Introduction to Electromagnetism
Prof Manoj K Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 08
Divergence of Electric Field and Gauss's Law

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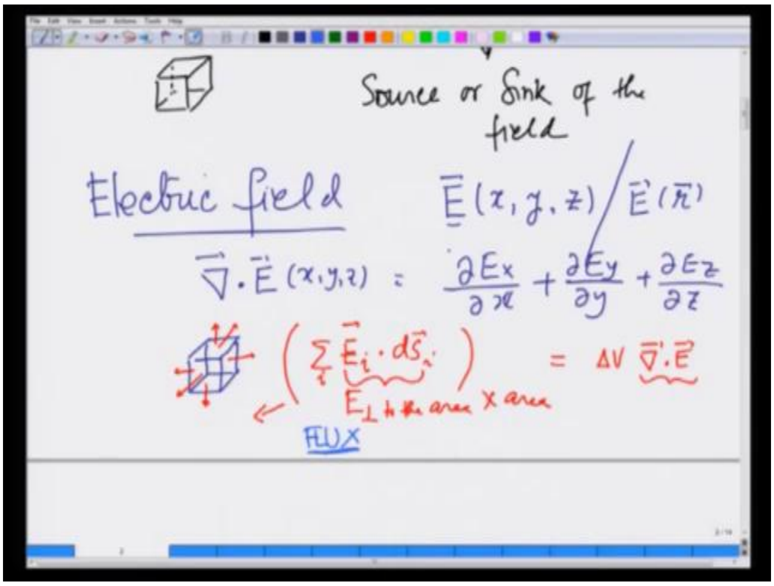

In the previous lecture for every vector field, we defined a quantity called the divergence. So, let me write this, we define a quantity called divergence that told us about whether in an enclosed volume, there was a net outflow of the vector field or net inflow for the vector field, and therefore it was related to source or sink of the field. We are now going to specialize to electric field and talk about the divergence of an electric field.

Recall that electric field is a vector quantity, which is a function of x, y and z; I can also write this as a function of vector r. So, is divergence is going to be the sum of the partial derivative of x component with respect to x, partial derivative of y component with respect to y and partial derivative of z component with respect to z. And again recalling how we define it, if I take close volume and take over all this surfaces, taking the area vector to be pointing away appointing out of the volume.

If I take sum of E fields on each surface dotted with the area and some over the entire all the surfaces, this what this means, I am taking the component, along the direction or perpendicular to the area and multiplying by the area. This was equal to this small volume, let us call it delta v times a divergence and this is what told us about, how it works as a divergence indicates, what a source or a sink is.

Recall that, if this quantity is 0, there is no divergence and therefore, a field or this some integration of the field over the area has to be non-zero in order for divergence to be non-zero and this in the case of electric field is defined as the flux. So, let us now calculate the divergence of the electric field due to a charge.

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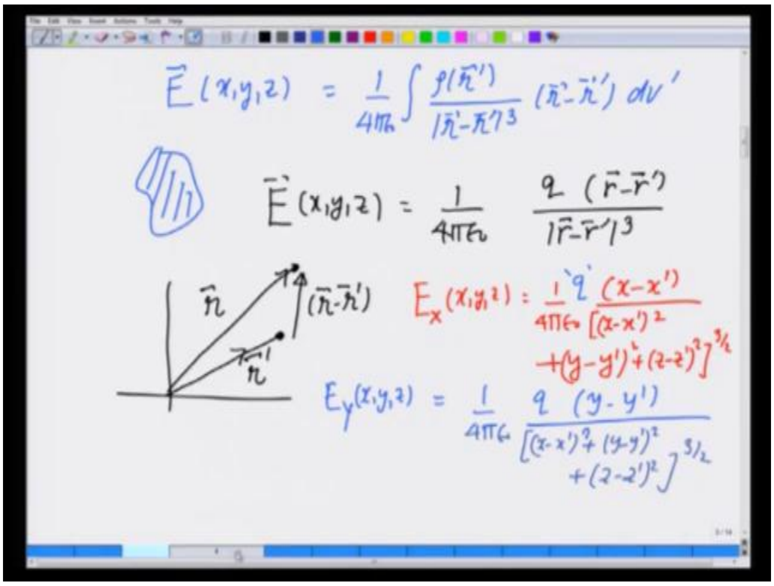

Electric field E at a point x, y, z is given as integration row r prime over r minus r prime cubed and deliberately writing it like this. So, that you get used to it d v prime; that is I am integrating over this prime volume in 1 over 4 pi Epsilon 0. To start with, let us write the electric field for a point charge, which is at x, y, z will be given as 1 over 4 pi Epsilon 0, q r minus r prime over r minus r prime cubed, where the position of the point charge is at r prime and I am calculating electric field at r, this vector being r minus r prime.

Let us take this divergence, before that, let me right the individual components of the electric field E x, x, y, z is going to be 1 over 4 pi Epsilon 0, x minus x prime divided by let us open this whole thing also is going to be x minus x prime square plus y minus y prime square plus z minus z prime square raise to 3 by 2. Similarly, E y at point x, y, z is going to be equal to 1 over 4 pi Epsilon 0 or they has to be q here, which has q, y minus y prime over this whole thing x minus x prime square plus y minus y prime square plus z minus z primes, raise to 3 by 2.

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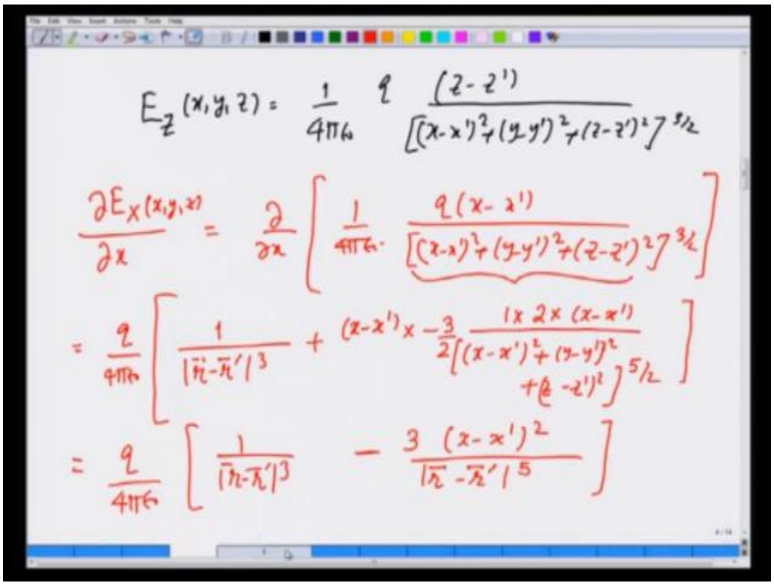

Quickly, I will write the z component also E z at x, y, z is going to be 1 over 4 Epsilon 0 q z minus z prime over x minus x prime square plus y minus y prime square plus z minus z prime square is to 3 by 2. Let us now calculate partial of E x with respect x. Notice that, this is E x is a function of x, y, z. So, I am taking a partial time with respect to x, which is partial derivative of 1 over 4 pi Epsilon 0, q x minus x prime over x minus x prime square plus y minus y prime square plus z minus z prime squared is to 3 by 2.

And it quickly do it, this comes out to be q over 4 pi Epsilon 0 is common. First term gives you 1 over, let me again for the lack of space write this is r minus r prime cubed plus x minus x prime. Now, I am taking partial derivative of this 1 over r minus r cube. So, this will give you times minus 3 by 2 1 over x minus x prime square plus y minus y prime square plus z minus z prime square raise to 5 by 2. And on top, I will get 2 times x minus x prime. So, this comes out to be q over 4 pi Epsilon 0, 1 over r minus r prime cubed minus 3 x minus x prime square divided by r minus r prime raise to 5.

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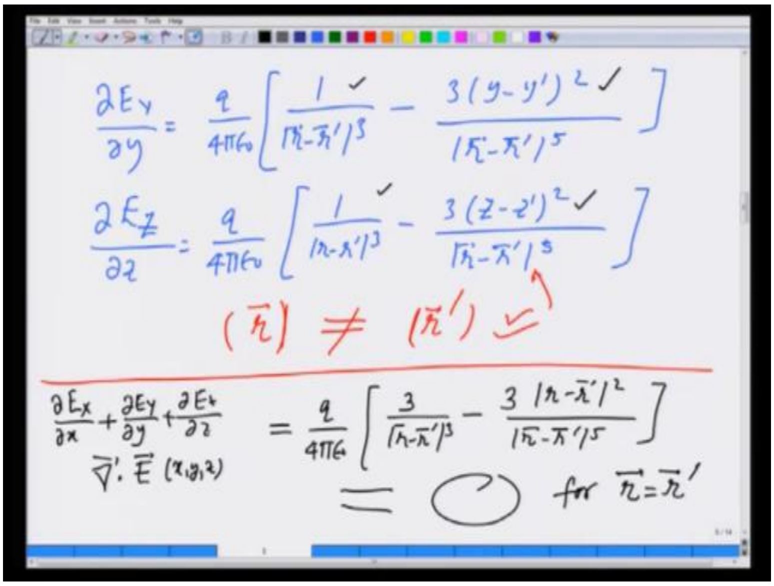

I can similarly calculate partial E y over partial y and this will come out to be q over 4 pi Epsilon 0, you can check this 1 over r minus r prime cube minus 3 y minus y prime square over r minus r prime raise to 5. And partial E z over partial z comes out be partial q over 4 pi Epsilon 0, 1 over r minus r prime cubed minus 3 z minus z prime square over r minus r prime raise to 5.

In all this, we have taken r vector is not equal to prime vector, because then this term, will vanish, whatever mathematics I am doing not be evaluated. If I add all this, so this commotion, you should remember. If I add all this, what I am going to get, I am going to get partial x over partial x over partial x plus partial E y over partial y plus partial E z over partial z, which is nothing but divergence of E, x, y, z is equal to these term together give me 3 over r minus r prime cubed.

So, I get q over 4 pi Epsilon 0, 3 over r minus r prime cubed minus 3 x minus x primes square plus y minus y prime square plus z minus z prime square gives me modulus r minus r prime square divided by r minus r prime raise to 5 and this is 0. So, as long as you are not sitting on the charge, the divergence of the field is 0.

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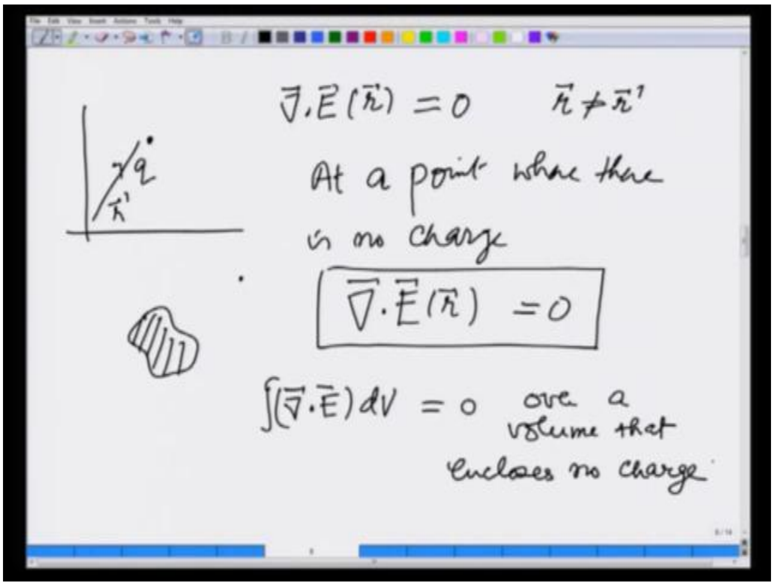

So, what we have learned is, if I take a point charge q at position r prime and calculate divergence of E at some point r, this is 0 for r not equal to r. So, now if I take charge distribution as long as I am away from the charge distribution; that means at a point at a point, where there is no charge, diversions of electrostatic field will always be 0. What it means is that, integration E over a volume, where there is no charge is 0, over a volume that enclose no charge.

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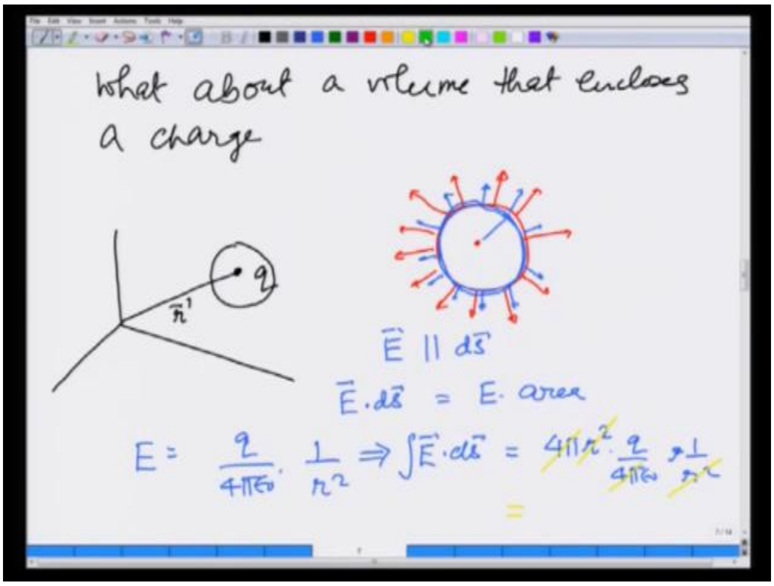

But, what about a volume that encloses a charge; let us now look at that, suppose I have charge q at r prime and I make a volume spherical volume to that is easier to handle around it. So, let us look at this charge and the volume around it, then I know flied lines are going go out like this everywhere and area as you all ready discussed also points away from the volume perpendicular to the surface. I will talk about this perpendicularity and all that little more later.

But, right now just take this d r, we are going to take the area is going to be take an every pointing away from the surface pointing out of the volume on the surface perpendicular to the surface. So, you see E and the surface are parallel everywhere and therefore, E dot d s is nothing but E dot d s is nothing but E times area there. Now, I know for a point charge E is q over 4 pi Epsilon 0, 1 over r square, where r is a distance from the centre.

So, it is value of E all over the surface, all over this spherical surface is the same and therefore, if I take E dot d s is going to be 4 pi r square q over 4 pi Epsilon 0 1 over r square, this cancels 4 pi cancels and this comes out to be q over Epsilon 0.

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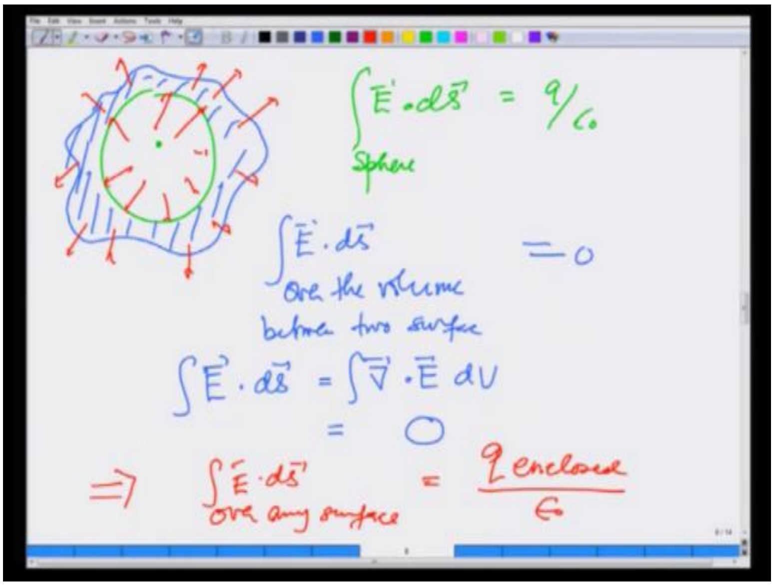

So, now I know for a point charge E dot d s around it over a sphere is equal to q over Epsilon 0 is a true for any this surface, let us see that. If I make an arbitrary surface, then E dot d s over the volume between two surfaces is going to be 0, why as you already seen that E dot d s integral is divergence of d v. And there is no charge in this region in between, if there is no charge in between, then this is going to be a 0. If this is 0; that means, whatever fluxes coming in, whatever electric filed lines of flux is coming in, say flux is going out, because this divergence is 0. This implies that E dot d s over any surface is q enclosed by that surface divided by Epsilon 0; that is surface need not be spherical.