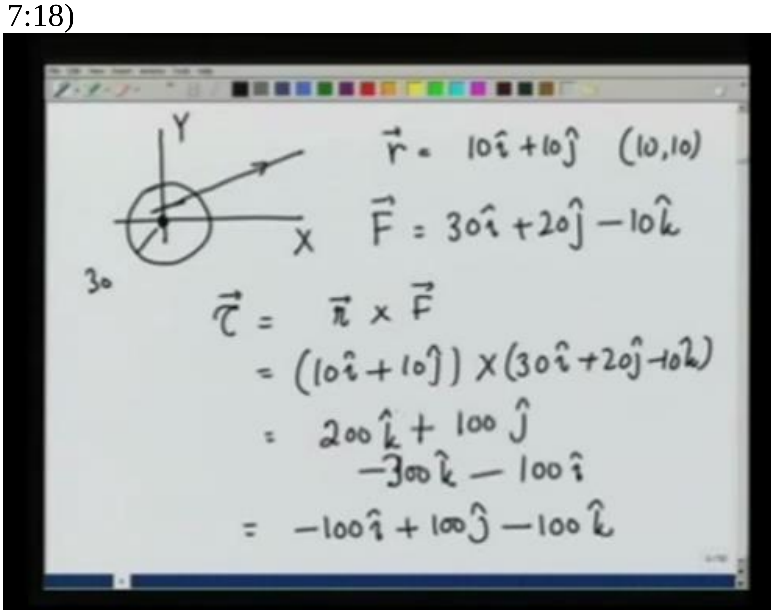

So as a simple example, let me take a disc which is free to rotate about the Z axis. Let services the X axis, this is the Y axis and Z axis is coming out of the plane. Let the radius of this be 30 cm and let me apply a force at a point R is equal to 10I + 10J. In XY notation, it is a point 10, 10. A force which is let us say 30 I + 20 J - 10 K. So I am applying a force at this point which is like this in the plane of XY and it has us the Z component also, - 10 K.

So therefore going into the plane of this board. The torque, total torque due to this force is going to be equal to R Cross F again I emphasise, recall from your previous lecture that this R could be anywhere along the line of action of this force. Right now we will take R to be 10 I + 10 J. So this is equal to 10 I + 10 J Cross 30 I + 20 J - 10 K. Let us work it out and it comes out to be 200 I crossed J which is K. I cross K is - J.

So - - + 100J. J Cross I is - K. So - 300 K. And J cross J is 0. J cross K is I. So - 100 I. So this comes out to me - 100I + 100J - 100K. That is the total torque. But I want to find the torque about the axis Z because this disc is free to rotate about axis Z.

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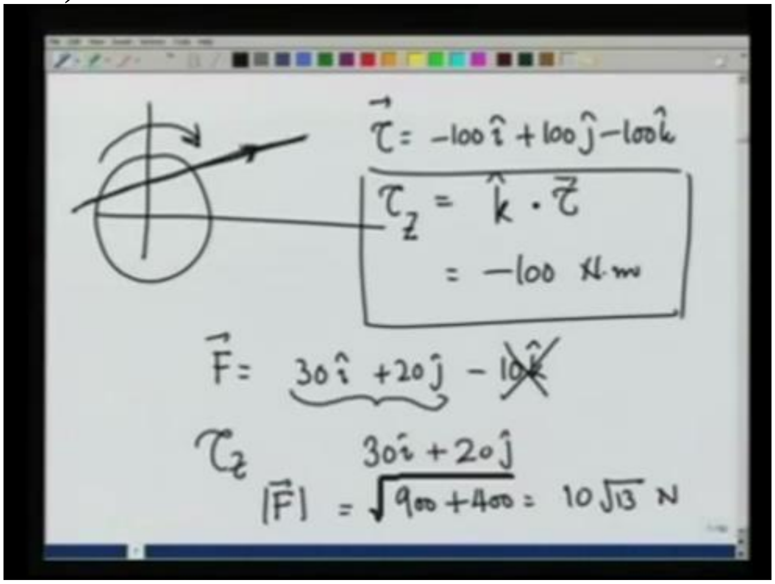

So that torque, let me make the picture again. Here is the disc. The torque came out to be - 100, is it - or +? - 100I + 100J - 100K torque vector. So talk about Z is going to be K dot torque and IS going to be - 100 Newton metres. - sign means that this is pointing in the direction of opposite to Z. Therefore the disc would tend to rotate with the right-hand rule in the clockwise direction.

This is the sense of rotation of the disc due to this torque. So torque about Z is coming out to be - 100 Newton metres. Let us see if this is consistent with what we have been learning in our 12th grade. So the line of force along which the force is working is this. The force is 30 I + 20 J - 10 K. As I said earlier, the only component of force that is responsible for rotation about the Z axis is that in the plane perpendicular to the axis. Only these 2 components.

So it is only these 2 components or the force in the plane of X and Y that is going to be responsible for the rotation of the disc. And therefore, 1st thing we do is for Tao about Z we ignore this force, the component of the force along the Z axis. It cannot apply any force about the Z axis. So Tao about Z is going to be given rise by 30 I + 20 J. What is the magnitude of the force? It is 30 square + 20 square.

So that comes out to be 900 + 401 that is 10 square root of 13 Newtons in this direction. I want to find the distance perpendicular to this force that is this distance D from the origin. Let us find that.

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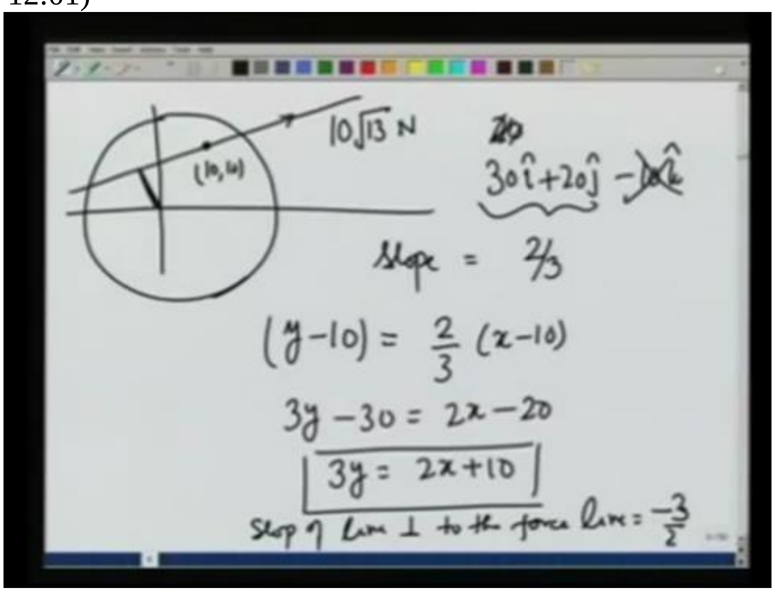

Let me make the picture again. So as far as the rotation of the disc is concerned, a force is acting on this of the magnitude 10 square root of 13 Newtons along this line which is given by the vector 20 sorry 30 I + 20 J - 10 K. This we have ignored. So in the plane XY, this is given by this. So this line has a slope of two thirds. And it is passing through the point 10, 10.

Therefore, the equation of this line along which the force is working is Y - 10 is equal to two thirds , that is the slope times X - 10. And therefore 3Y - 30 is equal to 2X - 20 or 3Y is equal to

2X + 10. I want to find this distance along a line perpendicular to this force passing through the origin. And therefore the slope of line perpendicular to the force line is going to be - 3 over 2.

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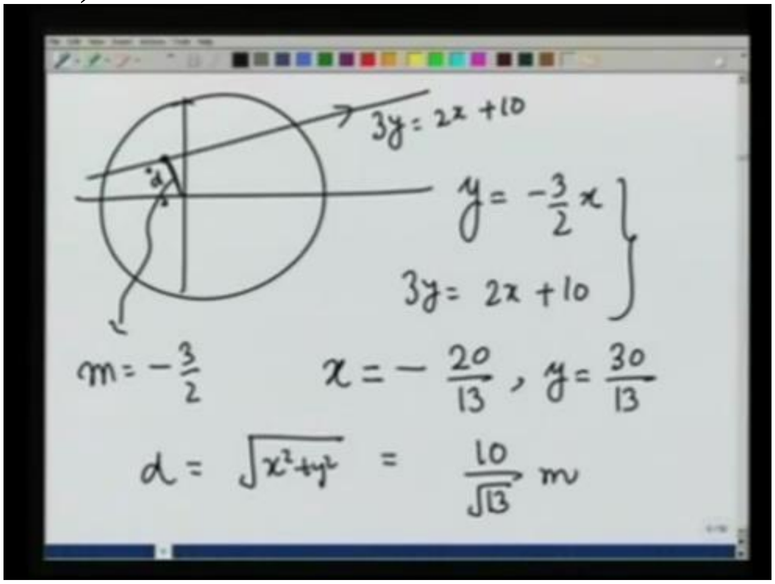

Let me make the figure again. This is the disc, here is the force acting and I am interested in this distance. The slope, the equation of this line is 3Y equals 2X + 10. Therefore the slope of this line M is going to be - 3 over 2 and it is passing through the origin. Therefore the equation of this line is going to be Y equals - 3 over 2X. The lines along which the force is working is 3Y equals 2X + 10.

And therefore I can find out what the intersection point is. You work it out and you will find that X comes out to be equal to - 20 over 13 and Y comes out to be equal to 30 over 13. And therefore distance D perpendicular from the origin for the axis of rotation to the force line is going to be equal to square root of X square + Y square which will be equal to 10 over root 13 metres.

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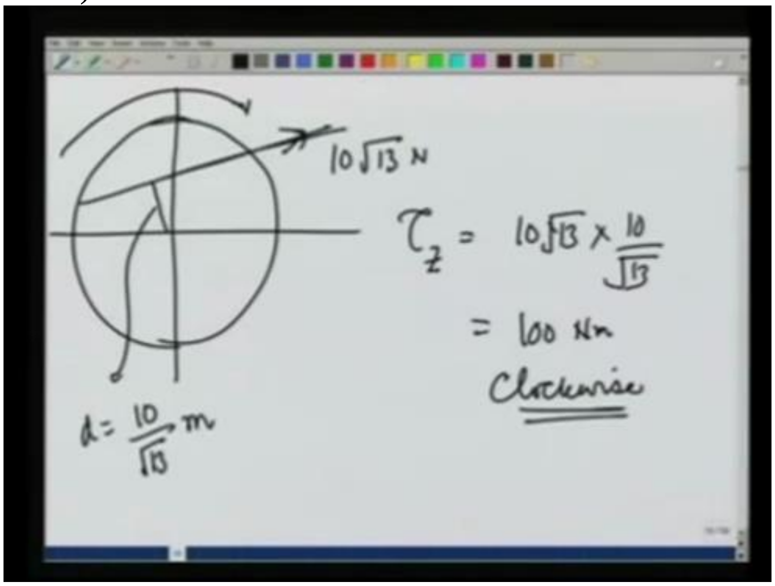

So we have F, we have D and therefore, the moment can be found out. The force is like this whose magnitude is 10 square root of 13 newtons. We have found this distance which is D is equal to 10 over 13 m. And therefore the torque about axis Z is going to be 10 root 13 times 10 over root 13 which is 100 Newton metres.

And because of the direction of the force being this way, the sense is clockwise. This is exactly the same as we found earlier which was coming out to be - 100 Newton metres. So you see the 2 definitions are consistent.

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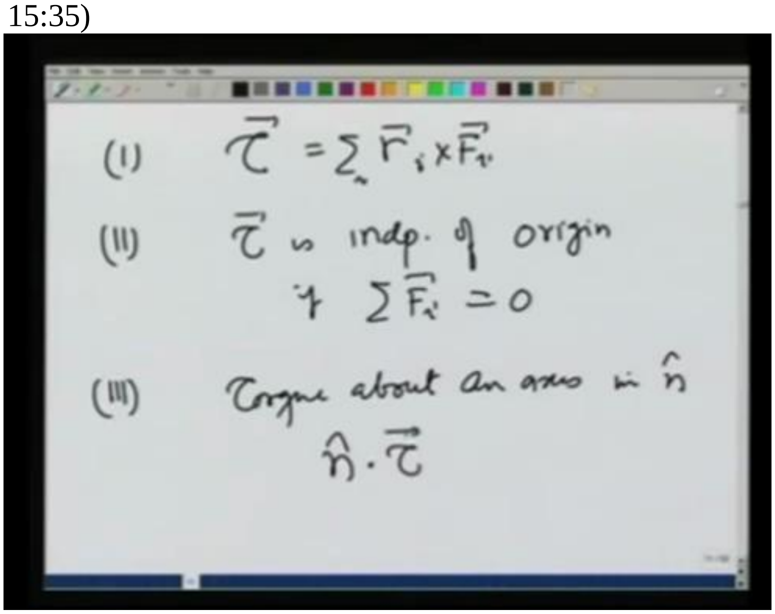

So let me now summarise what we have learnt about torques. Torque one, is equal to RIFI summed over. Two, torque is independent of origin if summation FI that is the total force on the system is 0. Third, that torque about an axis in vector direction unit vector N is N dot Tao.