Introduction to Electromagnetism
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Lecture - 09
Curl of a Field – I

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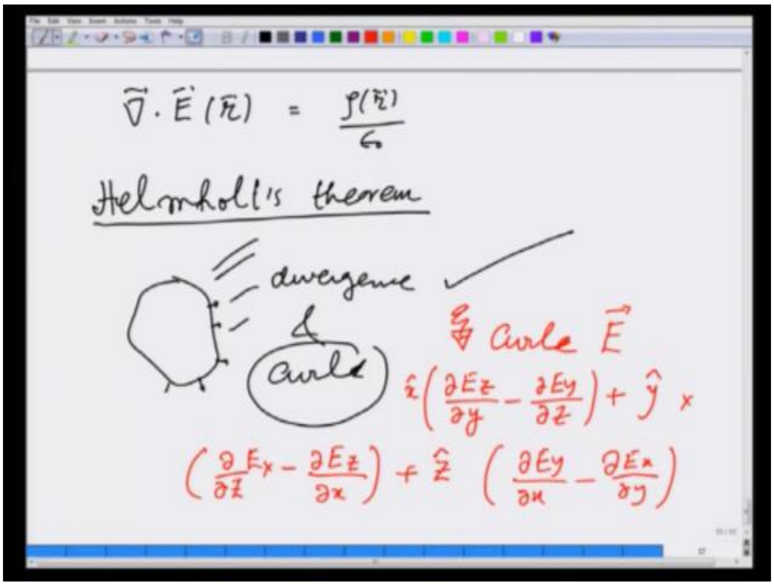

Recall that in the previous lecture, we obtained the divergence of electric field as rho r over epsilon 0. Also recall, before I started this mathematical treatment I had talked about Helmholtz's theorem without proof that said that if I know a vector field on a boundary or rather it is perpendicular component on a boundary. And if I know divergence and curl of this field, then I can calculate the field over the entire volume outside. So, this we have defined, now we want to understand the meaning of curl.

I had written when I had introduced this curl of that is again a specialized to E as x component partial E z over partial y minus partial E y over partial z plus y component being partial with respect to z of E x minus partial E z partial x plus z component being partial E y partial x minus E x E y. How does this quantity come about and what does it mean? In this lecture, this is what we will try to understand, so that you had a feeling for a curl. Hopefully by now, you do have a feeling of divergence diverging feel, flux coming in going out that we have discussed, let us understand the meaning of this.

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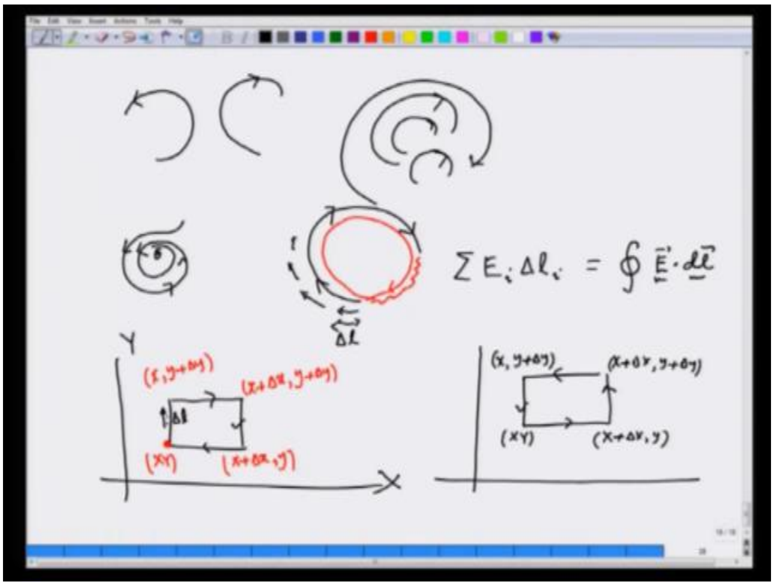

By in daily language, by curl you understand that something is going around, people have curly hair or if there is a sink and water goes in it, it goes like this. So, the velocity field becomes curly and we want to give a precise mathematical definition. So, suppose there is a vector field that is curling around, let us take one particular line of this vector field and calculate the following integral. I will move along the line, taking small segments delta l and calculate E on i'th segment, multiply by delta l along the lines and add it and make this path closed.

So, although they may not be any field out here, but I will make this path closed and let us see, what does it come out to be? For simplicity, again I am going to take a rectangular path, let us say in the x y plane. So, please understand what I am doing. I am taking E at each point, taking this component along the path and multiplying by the small delta l and summing it all around, which I can also write as what is called a line integral over a closed path E dot d l and this automatically gives the component of E along d l that I am taking.

So, let us do it for this small path in the x y plane, let us take the lower most corner of this to be x y. Let this be x plus delta x still at y, x plus delta x y plus delta y and x y plus delta y and usually, when we do this going around a path we take travelling direction to be positive if you are going counter clockwise, so I am going to take it like this. This is x y, x plus delta x y, x plus delta x y plus delta y, x y plus delta y and let us calculate this integral over this entire path.

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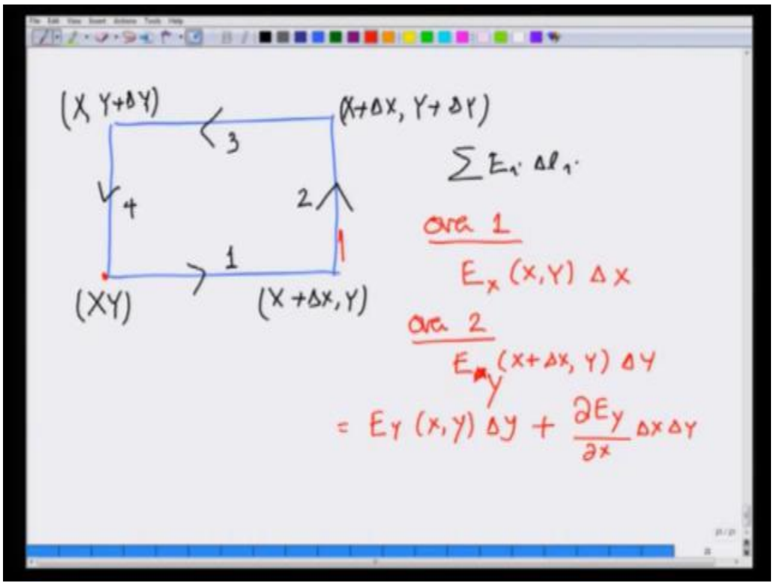

So, here is my path made in a bigger way and let me write the points again x y, x plus delta x y, x plus delta x y plus delta y, x y plus delta y and let us calculate traversing in a counter clockwise way. The sum E i delta l i, this is my path 1, 2, 3 and 4. Over 1, the value of E i delta L is going to be E x and let us take this previous point here to be representing E x E x y delta x. Over 2, it is going to be E x x plus delta x y delta y, it is not E x, because now I am going in the y direction, so it should be E y, which I can write as E y at x y delta y plus change in y with respect to x by the time I move delta x delta y. So, I have calculated over these two paths.

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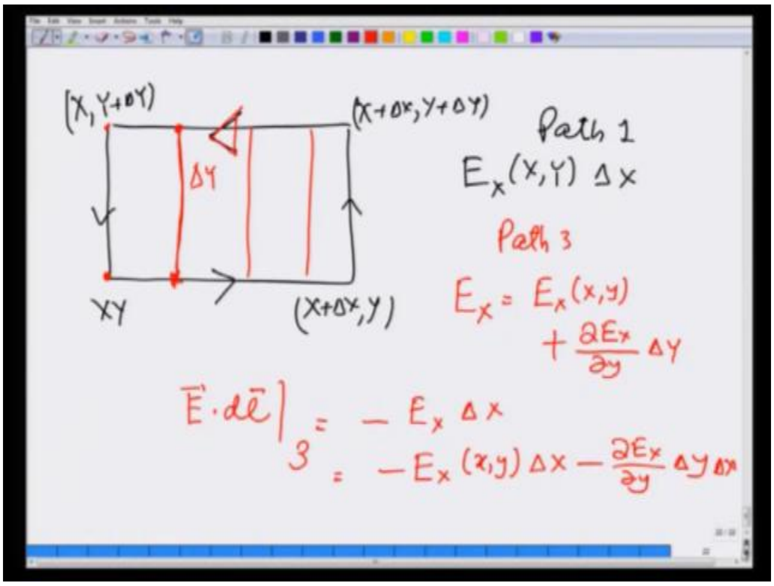

Now, let us go for to path number 3. I will again make it x y, x plus delta x y, x plus delta x y plus delta y and x y plus delta y. Now, I am going to calculate over this path number 3. Notice that, for path number 1 we had E x at x y delta x path 1. When I calculate it over path 3, for each value of x the corresponding value on path 3, the x value remains the same, the y value has changed by delta y. So, I am going to calculate for path 3, where E x is going to be E x x y plus partial E x over partial y delta y.

For each x, y has definitely changed at each point and we are taking the x value to be out here, same thing for this. So, E dot d l over 3 is going to be, now notice that d l is in the negative x direction. So, this is going to be minus E x delta x, which comes out to be minus E x x y delta x minus partial E x by partial y delta y delta x.

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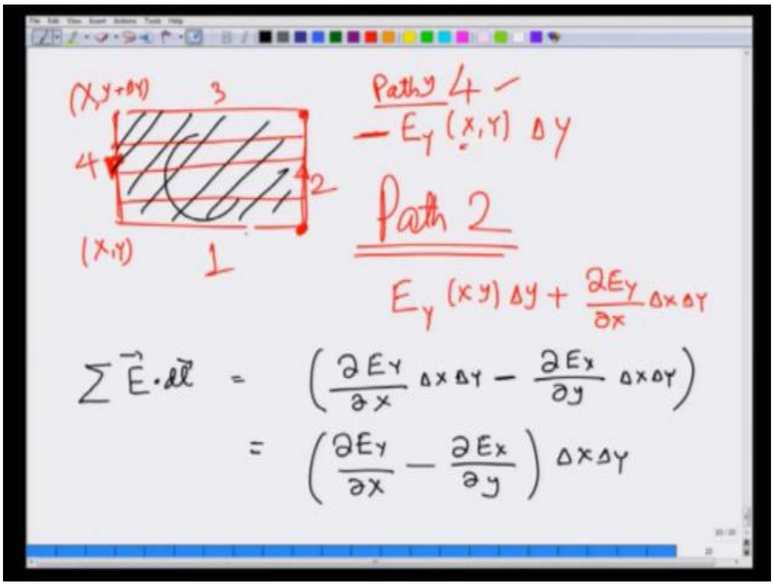

Similarly, over path 4 which is this path, this is x y plus delta y, x y everything is over at point x. So, I am going to have E y x y delta y and that is in the negative direction, so minus, because now the displacement is the negative y direction. So, for each point here, the only difference is that x is reduced by delta x, so it is at x and y, path 4. If I add all these together 1, 2, 3, 4 if I add all these together for a moment I will just show you, what it look like in path 1.

This was path 3, which became this. Path 2 and path 4 I have done, so 1, 2, 3, 4 let us calculate over path number 2, I had calculated over path 2. So, path 2 and path 1 and path 3 and path 4, when they are added together, path 2 was E y x y delta y plus partial E y partial x delta x delta y. If I add all these together, what you are going to get is that the integral E dot d l over this entire path is going to come out to be partial E y over partial x delta x delta y minus partial E x over partial y delta x delta y.

You can go to back to the previous slides and see that every other all other terms cancelled, which I am going to write to as E y partial x minus partial E x partial y delta x delta y. What is delta x delta y? Delta x delta y is nothing but the area of this small rectangle.

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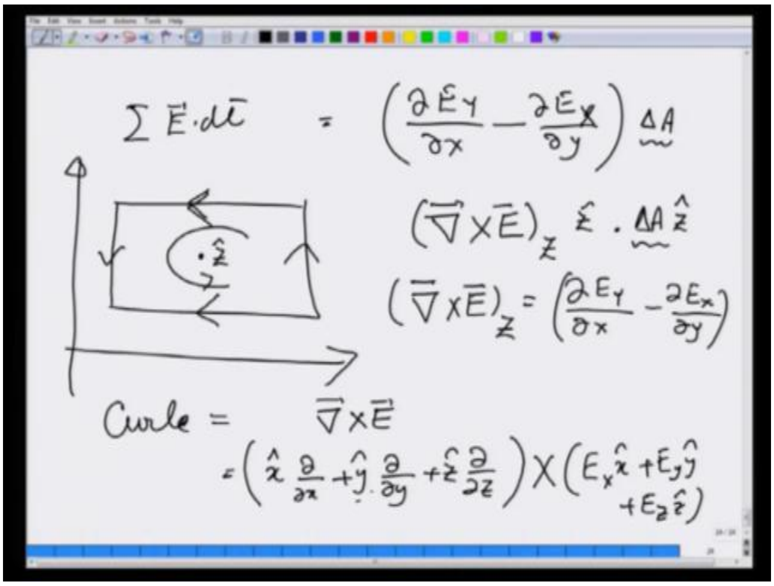

So, summation E dot d l over this rectangle is equal to partial E y partial x minus partial E x partial y, I will write x clearly times this small area. Taking the right hand convention; that means, if I curl my fingers around the path, the thumb gives me the direction of the area. So, if I go around like this, this is the x y plane remember x and y, thumb gives the direction of area I will take that convention, so that is a z direction. So, I can write this area, if I write it as a vector as delta A z vector.

You are not unfamiliar with the vector notation. So, earlier also when we were calculating divergence, we were taking area direction to be coming out of the volume. So, now, I am going to use this convention that, if I on a path, if I curl on a closed path, if I curl my fingers around the path the thumb gives me the direction of the area. So, I can write this as delta A z and this I am going to define as curl of E z component z dot z dot.

So, curl of E z component comes out to be partial E y over partial x minus partial E x over partial y. You can similarly calculate other components and I will leave this as an exercise for you, and then the curl is defined as curl of E which is symbolically x partial x plus y partial y plus z partial z which we… This is something which we had introduced earlier. Cross product with E x x plus E y y plus E z z, I will first take the cross product of unit vectors, and then do the differential operation and that will give you the curl.