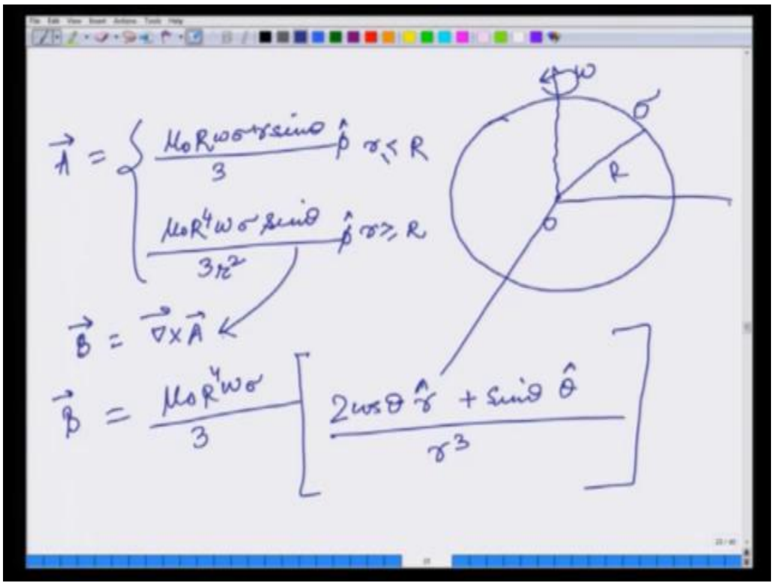
Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 77
Assignment - 06
Problems 5 - 8

Just now four problems of assignment 6 have been solved by Shailendar Kumar, the next four problems are going to be solved by Anmol Takur, who is also a PhD student in our department here at IIT, Kanpur. So, here is Anmol Takur.

Student: Thanks sir for the introduction, so I am going to do the remaining four problems of the assignment number 6. So, the problem number 5 is I am going to state the problem first. So, the problem is a sphere carries surface charge density sigma.

(Refer Slide Time: 00:42)



So, consider a sphere it carries a surface charge into sigma over it is surface, it is center is at the origin. So, this is the center origin and it is rotating about the z axis with angular speed omega which is omega z cap, good. So, let me first draw it, so it is rotating like this omega z cap, so the pointing vector just outside the surface is given as. So, we are supposed to calculate the pointing vector for this system.

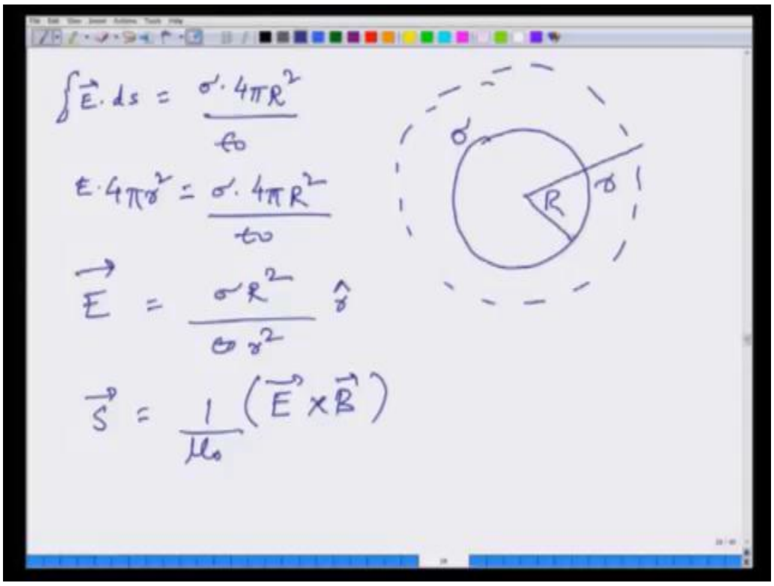
So, for solving this system I will refer to lecture number 25 in which instructor has clearly stated how to calculate the magnetic field for such systems. I am directly writing the vector potential for this spinning sphere from there we will calculate the magnetic field. We will also calculate the electric field for this system and then, on we will move to calculate the pointing vector.

So, if you look back the notes in lecture number 25, you will find that A for such system that is a vector potential is given as mu naught capital R is the radius. Let me state it here, R is the radius mu naught capital R omega sigma r sin theta, r is the distance where we are about to calculate the vector potential divided by 3. This is for r less than equal to capital R and mu naught R to the power 4 omega sigma sin theta divided by 3 r square, for r greater than equal to R.

So, in our problem we are supposed to calculate just outside. So, we will use this assumption later on. For now, we will use small r and capital R, later on we will put a small r equals to capital R, so just remind this. So, once we get A we can calculate B that is a magnetic field which is given by curl of this A, I am leaving this exercise for you, you can find out curl of A in spherical polar coordinate systems.

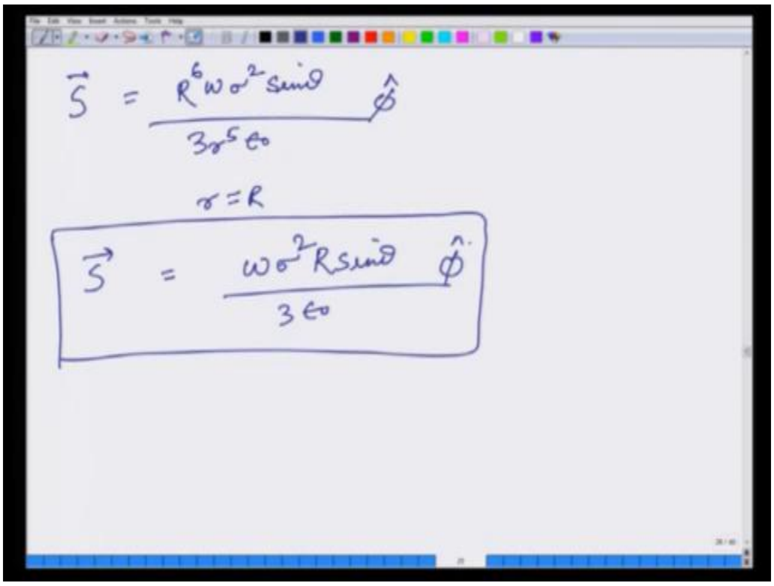
So, just I missed it, the direction of A is phi cap. So, B is curl of A you can find it anyway that how curl is done in spherical coordinate system, I am just writing the final formula for B just outside means outside, r greater than R. So, B will be B is equals to mu naught R to the power 4 omega sigma by 3 2 cos theta r cap plus sin theta theta cap by r cube. This will be the value of B for this system, you can just outside r greater than R that I am taking this for calculating this. So, once we are done with B, we will now calculate A the electric field due to this sphere.

(Refer Slide Time: 04:04)



So, the electric field we can use simple the Gauss law, I am again drawing this sphere, this is sigma just take a Gaussian surface at a distance r. Now, you can easily write it E dot d s is sigma which is give. You can write it as sigma into 4 pi R square, where R is the radius you can write it as like this, R is the radius divided by Epsilon and this can be easily done by E into 4 pi r square; obviously, E will be radially outwards which is given by equal to sigma into 4 pi R square by Epsilon naught and which gives you E is equal to sigma R square by Epsilon naught small r square r cap E. So, we have got E, we have got B, so pointing vector is given by S is 1 over mu naught E cross B.

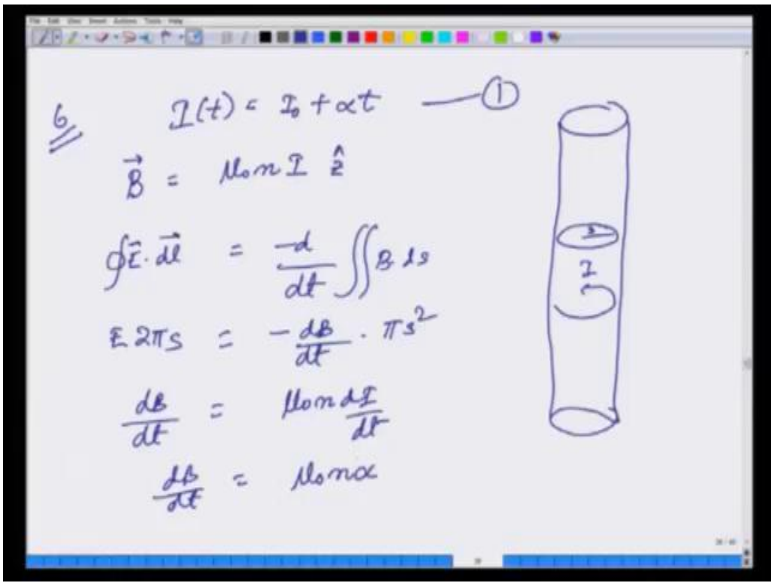
(Refer Slide Time: 05:19)



So, it is now vector product only and we will calculate it. You just calculate it I am writing the final form, just take care of every sign S will be R to the power 6 omega sigma square sin theta divided by 3 r to the power 5 Epsilon naught phi cap. Now, we are using the, what is given in question, it is given that just outside the sphere. So, put small r is equal to R just outside and then, it will reduces S equals to omega sigma square R sin theta divided by 3 Epsilon naught.

So, this is the answer pointing vector due to and the direction is phi. So, this is the solution, you can see it is option number be of the solution set or for question set, so this was the answer. Now, we will move to question number 6.

(Refer Slide Time: 06:19)

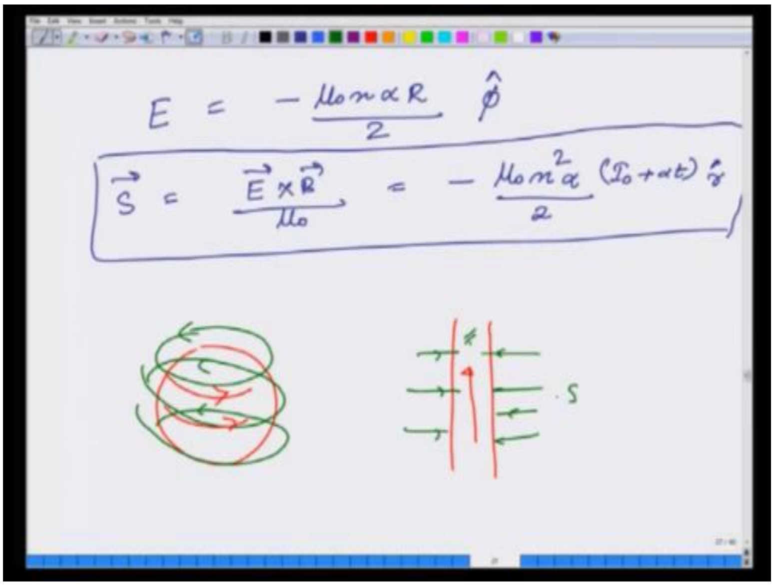


So, question number 6 states, a long solenoid with radius r has n turns per unit length and is carrying a current I naught at time t equal to 0. So, the problem is like that, we are having some long solenoid. So, I is in phi direction, take it. I is moving around I direction and whereas, and the current is changing means the current is given as I as a function of t is equals to I naught plus alpha s t linearly increasing, just outside the cylindrical surface, changes I equals to I naught plus alpha t then pointing vector just inside the cylindrical surface of solenoid will be.

Again you see that we are supposed to calculate just inside, so we will put the small r equals to capital R later. So, again we have to calculate E B and then, cross product of E and B. So, we very well know that for long solenoid, infinitely long solenoid magnetic field is given as mu naught n I z cap that I is in phi direction, so mu naught n I z cap. Now, I is a function of t, so change in current will actually change the magnetic field and this change in magnetic field with respect to time will induce electric field in fact.

So, this is given as the electric field given as E dot d l is equal to minus d by d t of B dot d s. Just take this small circle as a of radius S and we can calculate it as E to 2 pi s, this will be equal to minus d B by d t dot pi s square and d B by d t will be mu naught n d I by d t and from equation 1, we can directly write it as d B by d t this is equals to mu naught n alpha.

(Refer Slide Time: 08:48)

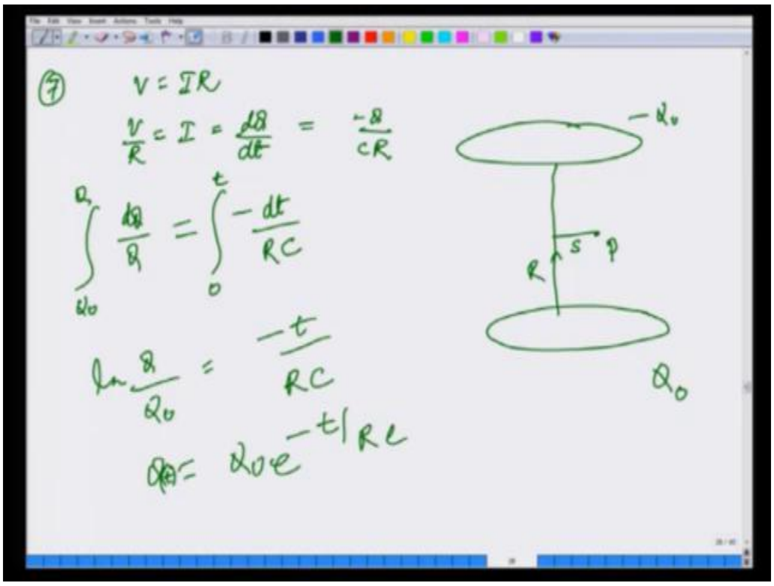


So, E is then given by equals to minus mu naught n alpha R by 2, where R is the radius of the solenoid actually it is like phi cap. Now, we have to calculate the pointing vector, B is already given in the previous slide. So, S is again given as E cross B divided by mu naught, which is given as minus mu naught n square alpha divided by 2 into I naught plus alpha t r cap. So, this is the radial involved, I will leave the physical explanation to sir he will explain you the physical explanation over this.

So, just now two problems have been solved, one of the problems was related to this sphere which is rotating and the other problem where there is a solenoid and the current is increasing. In the previous case, we found that the pointing vector is in the phi direction therefore, it just goes around. You can see that it winds on itself and there is no energy going in or going out. On the other hand, in the solenoid energy is coming in the pointing vector comes in.

As you recall pointing vector actually is a physical quantity which indicates how much energy per unit area is flowing. Now, in the previous case in the case of a sphere you can see since it is winding around or itself there is no energy going out or coming into this system. On the other hand, in the solenoid as the current increases, so does the field inside and if the B field is increasing the energy stored in that magnetic field is also increasing and that is brought in by this pointing vector, now we will solve the next two problems.

(Refer Slide Time: 11:08)



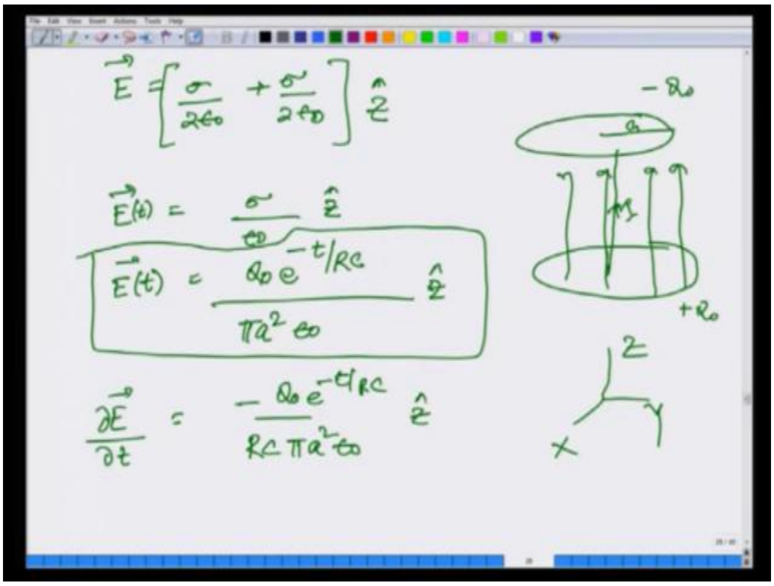
Student: The third question is consider two large circular plates of radius A forming a capacitor with the distance between the plates being d. So, it is like this, it is there are two plates, both of one is carrying charge Q 0, the next one is carrying some minus Q 0 and they are at distance R apart and we are supposed to calculate somewhere here. It is point P and we are supposed to calculate, if they are connected by a thin wire passing through their center, a current start flowing through the wire.

Obviously, there will be a current flowing from here to there from lower plate to upper plate. If the resistance of the wire is R, so the resistance of this wire is R given. What will be the magnetic field at a distance S from the wire at a point P? And this point is S apart, so this is, in the sense is S, this is minus Q 0. So, how to proceed? So, we will proceed like this, this is problem number 7 we know V equals to I R and we know V by R equals to I which is equals to d Q by d t and which gives us Q.

And this we can also solve as equals to minus Q by C R, charge is decreasing that is why we are taking the Q as minus Q, because charge is flowing from here to there that is a minus Q. So, this gives you d Q by Q this is equals to minus d t by R C and you can calculate it when t equal to 0 you are having Q 0 when t equal to sometime t this is Q. So, you can calculate l n Q by Q 0 is equals to minus t by R C which gives my Q equals to Q 0 e to the power minus t by R C.

This means the charge is Q at any time t, take it as t which means my charge from this plate lower plate it is like decreasing with time, which is like current is increasing, which means same. So, now, we will calculate E electric field first.

(Refer Slide Time: 13:23)



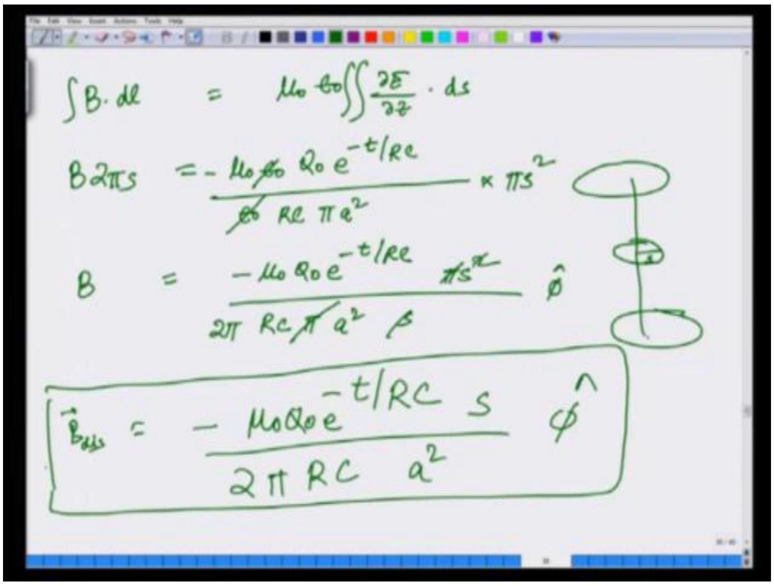
So, these are my plates, carrying charge plus Q 0 and minus Q 0, now at any point we know there is; obviously, there will be electrical pointing like this. So, at any point we know the electric field for a sheet or is given as E is equals to sigma by 2 Epsilon naught due to first plate and plus sigma by 2 Epsilon naught due to the second one and these all accumulates in z cap, where I am defining my z, where my coordinate system is like this z x y, so it is going in plus z direction.

So, my E field will be sigma by Epsilon naught z cap, now you can write E at any time t, because the charge is decaying. So, we can write it as E t and sigma we can write it as at any time t the charge left here is like Q naught E to the power minus t by R C by the area and the area is pi a square Epsilon naught z cap. So, this is my electric field, so one thing you now note here that for calculating magnetic field this change in electric field will induce some magnetic field.

So, we will first calculate the magnetic field due to this change in electric field and the second contribution to the magnetic field will be due to this wire which is carrying current I. So, there will be two contribution to magnetic field, first due to this time varying electric field and the second due to this current which is flowing through the wire. So, you will calculate one by one both here, so first I am going to calculate the magnetic field induced due to this time dependent electric field.

So, from here we can directly induce del of E by del of t, so this will be minus 1 by R C pi a square Epsilon naught let us not write as minus 1 I can simply multiply it by Q naught E to the power minus t by R C. So, this is the time this is d v by d t z cap for the direction.

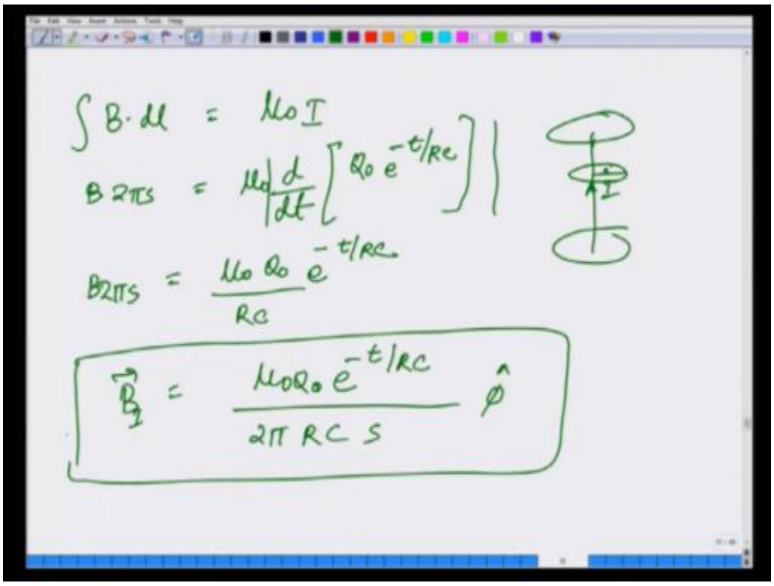
(Refer Slide Time: 15:57)



Now, we will calculate the magnetic field due to this time varying electric field. So, which is given by B dot d l this is equal to mu naught Epsilon naught d E by d t dot d s. So, this is the magnetic field due to time varying electric field I will again draw the let us take some S which is given here. So, B dot d l will be like d l is in phi direction, so it gives phi direction, one thing to note here that the direction. So, this is B into 2 pi s this will be mu naught Epsilon naught, we have just calculated d by d t in our previous slide and this is equals to...

So, let me write it here Q 0 there is a minus sign e to the power minus t by R C divided by Epsilon naught R C pi a square from here Epsilon, Epsilon would cancel and there will be a d s. So, this will be pi s square will appear here, so which gives my B is equal to minus mu naught Q 0 e to the power minus t by R C divided by 2 pi R C pi a square pi s square. So, this pi will cancel out this pi and there is a factor of s which will also come this s will cancel out this s and this will be in phi cap. So, this is my B vector is minus mu 0 Q 0 e to the power minus t by R C divided by 2 pi R C a square phi cap. So, this is the contribution due to time varying electric field.

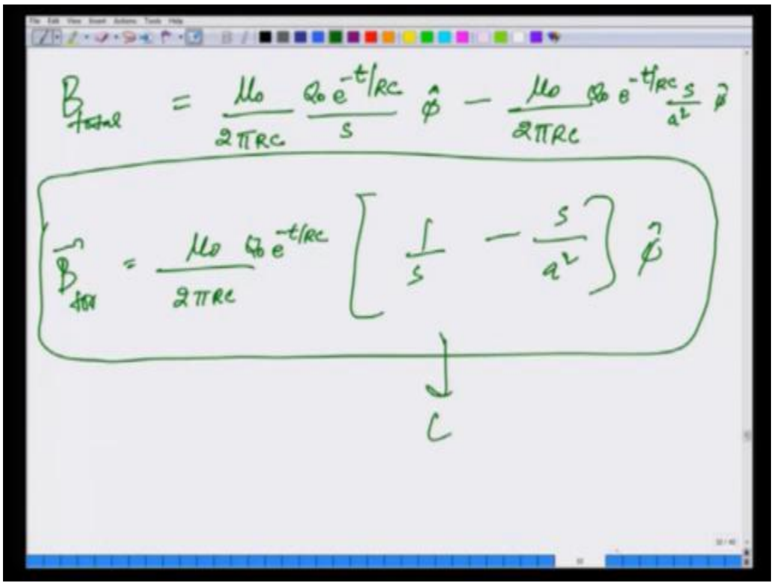
(Refer Slide Time: 18:19)



Now, the second contribution let us write it as a B displacement, you can say is B displacement, this is the first one. Now, we will calculate the second B contribution to be which will be again due to the current carrying through it, again make the same loop of s and again use the amperes law B dot d l equals to mu naught I enclosed the current is increasing in this direction you know. So, this is B into 2 pi s which is equals to mu naught I is d q by d t. So, this is for being precise let me write it d by d t Q naught e to the power minus tau by R C and I will take the mod because I is increasing in fact.

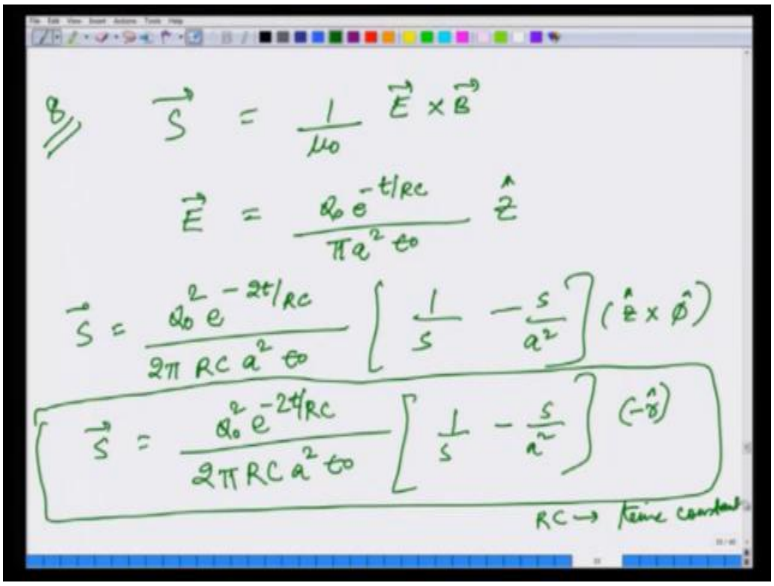
So, this will be mu naught Q 0 by R C e to the power minus tau by R C B to 2 pi s. So, B vector will be mu naught Q naught e to the power minus t by R C 2 pi R C s phi cap, this is due to let say the current I. So, B due to I, I am just writing it like this.

(Refer Slide Time: 19:37)



Now, total B we have to calculate B dot, so B total. So, B total will be mu naught by 2 pi R C Q naught e to the power minus t by R C by s phi cap minus mu naught by 2 pi R C Q naught e to the power minus t by R C s by a square phi cap. So, take the quantity which are common here, so this is mu naught by 2 pi R C Q naught e to the power minus t by R C is common I think and this we left with 1 by s minus s by a square phi cap. So, this is B vector, since we have calculated B now this is done and you can see option C satisfied this.

(Refer Slide Time: 20:42)



Now, question number 8 just we have to calculate the pointing vector for this system. So, we have done with we have calculated E, we have calculated B, now we will write s is 1 by mu naught E cross B as simple as this. So, we know just E equals to Q naught e to the power minus tau by R C pi a square Epsilon naught. So, this is the electric field in z cap, now s equals to s will be just multiply it like this. So, this will be Q naught square e to the power minus 2 t by R C divided by 2 pi R C a square Epsilon naught 1 by s minus s by a square and this is z cap cross phi cap which is s vector it is, which is s vector equals to Q naught s square e to the power minus 2 t by R C divided by 2 pi R C a square Epsilon naught 1 by s minus s by a square and z cross phi is minus r cap.

Now, this is the pointing vector you can see options from your question set, it is matching with one of the options. One thing I will mention that R C is call the time constant sometime, time constant of transient circuit.