Introduction to Electromagnetism

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Lecture – 78
Problem set - 07

In this final session, we are going to solve problems based on the last set of lectures taken in week 7. I am not going to call it home assignment, because this was not assigned to be submitted.

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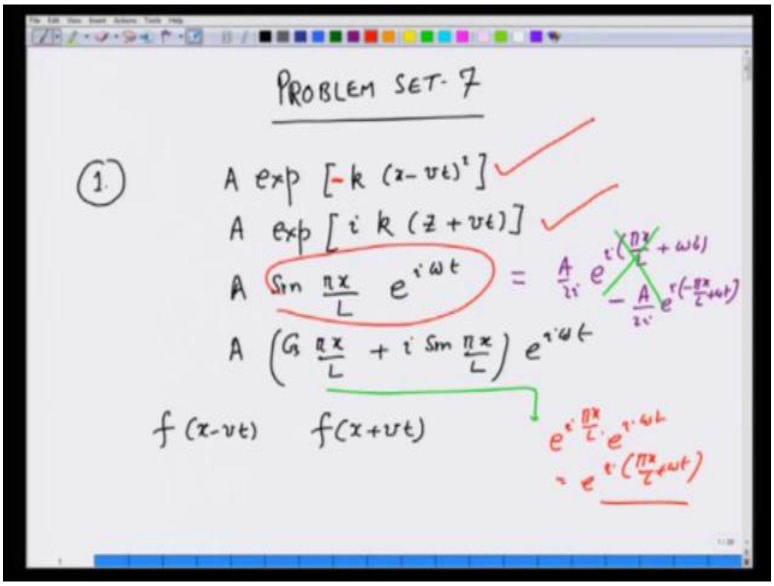

So, this I am going to call this problem set and the problems in this set have been chosen to convey to you, how whatever we developed can be applied in the real life situations. So, to start with, in this problem recall that, we had given certain forms for waves, which are traveling. So, in this problem, we are going to ask of the four forms given, e exponential A is a constant, some constant k x minus v t square one form.

Other form is A exponential i k z plus v t, third form which is given as A sin pi x over L, e raise to i, some omega t and fourth form is cosine pi x over L plus i sin pi x over L, e raise to I omega t. Of these four forms given, which ones represent travelling wave and which one does not. So, as we had yet seen in the lecture that any function of the form x minus v t or a function of the form x plus v t represents a wave, which is travelling.

And therefore, the first form which is a function of x minus v t represents a pulse or a wave, which is travelling to the right. In fact, to make it better, I should put a minus sign here, so that the wave dies down as you go from the center. Similarly, the second one has z, which is distance plus v t. So, this is also a travelling wave, which is travelling to the left or to the negative z direction. Third one cannot be combined in the form of x plus v t alone or x minus v t alone.

In fact, if you expand it, it will be A by 2, e raise to i, pi x by L plus omega t minus, so 2 i A over 2 i, e raise to i minus pi x over L plus omega t. So, it is a combination of two functions or two ways, one which is travelling to the right and one, which is travelling to the left. So, this is not a travelling wave. So, let me cut this out, this is not a travelling wave and final one is e raise to i, pi x over L times e raise to i, omega t, which is e raise to i, pi x over L plus omega t. And this again is of the form f a function x plus v t and therefore, it represents a travelling wave.

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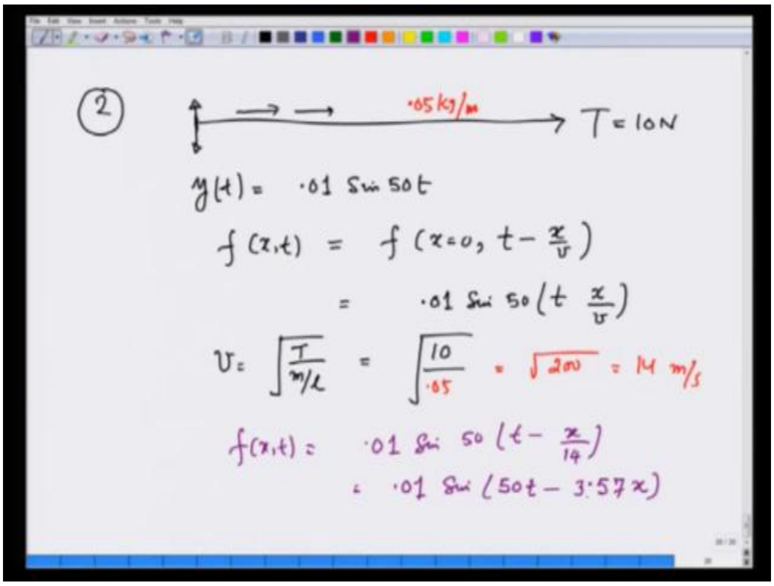

Question number 2; says a long stressed string with tension 10 Newton's, so we have to consider a long string, which has tension T equals 10 Newton's is held at one end. So, we are holding one end of it and this end is being moved in a displacement. So, this is being moved up and down as a function of time y t equals point o 1 sin of 50 t. So, it is frequency of moment is 50 per second. What we want to know is; obviously when I move it, there is going to be wave travelling down, how would this wave be represented.

Now, recall, since the wave is travelling to the right, this is going to be a function of x and t and we had written this as f at x equals 0 at x is going to be, the displacement is going to be what it was at a time x over v earlier that at time t. So, I am looking at displacement at point x at time t, it is going to be equal to what the displacement was at x equals 0 at time t minus x minus v.

Now, we know what to do. At x equals 0, I know the displacement. So, this is going to be 0.01 sin of 50, instead of t, I am going to substitute t minus x over v and v, the speed of wave on a travelling string is square root T divided by mass per unit length. And that we have been given as t is given to be 10 Newton's, I forgot to mention mass per unit length of this is given to be 0.05 kilograms per meter.

So, this is going to be 0.05, which is square root of 1000 or 5200 or roughly 14 meters per second. And therefore, the wave the final answer for f x t for the travelling wave is going to be 0.01, whatever units those are, sin of 50 t minus x over 14 or 0.01 sin over of 50 t minus 3.57 x; that is the answer.

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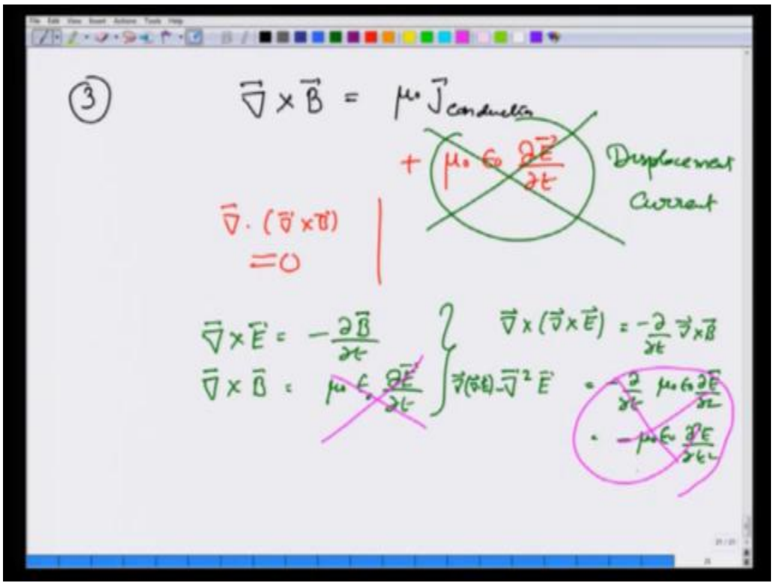

Question number 3, relates to if a changing electric field did not give rise to a magnetic field will f to magnetic waves exist. So, what we are asking is, you know from the Bio Savart law that curl of B is equal to mu J conduction. This was before Maxwell arrived on the scene and what Maxwell said is no, no, no, this cannot be true in general, in general I have got to add one more term to this. So, that continuity equation is satisfied.

Because, then if I take divergence of curl of B, which is identically 0, right hand side also gives me 0 and this is the term that is known as the displacement current. And the interpretation is that the changing electric field gives rise to a magnetic field. The question we are asking is, if a changing magnetic field did not give rise to magnetic field; that means, if or changing electric field did not give rise to a magnetic field; that means, this term did not exists would Electromagnetic waves be there.

The answer is no, because if you recall, how electromagnetic waves arises. We combine the Faradays law, curl of E, which is minus partial of B with respect to t and in free space we say curl of B is equal to mu 0 Epsilon 0 partial of E with respect to t. When, we combine these two by taking curl of E, which is equal to minus d by d t of curl of B. It gives me del square E, gradient of divergence of E minus z square E equals minus d by d t of curl of B, which is mu 0 Epsilon 0, d E, d t which is the second derivative minus mu 0 Epsilon 0 d 2 E by d t square.

If this term was not there, if the second equation was not there; that means, if the electric field, changing electric field did not give rise to a magnetic field, this term will not be there. And therefore, I will not get a wave equation; I will not get a propagative wave solution. Recall from the lecture, where I qualitatively described how E and B sustain each other in propagating space, it was precisely this term that gave rise to B and changing B, then gave rise to E and they sustain each other. So, if this term is not there, there will be no electromagnetic waves, so without displacement, no electromagnetic waves.

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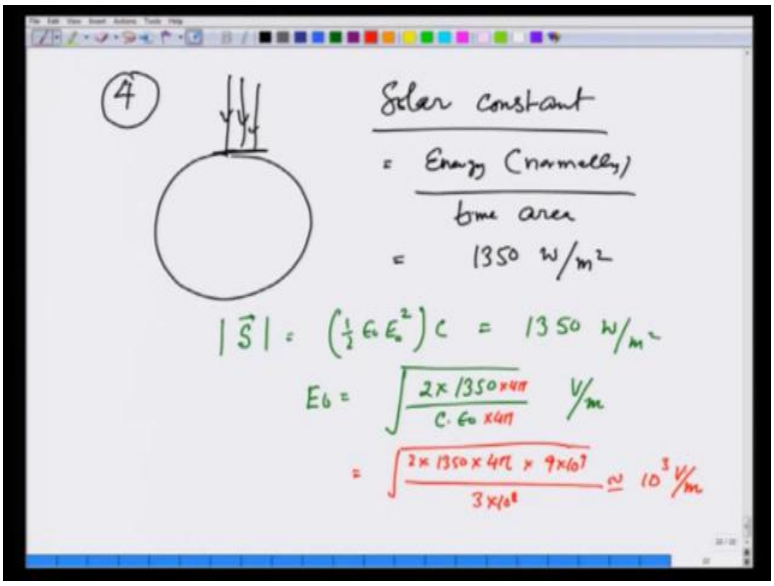

Question number 4; solar energy that falls on earth has so if I take solar energy falling on the earth, it has a value which is known as solar constant, which is defined at energy falling normally. That means, at an angle perpendicular to the surface of the earth at that time per unit time, per unit area, this is roughly 1350 watts per meter square. So, what this question ask is, solar energy falling per unit time, per unit area for the earth at normal incidence is about 1350 watts per meter square.

What is the corresponding value of the electric field for solar radiation? I had solved a similar problem in the lectures for a 60 watt bulb, now we are asking for the solar radiation. So, I know that the energy per unit area, per unit time or power per unit area is the pointing vector and it is magnitude is equal to 1 half Epsilon 0, E 0; that is the amplitude of the electric field square times c.

And this is given to be 1350 watts per meter square and therefore, E 0 is going to be equal to 2 times 1350 divided by c, Epsilon 0, square root volts per meter. I can make the calculation easy, if I multiply by 4 pi here and 4 pi here, because I know 1 over 4 pi Epsilon 0 is 9 times 10 raise to 9. And therefore, I can write this as 2 times 1350 times 4 pi times 9 times 10 raise to 9 divided by c, which is 3 times 10 raise to 8 and this roughly comes out to be 10 raise to 3 volts per meter and that is the answer.

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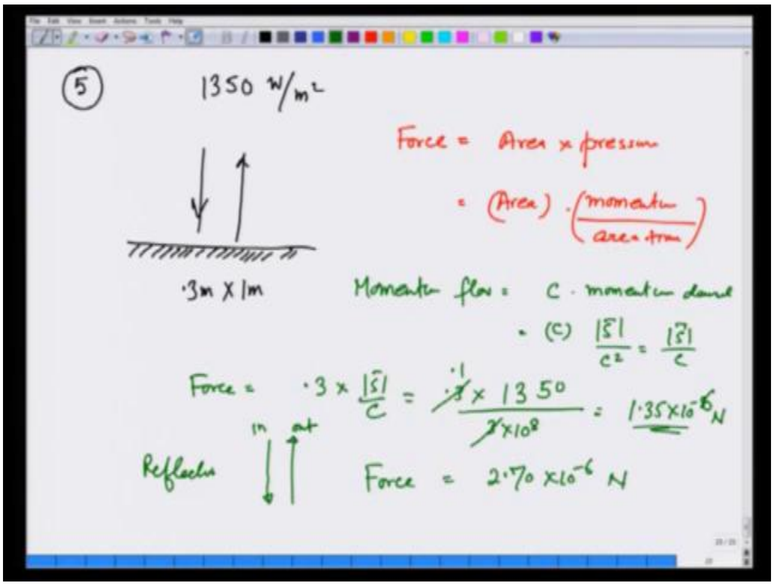

Next question number 5; we are again coming back to solar radiation, which has an intensity of 1350 watts per meter square. And the question we are asking is, if this radiation falls normally on a mirror and gets reflected and suppose the mirror size is, you know it is a large mirror, so 0.3 meters; that is roughly 1 fourth times 1 meter. So, it is like a mirror you put on the wall.

If it falls on it and reflects, what is the force experienced by the mirror, you know we have done in the lectures that electromagnetic radiation carries momentum and whenever it strikes a surface, that momentum is going to be transferred and that exerts a force or develops a pressure. So, force is, what we want to know that is going to be equal to the area times the pressure and what is pressure? Pressure is momentum transfer per unit area, per unit time or momentum flow.

Now, momentum flow is going to be equal to, so let us write it in a different color. Momentum flow is going to be c, the speed of light with which it flows times the momentum density, which is nothing but, c times modulus of S. We are just talking about the numbers divided by c square or mod S over c; that is a momentum flow. So, this is going to be the pressure, if the radiation gets absorbed, if it is get reflected, then the momentum change is going to be twice as much.

So, if it got absorbed, then the force would be area, which is 0.3 meters per or 3 meters square times S over c, which is point 3 times, S we have already calculated is already given 1350 divided by 3 times 10 raise to 8. So, this is 0.1, this becomes 135 times 10 raise to minus 8 or 1.35 times 10 raise to minus 8 minus 6 Newton's. If the radiation got absorbs, suppose I had a black sheet, but it is getting reflected.

So, reflection means, there is a momentum coming in and momentum going out. So, in, out and if I subtract one from the other, I get twice as much value. So, force in that case is going to be 2 times this number, which is 2.70 times 10 raise to minus 6 Newton's. As I had already remarked in the lectures, the force by radiation is very small, radiation pressure is very small as you see indeed that it is a very small number, even on a large 0.3 meter square area mirror.

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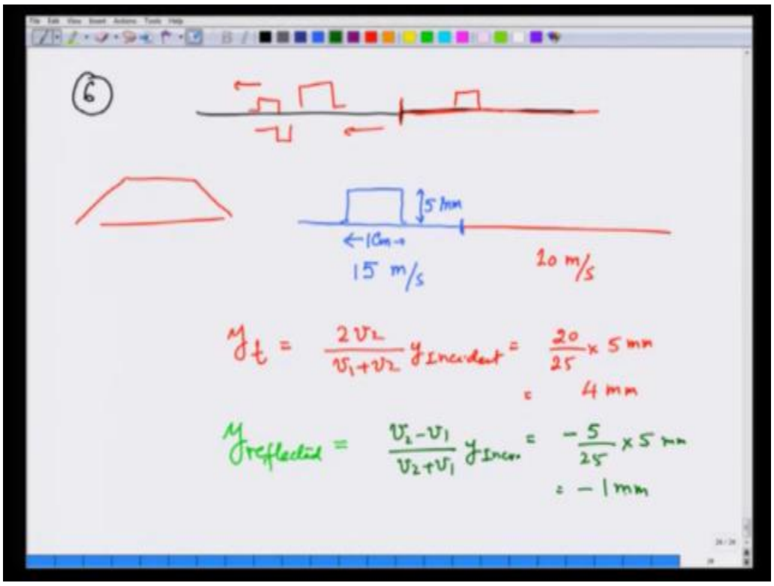

Next, two problems are going to be on the reflection of waves on a boundary. So, I had also done, this is problem number 6; reflection of wave on a string. So, suppose there are two strings, let me make the other string in a different color. Suppose, there are two strings, so that there is a boundary, on the boundary, whatever wave comes in… Suppose, there is a pulse coming in, it gets transmitted and it gets reflected also.

Depending on what the ratio velocities are, it can get reflected either with the same phase or opposite phase. So, the question is, suppose there is a square pulse, we are taking idealization, where the pulse suddenly rises becomes a constant and goes down. In non-ideal world, the pulse would be more like slowly rising, going, staying there and coming down. But, we are idealizing it and this idealization, we are saying, suppose there is a string and another string attached to it.

And there is a pulse coming in, an ideal pulse, square pulse, which has a width of 1 centimeter and a height of 5 millimeters and this comes in. And when it comes to the boundary, it gets reflected as well as transmitted. Reflection comes, because the velocities are not matching. And what we want to know is that, what will be the shape of the pulse of the transmitted pulse and the reflected pulse. And the velocities are given to be for the blue string, it is 15 meters per and for the red string, it is 10 meters per second.

So, first thing very easy to calculate is the amplitude, the transmitted amplitude is nothing but, 2 v 2 divided by v 1 plus v 2 y incident. This we had already calculated in the lecture. So, this going to be 20 divided by 25 times 5 m m and that comes out to be 4 m m. So, the transmitted pulse is going to be having a slightly less amplitude and how about y reflected, y reflected is nothing but, v 2 minus v 1 over v 2 plus v 1 y incident. And this is going to be minus 5 over 25 times 5 m m, which is minus 4 m m. So, what it tells you is that, when the wave is getting reflected it is phase changes is going to point the other way.

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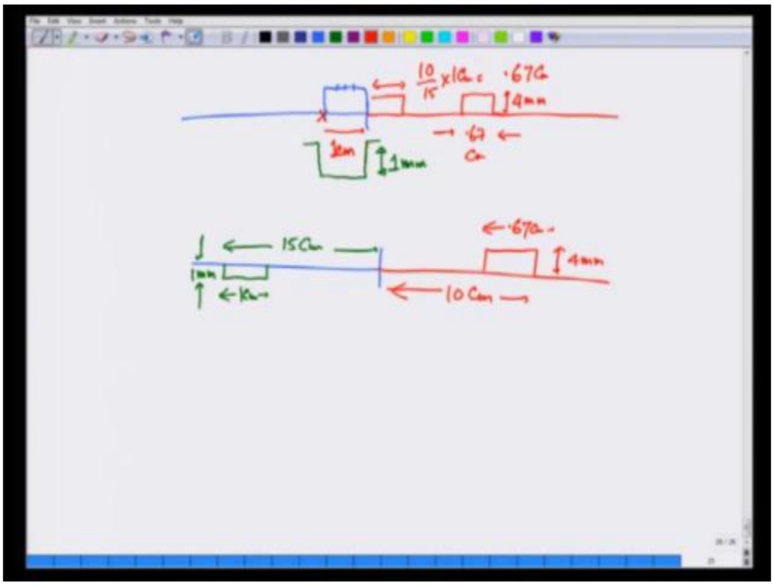

How about the overall shape? So, here is the pulse coming and it is getting transmitted, after sometime as it gets transmitted. Now, imagine this pulse coming in here, whatever amplitude is coming in is getting reflected and it is getting transmitted also. However, whatever comes on the red side, it goes little slow. So, by the time this rear end comes here; that means, it has travelled 1 centimeter. This fellow, because the pulse in the transmitted the other string is moving slower, this fellow would have moved only 10, 15th of that 1 centimeter and that means, 0.67 centimeters.

So, in the red string, the pulse is going look little squeezed, it is width is going to be only 0.67 centimeters. Why because, the front end could not move that far by the time, the rear end came to the boundary and it is height is 4 millimeters. On the other hand for the reflected wave, the size is going to be the same, because the velocities are the same. So, let me make with green, it is going to be the same width and the height is going to be 1 millimeter, because of the reflected amplitude is less.

So, after some time, when I look at the two pulses, now the original pulse would be gone and if I look at the reflected and transmitted pulse, what I am going to see is suppose the reflected pulse has moved, the front end of it has moved 15 centimeters. So, this is 15 centimeters and this obviously, is 1 centimeter and this height is 1 millimeter. The transmitted pulse would have moved only 10 centimeters, the front end would have moved only 10 centimeters and it is width is going to be 0.67 centimeters and height is 4 millimeters. So, this is how they are going to look, the reflected pulse is going to be much farther than the transmitted pulse. In fact, it is 1 and a half times farther. So, that is the solution of this problem.

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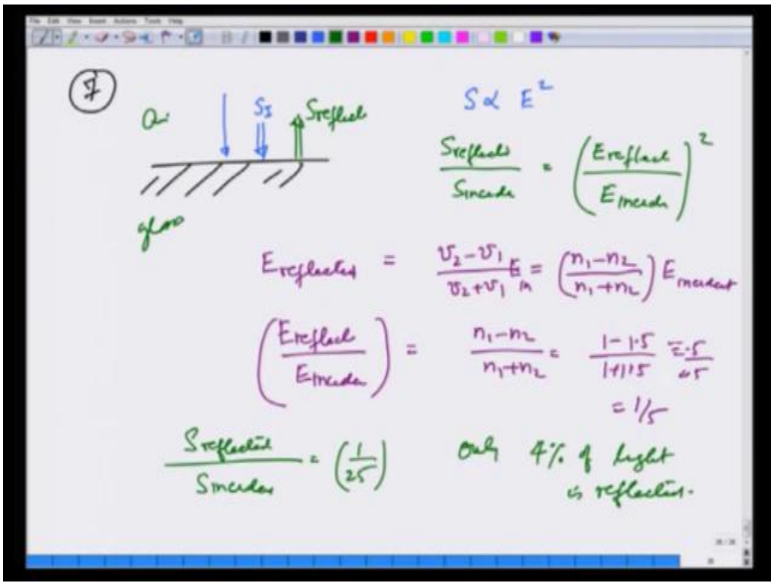

Next problem number 7 is light is incident normally on glass slab. So, I have a glass slab and light is incident normally. What about the how what percentage of light is reflected that is what you want to know. So, that means, if there is some intensity coming in, some pointing vector S incident pointing vector gives you the amount of power coming per unit area. What will be the power going out, S reflected; that is you want what we want to know.

Now, I know it in the same medium here, then S is proportional to E square. So, S reflected divided by S incident is going to be E reflected over E incident square. So, if I know E reflected by E incident, I am done. I know that E reflected for whatever we derived in the lecture through boundary conditions is nothing but, earlier we wrote it for the string v 2 minus v 1 over v 2 plus v 1, which could also be written as n 1 minus n 2 over n 1 plus n 2 times E incident.

You can see a well known fact that is always told you that if n 2 is larger than n 1, then the reflected light has a opposite phase and we see that, if n 2 is larger than n 1, E reflected would be minus of E incident. So, in this case therefore, E reflected over E incident is equal to n 1 minus n 2 over n 1 plus n 2, which is, if I just take the modulus, because I am going to take the square finally is all just do it 1 minus 1.5 over 1 plus 1.5, which is 0.5 over 2.5, which is one-fifth.

And therefore, S reflected over S incident is going to be equal to 1 over 25, where that is equal to E reflected by E incident square and therefore, S reflected is only 125th of E incident or only 4 percent of light is reflected.

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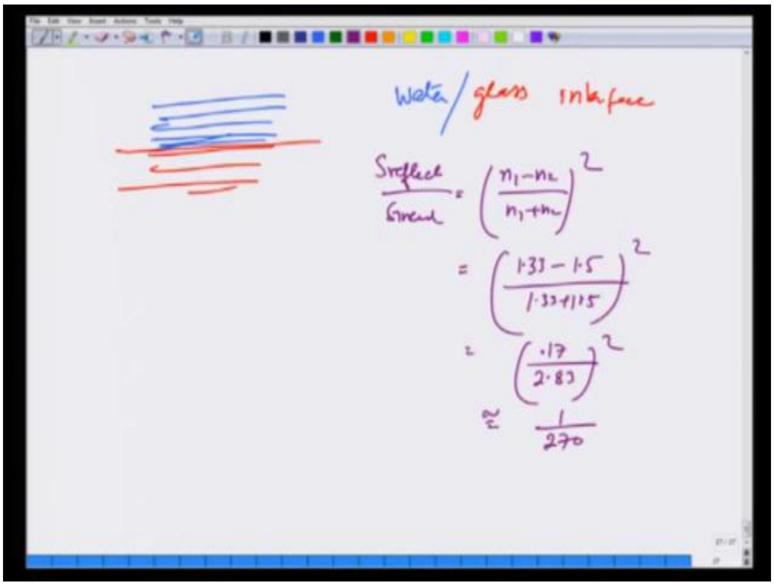

What if instead of taking air and glass interface, if I took water glass interface, so let us see, if instead of that, suppose I had water on top and glass at the bottom, at this interface, what will be the reflection reflected light. So, now, I am taking water glass interface. In this case, S reflected over S incident, which is again n 1 minus n 2 over n 1 plus n 2 square will be equal to 1.33, suppose water refractive index is 1.33 minus 1.5 over 1.33 plus 1.5 square, which is 0.17 over 2.83 square and you calculate this, this comes out to be 1 over roughly 1 over 270.

So, less than 2 percent almost you know less than less than 2 percent of the light is reflected. You can see, if the refractive index matches, then the reflected light is less. In fact, if the refractive index becomes equal, then there will be no reflected light. This is what is sometimes called and you will hear this technical term as you move further in your profession is impedance matching.

If you match the impedance reflected light is very low, sometimes you know you see in electronic instruments or you know the impedance is not matching and therefore, there is a lot of noise or something. That is because there is a lot of reflection taking place. The moment you match the impedance or refractive index or velocities of the two mediums the reflection is very rare. So, for effective transmission, it is important that, wherever there is a joint, impedance matching is done.

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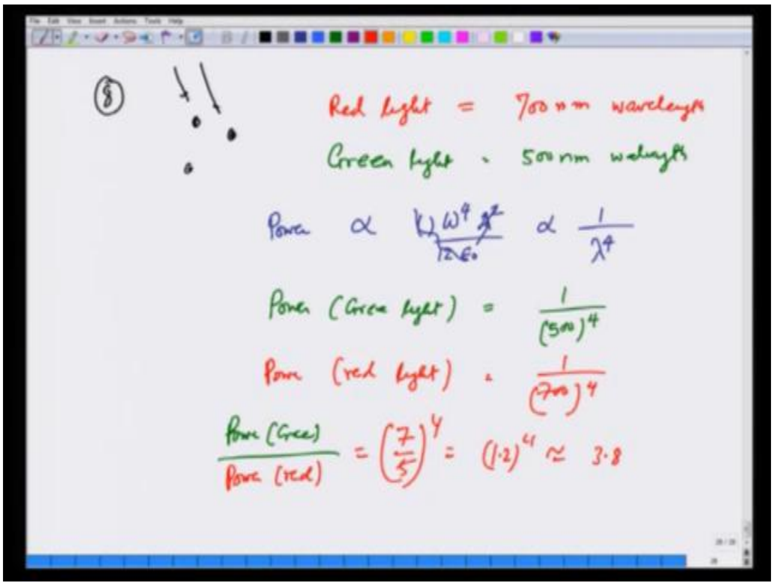

Finally, I am going to do this problem, suppose there are you know these particles of different in kinds air and light comes on them and it starts making the electron and those oscillate with the same frequency. So, what we want to know is given the same size of the particle, same amplitude of these electrons oscillating. Suppose one particle is giving out red light, which is roughly 700 hundred nanometers wavelength and another particle is giving out green light, which is roughly 500 nanometers wavelength.

Otherwise, these particles are the same, the amplitude of you know electrons oscillating and everything is the same what will be the difference in the power, which is coming out of these particles. So, my lectures in radiation, you saw that the power coming out is proportional to omega raise to 4, if rest of the things are the same. This is off course depends on the amplitude and there rise coefficients Epsilon 0 12 and all those things, but we are not worried about that, because all those things are the same.

In this case everything else being identical the power is proportional to omega raise to 4 or it is proportional to 1 over lambda raise to 4. And therefore, power for green light is going to be 1 over 500 nanometers raise to 4 and power for red light is going to be 1 over 700 raise to 4. So, power of green over power of red is going to be 7 over 5 raise to 4, which is 1.2 raise to 4, roughly equal to 3.8.

So, green light is going to come out as 3.8 times larger than red light; that much power more power is going to be there and this is the explanation for blue sky also. Remember, when this light is coming in the particles, it makes these dipoles and the blue light is going to be radiated much more. So, that power gets scattered much more and you see it coming from all over the place, red does not get scattered that that much. So, therefore, it comes directly to you.

Thank you.