Engineering Mechanics
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Module 1
Lecture No 11
Calculating torques and couple moments-I

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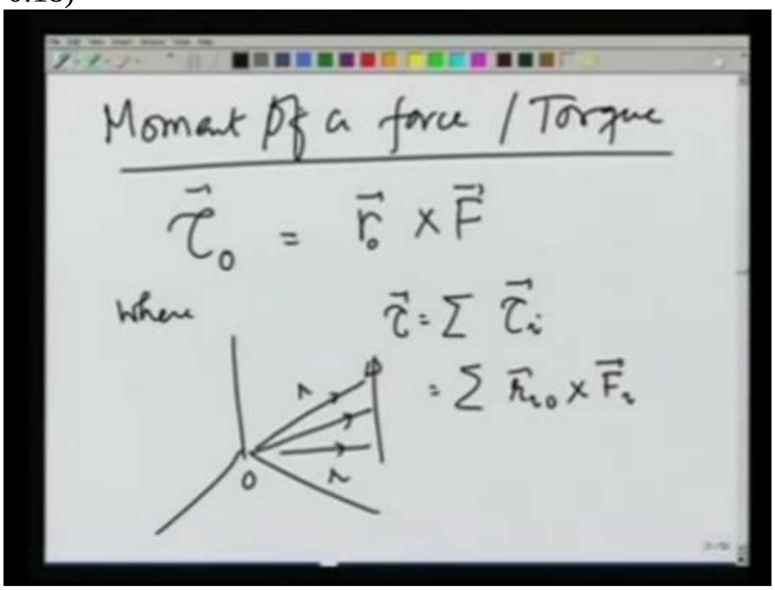

As indicated earlier, the moment of a force or torque is given by R Cross F where R is a vector from the origin to any point on the line of action of force. If there are many many forces, then the total torque is going to be equal to sum of all these torques which is going to be summation RIO Cross FI. Let us see that through an example.

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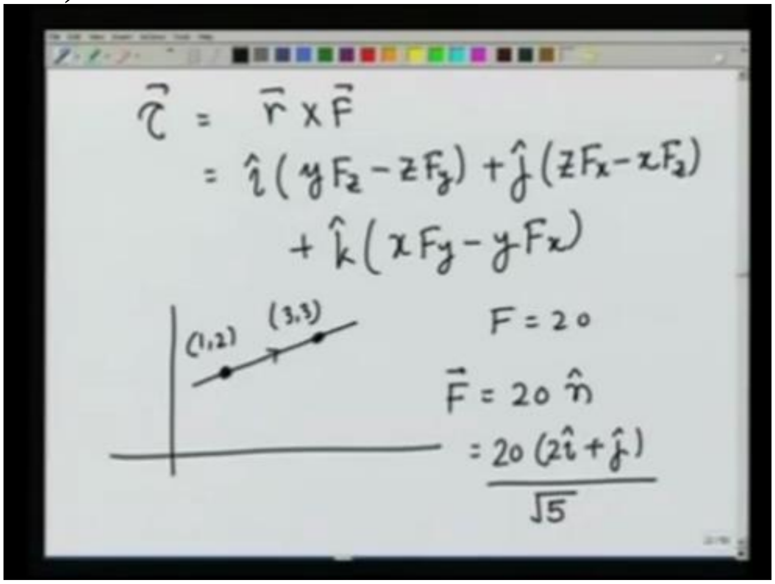

So if I am calculating the moment Tao which is R, I am dropping O now. It is understood that it is depending on the origin. Cross F is going to be equal to its component X direction is going to be equal to YFZ - ZFY + component in the Y direction is going to be ZFX - XFZ + the component in the Z direction is going to be XFY - YFX. So let us take a simple two-dimensional example.

Let there be a force of 20 newtons acting along the line which is passing through say 1, 2 and 3, 3 so that if I were to write the forces as vector, it would be 20 times a unit vector in this direction which I can write as 20 times, a vector in this direction would be 2I + J divided by square of the magnitude of this vector 2I + J which will be square root of 5. So this is the force that is being applied.

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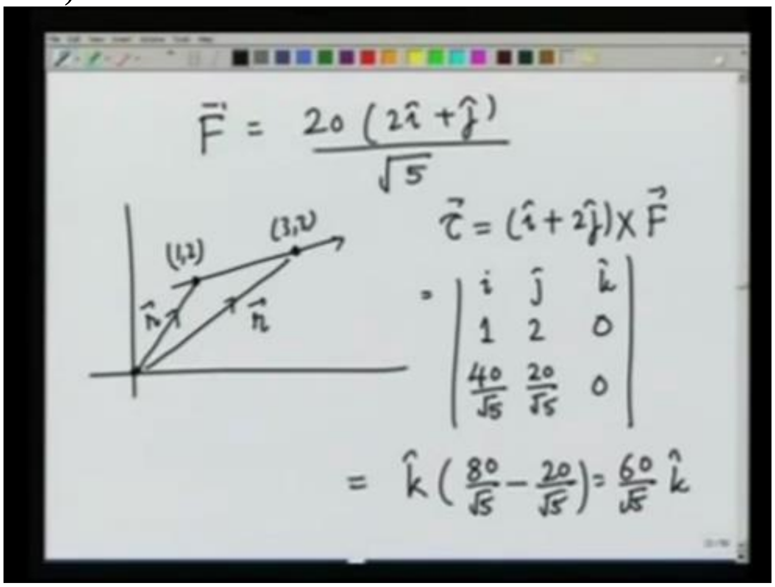

So the force that we are applying is F equals 20 2I + J over square root of 5. It is being applied along the line which is passing through 1, 2 and 3, 3. Letters calculate the torque about the origin by 1st taking R from 0 to 1, 2 so that this vector is going to be I + 2J and we are crossing it with F which is going to be nothing but IJK the determinant way. I is 1, 2, 0. For the force, it is 40 over root 5, J component is 20 over root 5 and 0.

This gives I component 0, J component 0 and K component is going to be 80 over root 5 - 20 over root 5 which is 60 over root 5K. That is the torque. What will torque be if I take R instead to be the vector from 0 in the origin to 3, 3. Let us calculated that.

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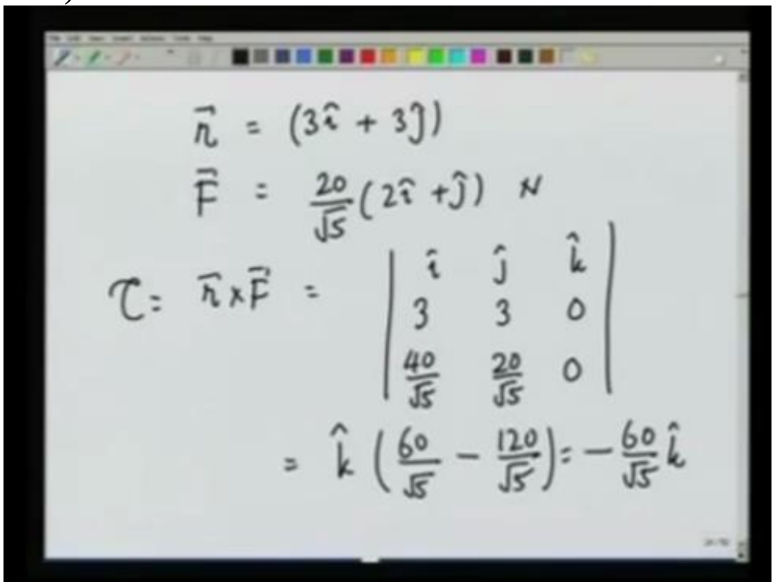

So that vector is going to be 3I + 3J. Force of course is 20 over root 5 2 I + J newtons. So I calculate torque which is R Cross F is going to be equal to IJK, this determinant. 3, 3, 0, 40 over root 5, 20 over root 5, 0. Again it gives only nonzero components in direction K is going to be equal to 60 over root 5 - 120 over root 5 which is - 60 over root 5K. The direction seems to be opposite. I must have made a mistake in the previous part.

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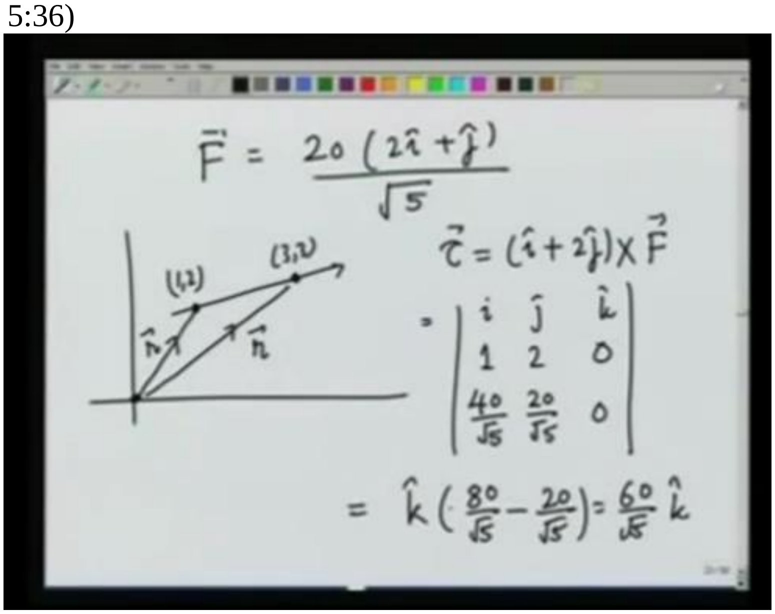

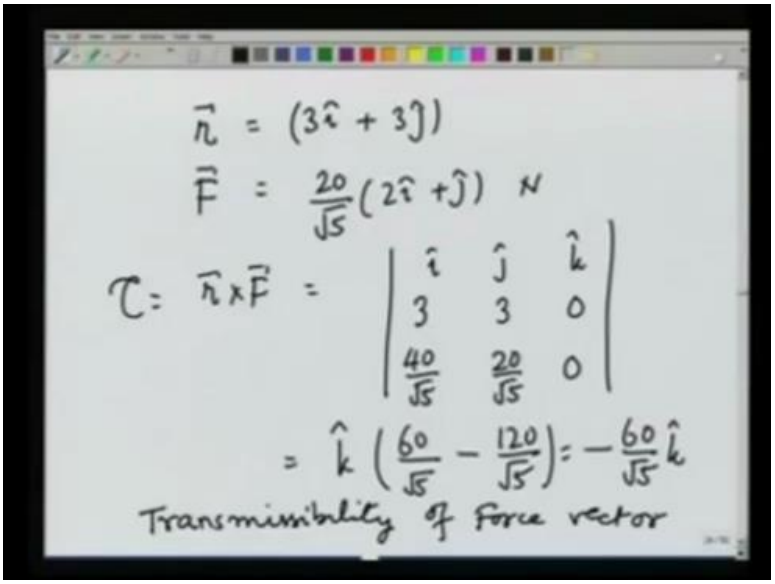

Let us check that. And yes indeed, there is a mistake. This should be -, this should be +. So in the previous part also, the answer was - 60 over root 5K and this part also the answer we get is - 60 over root 5K. And therefore you see that no matter where I take that vector R to be along the line of application of the force, the torque comes out to be the same. This is also an example of transmissibility of force vector that we talked about in the previous lecture.

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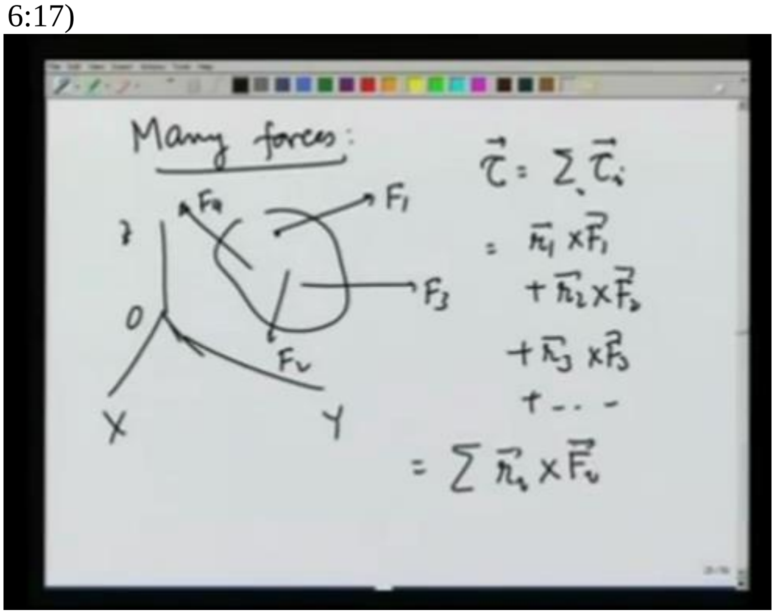

As I said earlier, if there are many forces on a body, origin X, Y, Z there are many forces working, F1, F2, F3, F4 and so on. Then the total torque would be equal to summation of individual torques which will be equal to R1 Cross F1 + R2 Cross F2 + R3 Cross F3 and so on is equal to summation RI Cross FI where RI is the vector from the origin do any point on the line of action of the force FI.

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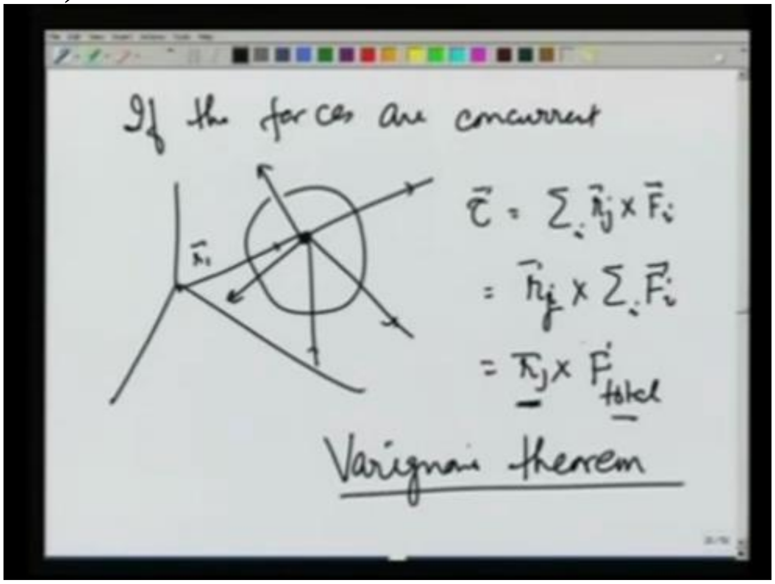

If the forces are concurrent, that means they all meet at some point, this may be one force, this may be the other force, this maybe the 3rd force, 4th force, 5th force and so on. Then the torque I can take this point where they meet to be the point where I am going to take the torque displacement and therefore this is going to be RJ. Let us call this RJ Cross FI.

It is equal to RJ can be taken out because it is same for all forces Cross FI which is nothing but RJ Cross F total. So if all the forces meet at a certain point, the total torque due to all the forces is equal to the torque of the total force Cross with the direction up to that point. This is known as Varignon's Theorem. And true only when all the forces are meeting at a certain point.

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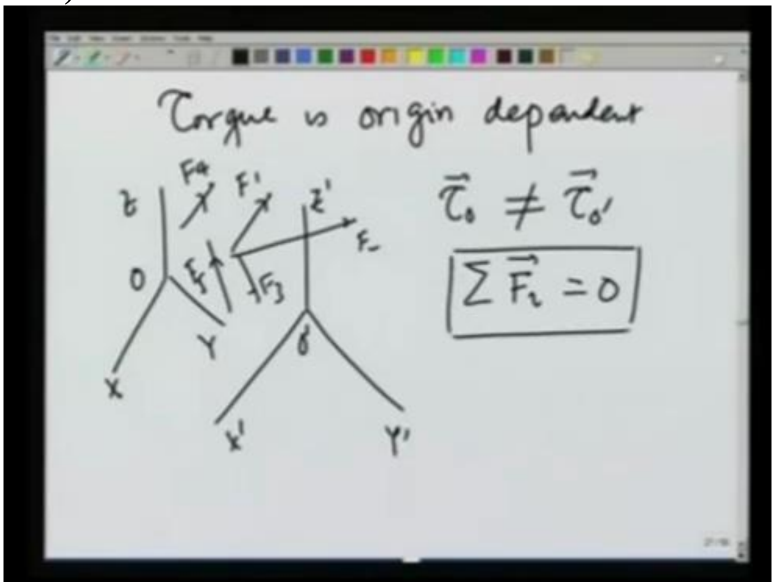

In general, since the torque is origin dependent, therefore if I take two particular frames O and O prime, X, Y, Z, X prime , Y prime, Z prime, Tao O is not going to be equal to Tao O prime for a given set of forces F1, F2, F3, F4 may not even Cross then F5 and so on. However, the result very special circumstances under which the 2 torques are equal. And that is when summation of FI is equal to 0. And I am going to show that to you.

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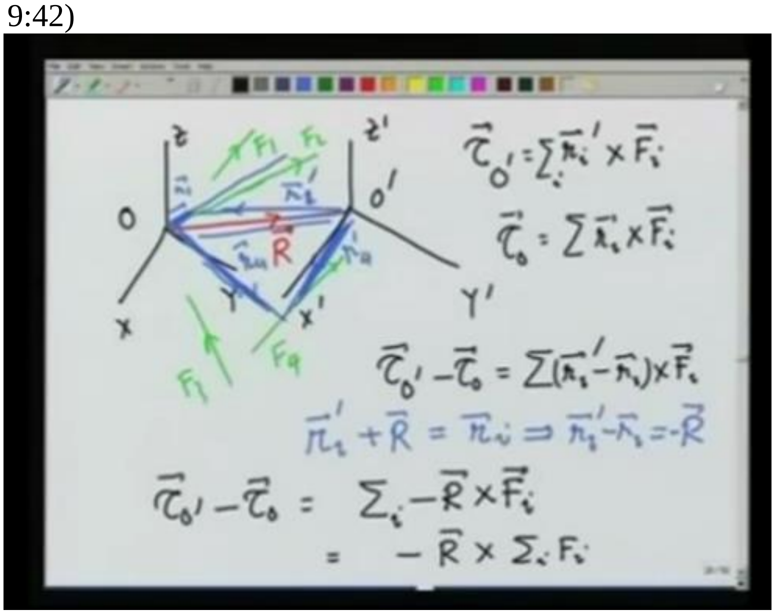

Let us take two different frames O, O prime, X, Y, Z, X prime, Y prime, Z prime. Let the distance vector displacement vector from O to O prime be R. I am going to draw in green, the forces. Let this be F1, let this be F2, let this be F3, let this be F4 and so on. Then the torque about O prime is going to be RI prime Cross FI summed over I. And torque about O is going to be summation RI Cross FI where RI and RI prime I take to be the vectors, let me draw them in blue, touching at the same point.

So let us say if I extend this force like this, this could be R1, this could be R2 and up to this point, this is R1. Or for F4, this is R4 and this R4 prime. Oh this is sorry, this should be R1 prime. So if I now take the difference between Tao O prime - Tao O, this is going to be equal to summation RI prime - RI Cross FI. And from the figure, it is clear that this is R4 prime, this is R and this is R. So RI prime + R is equal to RI.

Like R4, this is R4 is equal to R + R4 prime. Similarly R1 is equal to R + R1 prime. So RI prime + R is in general RI. And therefore RI prime - RI is equal to R with a negative sign. And therefore Tao O prime - Tao O is equal to summation I - R Cross FI is cross summation. R being the same vector, I can write this as - R cross summation IFI. That is the difference between the torques taken about point O.

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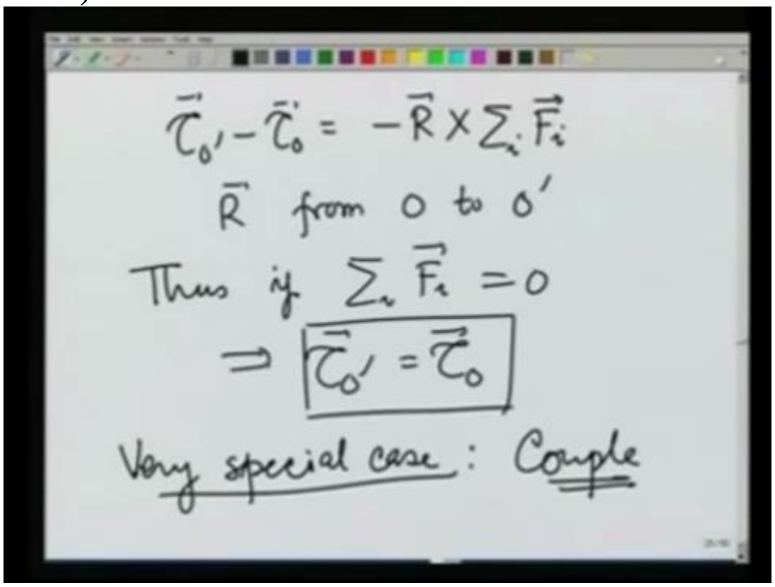

So let us write this again. Tao O prime - Tao O is equal to - R cross summation IFI. R's are better from O to O prime. Thus if A total for summation FI is equal to 0, this implies how O Prime is going to be equal to Tao O. The torque becomes independent of the origin because I took O and O prime arbitrarily. This is what I promised you earlier, I will show that if the total force on a system is 0, then the torque is independent of the origin.

No matter about which point I take the torque, it will always come out to be the same. A very special case of this is something called a couple which is used in engineering mechanics quite a lot. And what is a couple?

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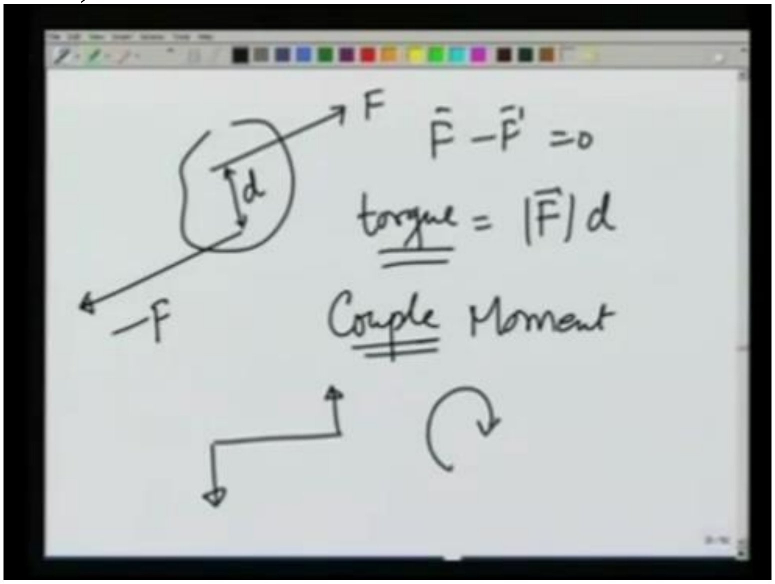

A couple is suppose I take a body, apply a force in one direction, apply a force in the opposite direction, - F and they are displaced parallely from each other by a distance D. Then net force F - S is equal to 0. And the torque need not be 0 because the forces are displaced from each other is going to be magnitude of F times D where D is the perpendicular distance between the 2. And this is known as a couple.

Or couple moment. It is indicated either by this symbol where these 2 arrows show the force applied or a symbol like this. So it is a pure moment without any force.

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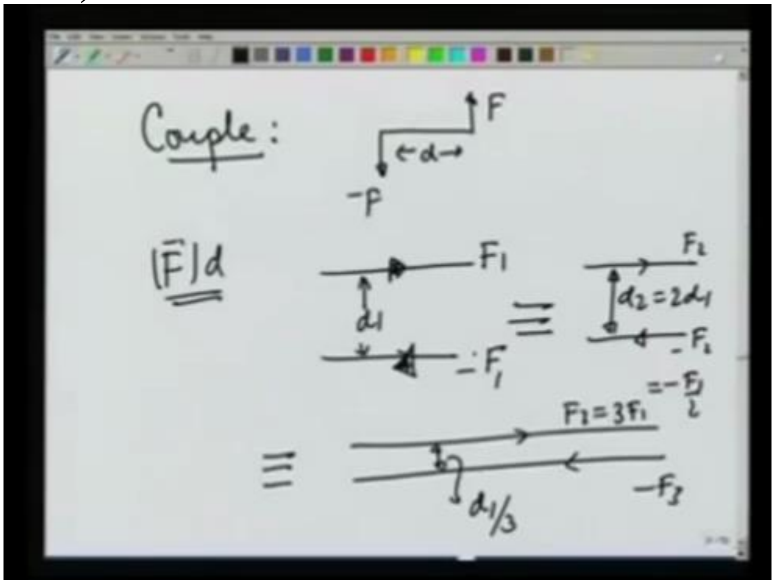

Thus a couple is a force or two forces F, - F displaced by a parallel distance between them D and its moment is given by FD. Since the net force is 0, a couple, the same couple can be generated by different kinds of forces. For example, I could have a force F1 going the other way, - F1 displaced by a distance D1 from each other. This would be equivalent to a force F2 which lets say is - F1 divided by 2 about the distance D2 which is twice D1 which will also be equivalent to a force F3 - F3 let us say which is equal to 3 times F1 but displaced from each other by a distance of D1 over 3.

The all 3 of them represent the same couple. Because the net force is 0, so there is no effect of the force. So let me just be reap what all did we do in this lecture.

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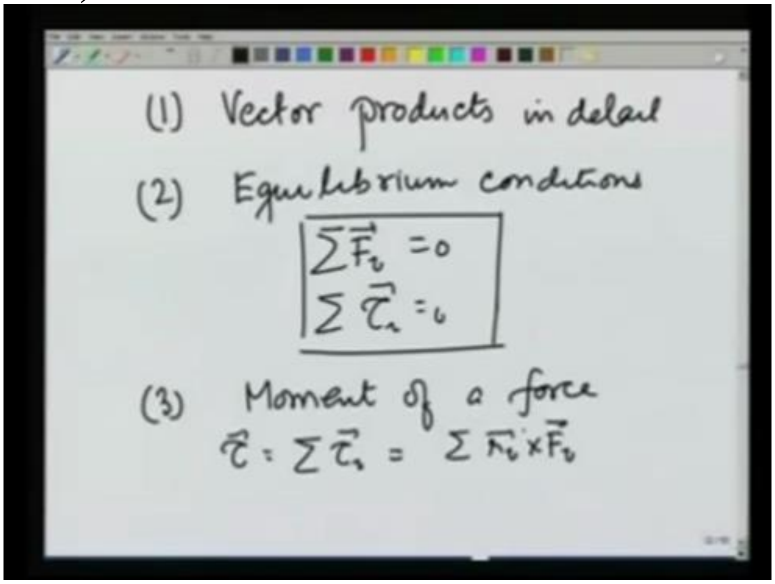

One, we looked at better products in detail. Then we looked at equilibrium conditions and found that summation FI is equal to 0 and summation torque I is equal to 0 are both necessary and sufficient condition to ensure equilibrium. Then we started looking at different aspects of equilibrium separately. And 3rd will look at moment of a force and found that Tao the total torque Tao I summed over is equal to RI Cross FI case origin dependent.

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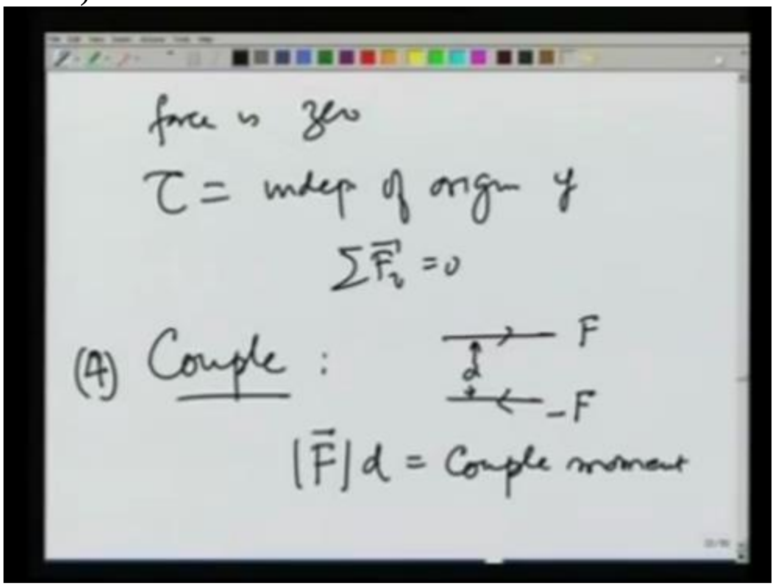

But becomes independent of origin is total force is 0. Let me rewrite this again. Tao is independent if summation FI is equal to 0. And then we looked at a very particular special kind of torque under these circumstances called a couple which is 2 forces equal and opposite forces separated by a parallel distance D.

In this case, the force magnitude times D is the couple moment and the direction you can find by right-hand rule or the cross product rule yourself. In the next lecture, we will look at how to define the moment about an axis or a couple about an axis and then go on to discuss other things about equilibrium.