Engineering Mechanics
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Module 1
Lecture No 10
Equilibrium of rigid bodies-Forces and torques

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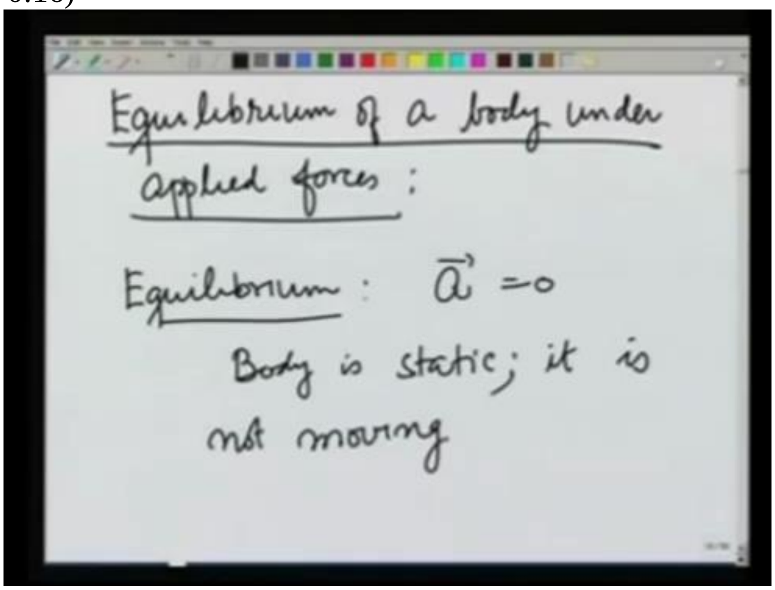

Now we are ready to look at equilibrium of a body under applied forces. As said in my previous lecture, by equilibrium, in general we would mean that the acceleration is 0 but here we would also mean that the body is static. It is not moving. So let us look at the conditions that are necessary and sufficient to make sure of that. Let us go step-by-step and look at a point.

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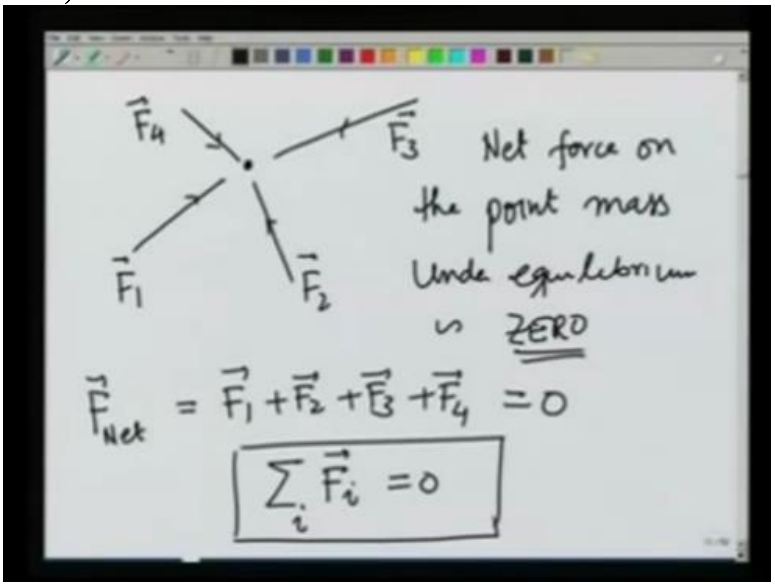

Suppose I have a point mass and I want it to remain stationary. But there are several forces applied on it. So this could be F1, this could be F2, this could be F3 and this could be F4. If the body does not move and its acceleration is 0, that means the net force on the point mass under equilibrium is 0. And that means, in this particular case that F1 + F2 + F3 + F4, the better sum of all these 4 forces gives me the net force and that must be 0.

In general, if there are more than 4 forces, there may be N forces. I should have one condition, summation over I all forces is equal to 0. This is one condition for equilibrium. So there could be more than one force, more than 4 forces. All sorts of directions, this direction, this direction. Sum of all that must be 0 in order for the point mass to be in equilibrium.

I have been talking about point mass because in that case we are all, it cannot do anything else. Just move. On the other hand, in engineering problems, we do not always have a point mass.

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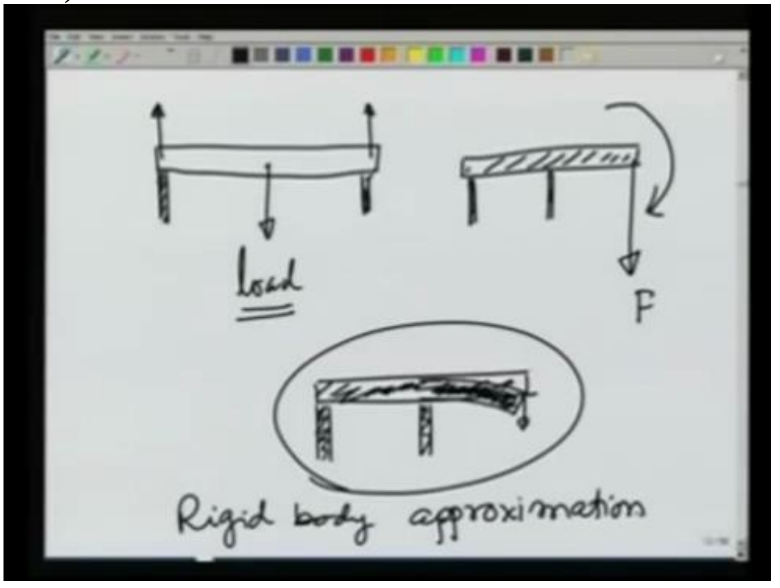

We have extended bodies. For example, I could have a beam that is supported on 2 pillars and there is a load here. Add the 3 forces that is one force, force applied by the support in this direction, force applied by the other support in this direction and the load. Not only these 3 forces should be 0, there should be some other conditions. What are those conditions? We are going to look at them.

When we apply force on an extended body, experience tells us that the extended body not only can move, it can have some more effect on the extended body. For example, if I put the support here and 1 support here and apply the force this way, you know that this body is going to rotate like this. Other effect that the force can have is again, looking at the same beam, being supported by 2 supports, 2 pillars, and if I apply a force here, it could bend like this.

So there are all sorts of things that a force can do besides just giving it a motion. It can make the body rotate or make it bend. In this course we are going to assume that this does not happen. The body is so rigid, its internal forces are so good that they adjust and do not let the body deform. This is called as rigid body approximation. The body is so rigid that it does not deform at all.

So the only 2 effects then the force can have is number 1 move it, number 2, rotate it. So we are going to consider these 2 things in this course for equilibrium.

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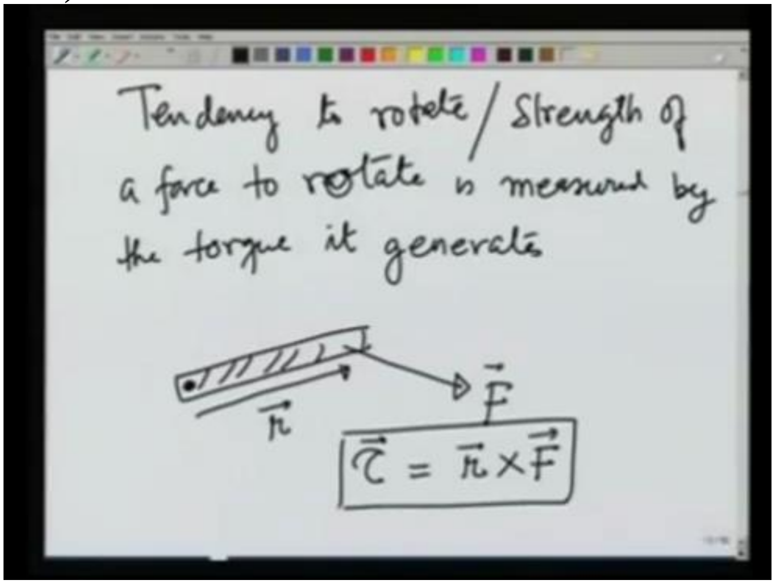

The tendency to rotate or strength of a force to rotate is measured by a quantity called the torque. By the torque it generates. For example suppose I have a body which is held at one point and if I apply a force in this direction, at distance or displacement R from the point at which it is held, then the torque is given by R cross F or the vector product of the displacement times the point at which the force is applied.

Experience tells us, farther the force from the point at which the body is held, more it rotates, more it tends to rotate. So therefore the distance is important. The vector product is important because if the force is applied in the direction of this point, if the force is along the same direction, it will not rotate. So vector product in that case could be 0. So this is the torque is defined.

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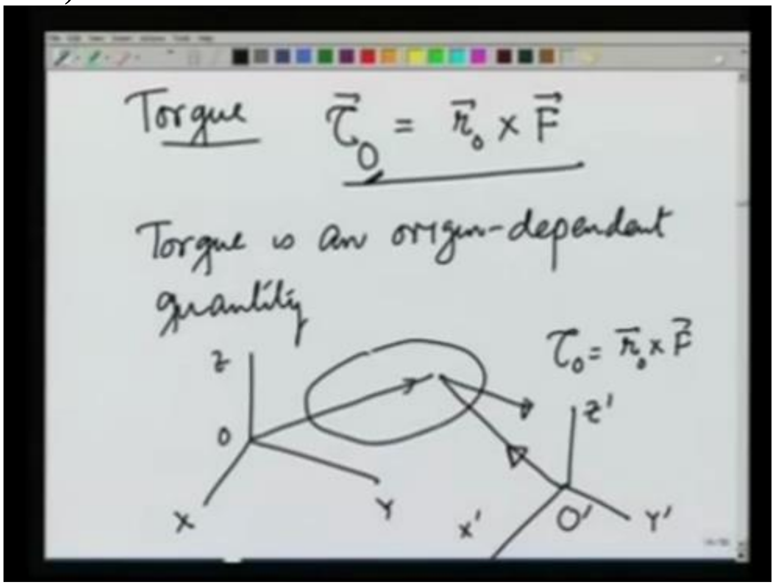

So let us write now, torque Tao from the origin O is going to be given as R measured from O Cross F. RO measures the point at which the force is being applied compared to the original or the point at which the body is being held. But naturally when you define torque like this, it is an origin dependent quantity. For example, let us take X, Y and Z axis.

Let us take the body here. If I apply a force at this point, in this origin, the torque R cross F, let us say RO. On the other hand, if I have a different frame of reference, centred at O prime, the torque you can see is going to be different.

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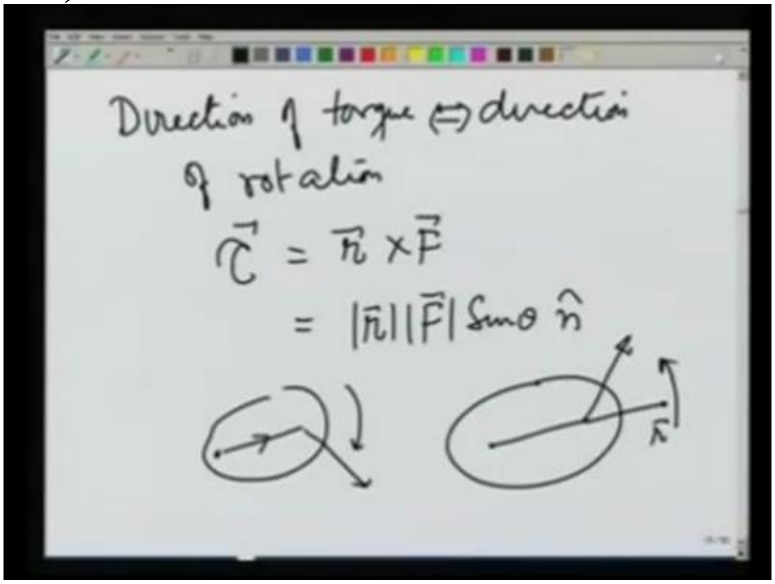

How do we relate the direction of torque to direction of rotation? So if torque is given by R cross F, it is going to be equal to magnitude of R magnitude of F sine of theta N where N is the vector that is given by this thumb when R is turning towards F through the smaller angle. The tendency of rotation is also the same. If N is the direction of torque, the fingers, right-hand fingers give me the direction of rotation as you can see.

If there is a body here, held here, if the force is this way, then R cross F is going to go into the board. So body you can see has a tendency to rotate this way. On the other hand, if the force is in this direction, then R cross F is coming out of the plane of this board. And the body has a tendency to rotate in this manner. So this is how we relate the torque direction and the direction of tendency of rotation.

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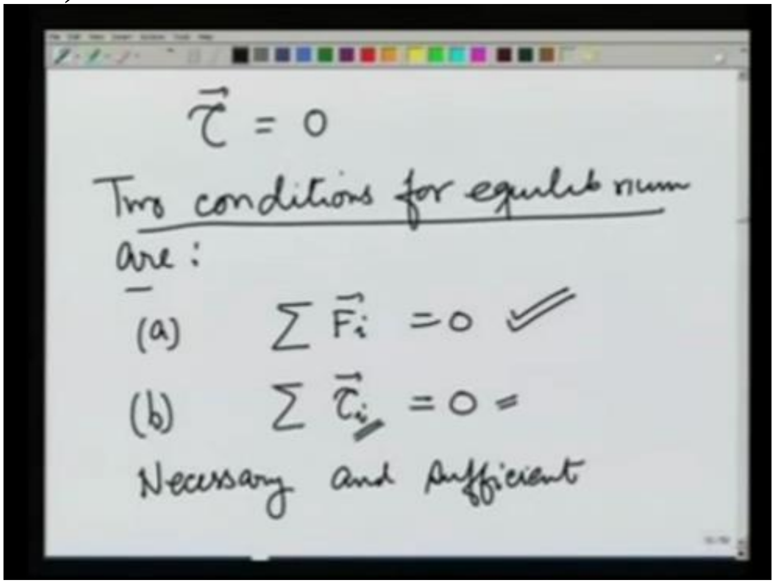

So if we want that in addition to not moving, the body also does not rotate for equilibrium, then the torque on the body should also be 0. So two conditions for are a-sum of all the forces FI is 0, then the body would not move. And b, some of all the torques applied on the body should also be 0. You may ask, I have just now said that torque depends on the origin and I have not put any origin here. Once you satisfy this condition that sum of all the forces is 0, the torque the net torque is going to be independent of the origin as we will show later.

So therefore, this condition is also independent of the origin once I make sure that summation of FI is 0. And these 2 conditions are necessary and also sufficient for equilibrium of a body. Let us take an example. A simple one to start with.

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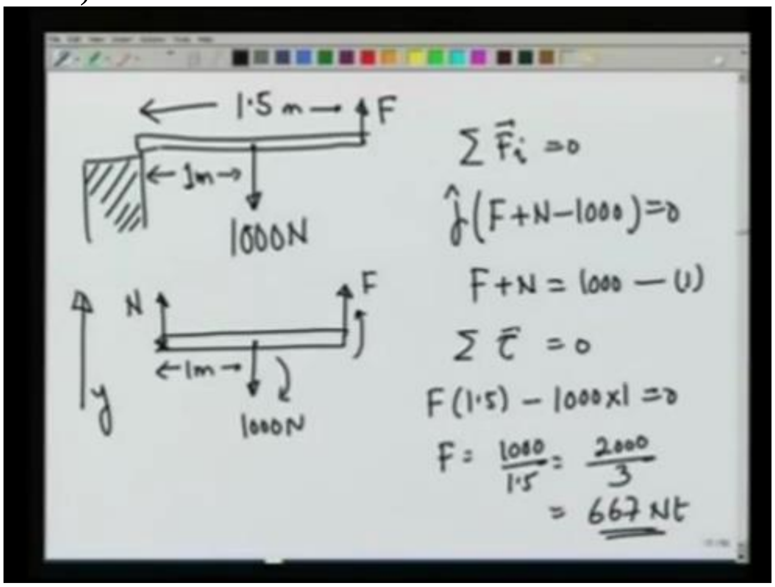

Suppose I have a beam which is being supported on a block of say cement. The beam is lightweight. So I can ignore its mass. And suppose it is holding a weight of 1000 newtons. What force F should I apply in order to hold this weight? Let us say this distance is 1.5 m and the load is being held at a distance of 1 m. If I look at the beam, the forces that are being applied on the beam are the force by me force of 1000 newtons at a distance of 1 m from the support and the support could also be applying a force. And…

So 1st condition that summation FI is equal to 0 gives me if I assume this to be the Y direction that J unit vector F + N - 1000 is equal to 0 or F + M is equal to 1000. That is condition 1. On the other hand, torque should also be 0 about any point I take. So let us take this point about which I am going to take the torque and then you see that 1000 newton gives me a torque clockwise.

The F gives me a torque counterclockwise and they should both balance each other. So if I write summation Tao is equal to 0. Since all the torques are in the same direction up or down, I can straightaway write this as F times 1.5 - 1000 times 1 should be equal to 0. Or the force is 1000 over 1.5 which is 2000 over 667 newtons approximately. So you see this is how I am going to apply the condition for equilibrium for the force and the torques.

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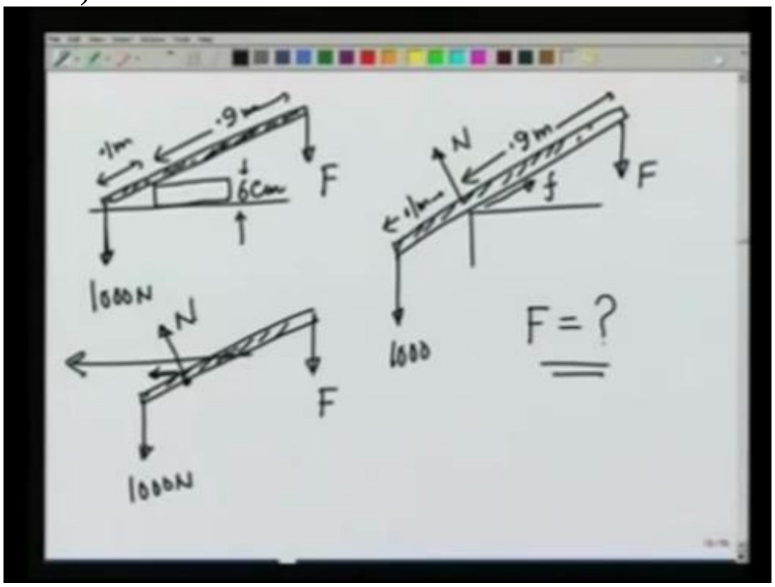

For the 2nd example, let us take the case of lever that we use every day. Let us suppose we have a very hard rod which is put on a brick on the ground. Let us say the height of the brick is 6 cm. Let this length be 0.1 m, let this length be 0.9 m and I am trying to lift a load which is pushing the rod down of 1000 newtons by applying a vertical force F. What all would happen?

Suppose initially I ignore the friction between the brick and the rod, then you can see that on the rod the forces that we are applying our number 1, the force downwards. Number 2, the force downwards because of this load, 1000 newtons and at this point, the brick is going to apply a force perpendicular to the rod in this direction. You can see right away if there is no friction this N is going to have a horizontal component that is not balanced.

So the rod is going to have a tendency to move this way. In order that the rod does not move, there should also be a fictional force between the rod and the brick. So we are going to assume that there is a sufficient frictional force that prevents the slipping to the left. So let there be a fictional force in this direction F. Let there be a normal reaction N, let there be a force that we are applying F, vertically down and let there be a force 1000 newtons here.

This distance is 0.1 meter, this distance is 0.9 meter and the friction is sufficient to prevent slipping. We want to know the value of F. How much force should I apply down in order to lift this weight of 1000 newtons.

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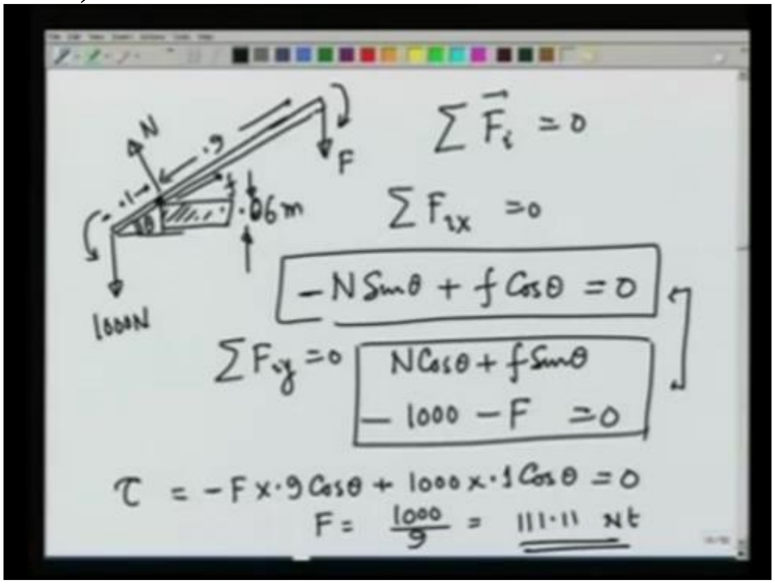

Again, we are going to apply the 2 conditions that we talked about. This is the brick of height 0.6 0.06 metre, 6 cm. Let this angle be theta. This is 0.1, this is 0.9. This is N. This way is frictional force F. This is F and this is 1000 newtons. So when I first write the force condition, summation FI is equal to 0, let us write its X and Y components. So summation FIX is equal to 0 gives me Nsine theta in the negative direction was F cosine of theta is equal to 0.

Similarly when I write summation FIY, the force component in Y direction is 0. This comes basically by writing force in terms of I and J and making each component of the vector 0. This gives me N cosine of theta less F sine of theta -1000 - S is equal to 0. So these are the 2 conditions for the force balance. How about the torque? For the torque, let us choose a convenient point because it does not matter which point I choose.

Let us choose the support right here as the point about which I will take the torque. Then you can see that this F has the tendency to take the rod clockwise and this F has the tendency to rotate counterclockwise and therefore two are working in opposite directions. The torque is all going to be this we take, this plane to be XY plane, torque is going to be in K or - K direction. And therefore I can just write the scalar numbers right away.

It is going to be equal to clockwise would give me a - K. So F times 0.9 cosine of theta with a - sign + 1000 times 0.1 cosine of theta is equal to 0. And therefore F comes out to be 1000 over 9 which is roughly 111.11 newtons and that is your answer. If you wish to find what N and F are, those can be found from these equations. You will require sine theta and cosine theta for that and that you can obtain from this triangle.

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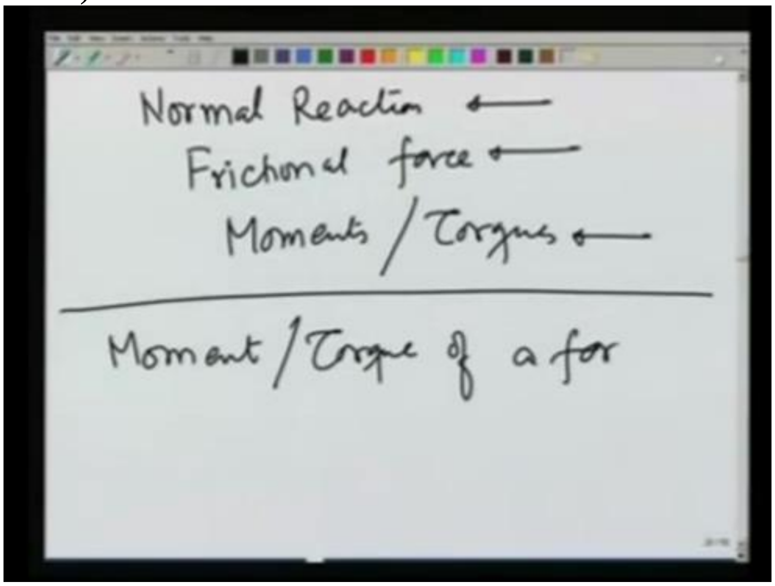

So in these 2 examples, you saw that I have used terms like normal reaction, frictional force, moments, torque, etc. so now we will specialise and look at each of these things more carefully. For example, if I take a support in which direction does it apply the normal reaction? In which direction are the frictional forces? What are the different types of frictional forces? We will study moments in slightly more detail to understand how it can affect the motion and what special moments are.

So we will take these topics one by one. I want to start by discussing moment of a force. Moment or torque of a force.