Introduction to Electromagnetism

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Module - 08
Lecture - 76
Assignment - 06
Problem 1- 4

In this session, we are going to solve assignment number six. And, as I had said earlier while solving assignment number five, these assignments will be solved by the teaching assistance who have been looking through the assignments, solving them, grading them. And therefore, you get a flavor from what kind of thinking students have on these problems. Today's assignment will be solved by Shailendra Kumar and Anmol Thakur. They are both Ph.D. students at the Physics department in IIT Kanpur. So, here is Shailendra Kumar.

So, the first question is based on Lenz's law. So, this is actually a straightforward dimensional analysis. But before I solve the question, I would like to tell you about the demonstration once again; which you can see in lecture forty nine in unit six.

So, we are going to solve question one of assignment six. In this question, as you have seen in the demonstration in lecture forty nine, so there is a tube of conducting material and magnet is dropping in. The flux; because of that magnet is changing. And, so eddy currents are creating in that tube. So, those eddy currents are actually opposing the motion of the magnet. There is another force which is gravitational pull acting downwards on the magnet. So, these two forces interpolate and after sometimes there is no acceleration acting on the magnet.

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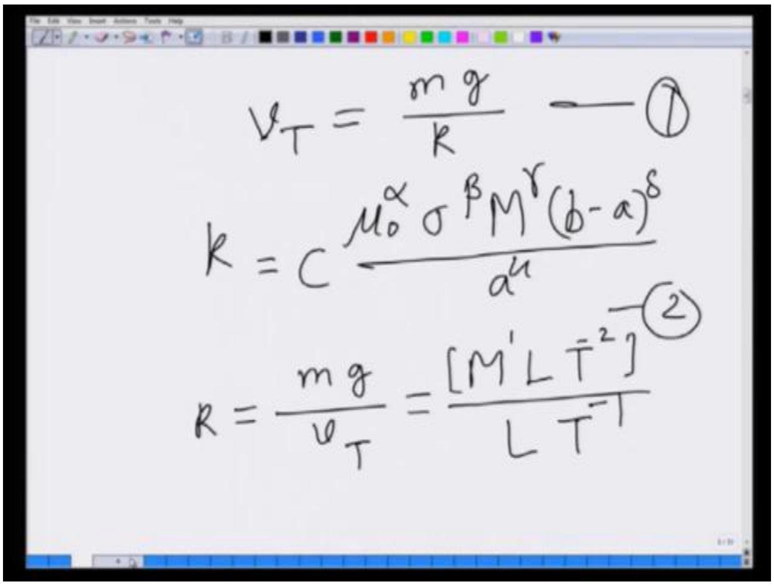

So, the magnet attempts a velocity; which is called terminal velocity. Terminal velocity is given by mass of the magnet times acceleration due to gravity divided by a drake co efficient. This drake co efficient is also given by mu naught alpha.

You can read the assignment, and there you can see this. Sigma beta magnetic moment of the magnet gamma, m gamma, then the thickness of the tube divided by inner radius of the tube power four. But, let us put a absolute constant, in the sense that it has no dimension. The C has no dimension. Now, let us call this relation one and this relation two. From the formula one, you can find the dimension of k. So, k becomes m g viability; which is mass one, g has the dimension L T minus two; then v is actually dimension of velocity, so which is L T minus one.

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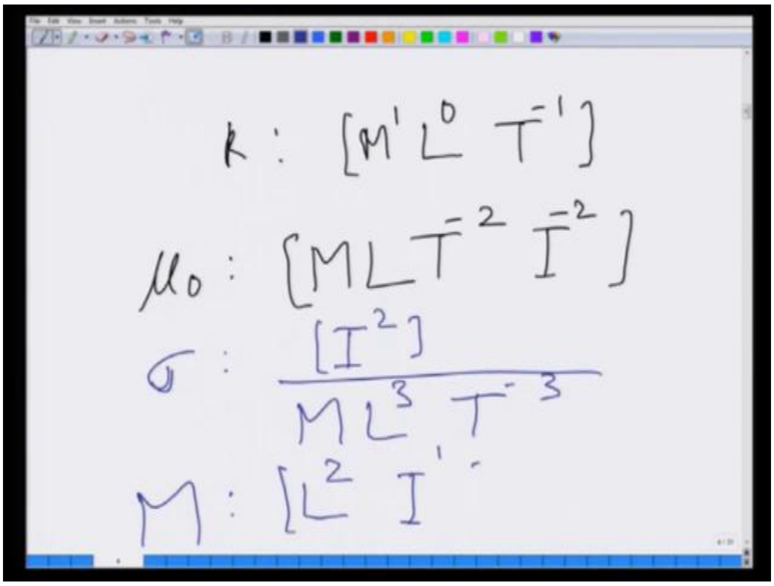

So, the effective dimensions we have, the dimension of k is M one L 0 and T minus one. From the second relation, which is k is equal to C mu alpha, this relation, I am going to write the dimensions of every quantity used here.

So, dimension of mu naught is M L T minus two I minus two. And, dimension of sigma; sigma is used here; it is I two M L three T minus three. And, dimension of magnetic moment is L two I one.

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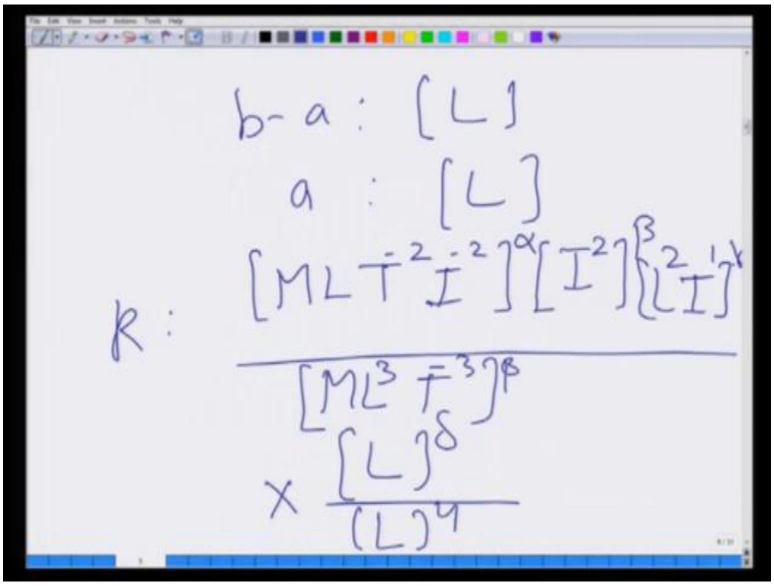

And, the dimension of b minus a; let us forward. It is L. And for a, it is also L. Now, we can write k like this. Dimension of k like this. For mu M L T minus two I minus two alpha and then for sigma it is I 2two divided by M L three T minus three beta. And, for this is again beta, here also beta; and for M, it is L 2 I 1 gamma. So, and for b minus a we will put L delta divided by, L for a, L 4. Now, we can explicitly write it like this. Dimension of k will be; it is actually M T minus 1. We have calculated it from first relation.

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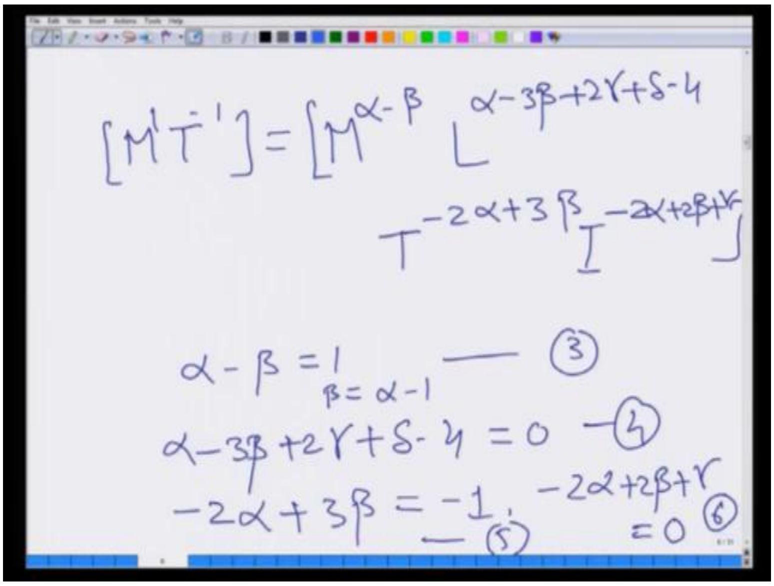

And from the second relation, the dimensions we get are like this; M alpha minus beta, then L alpha minus 3 beta plus 2 gamma plus delta minus 4. For T, we are getting minus 2 alpha plus 3 beta; and for I, we are getting minus 2 alpha plus 2 beta plus gamma. Now, we can compare the dimensions. For dimensions of M, we are getting alpha minus beta is equal to 1; call it relation 3. And for length, here you see that there is no L. So, alpha minus 3 beta plus 2 gamma plus delta minus 4 is equal to 0. Dimensions of T will give us minus 2 alpha plus 3 beta is equal to minus 1. And for I, we will get minus 2 alpha plus 2 beta plus gamma is equal to 0.

We have to solve these equations. So from equations three, we see that beta is equal to alpha minus one. You can put this equation in this equation; equation five. So, we have got four equations here. Now, these equations have got four unknowns; four equations. So, we can solve these equations easily.

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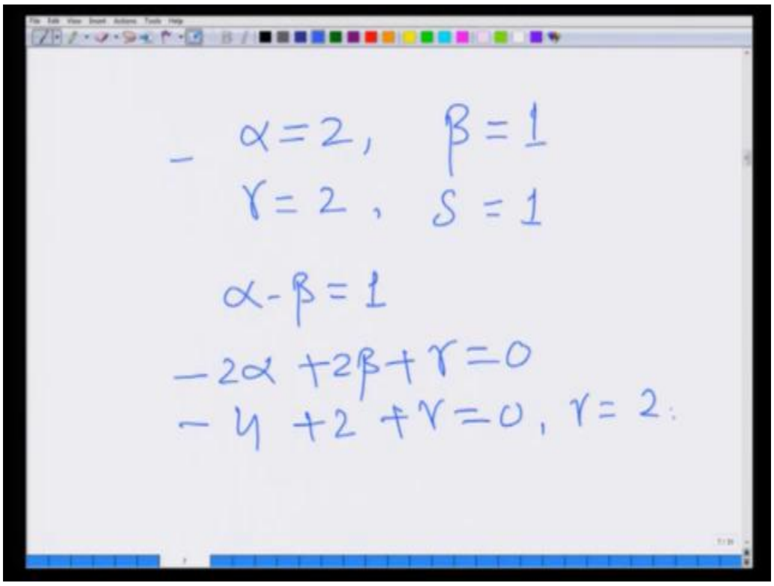

So, I am writing the solution here. So for alpha, we will get 2; beta, we will get 1; gamma, we will get 2 and delta, we will get one. We can check these answers by using like; our third equation is alpha minus beta is equal to 1. If you put alpha is equal 2, beta is equal to one, you can see that this is satisfied. Another equation, we can check gamma as well. So, we have equation; minus 2 alpha plus 2 beta plus gamma is equal to 0.

If you put alpha is equal to two here, it will become minus four. And if you put beta is equal to one, we will get two. So, you get gamma is equal to two. Similarly, you can verify it for delta. In the second problem, the non-conservativeness of the field is shown.

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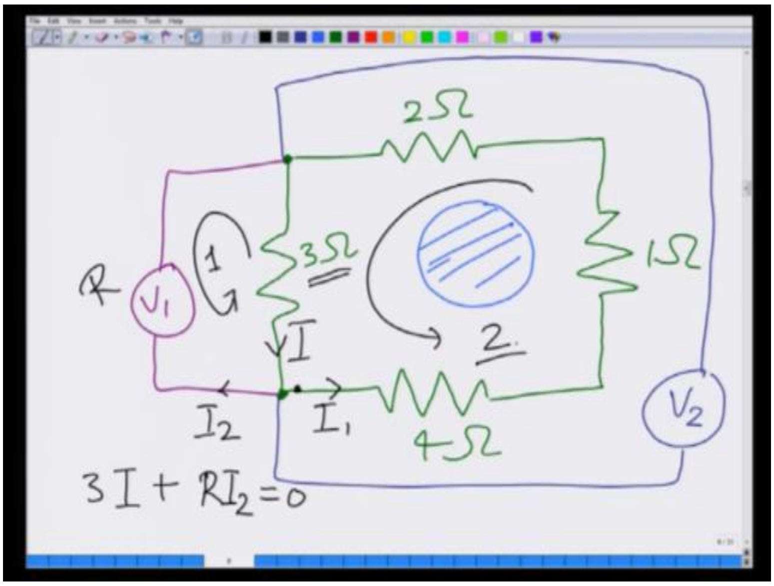

So, this is the diagram, here is a solenoid. And so because of this solenoid an electric field, an e m f is produced actually. And that e m f; so, you start from any point. let us start from this point. And starting from this point, you move around the loop. you will come to this path and you will see that you have gained a 20 volt; some volt, when you complete the loop. But, if it is conservative, then the gain should be 0. But, here there is some gain; which is 20 voltage as given in the problem.

So, this is not conservative. It is also been shown in the lectures. So, we want to solve the reading if we measure the potential drop across this resistor, this resistor. And, if we…and what will be the voltage reading? We measure the potential across these three resistors. So, we will use Kirchhoff's law for this.

So, let us make a loop here. So, in this loop assume that there is current flowing in this direction, in which one part will go here; let us say this is I, I one and this is I two. So, let us consider two loops; one loop in this and another loop here. Now, we can write Kirchhoff's equation for this loop. The equation will be; before that let us assume that this resistance of the volta meter is R. Then, we can write the Kirchhoff's equation like this.

The drop across this resistor will be three I and the drop across this resistance will be R times I two. So in this loop, this should be 0. So, this is 0 because this loop is outside this circuit. So, there is no flux changing at all. So, that is why we are putting it equal to 0. Then, let us move on to this loop. Let us call it loop 2.

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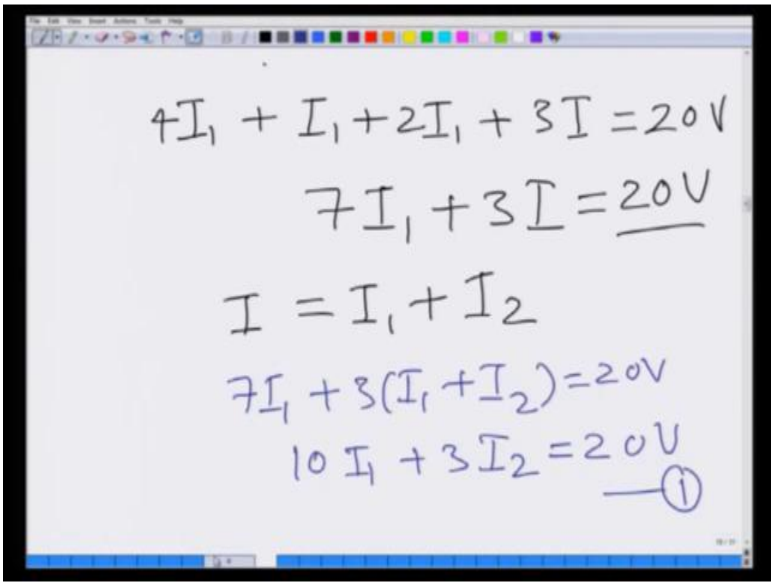

And for this loop, let us write the equation. The equation will be like four times I one, then I one current is flowing through one ohm resistance. So, that will be I one. then, voltage across two ohm will be 2 I 1, then voltage across 3 ohm will be 3 I; which is total 20 volt.

Now, this equation becomes 4 I 5 I. So, total becomes seven I 1 plus 3 I is equal to 20 volt. Now, we can see from here that I is actually I one plus I 2. So, let us write I is equal to I 1 plus I 2. So, this equation becomes seven I 1 plus 3 I 1 plus I 2, which is like 10 I 1 plus 3 I 2 is equal to 20 volt. Let us call it again equation one.

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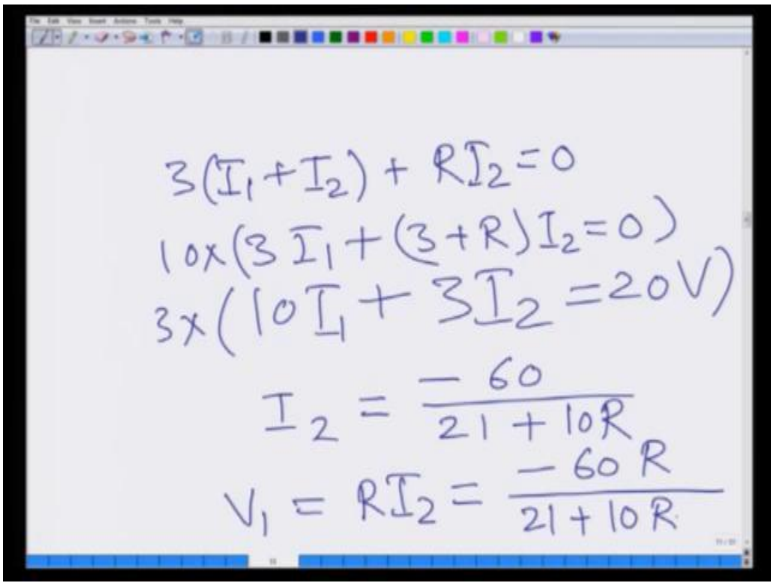

And from the previous equation in loop one, this 3 I plus R I 2 is equal to 0. We can write it like 3 I 1 plus I 2 plus R I 2 is equal to 0; which is 3 I 1 plus 3 plus R I 2 is equal to 0. Let me write the previous equation here again. So, 10 I 1 plus 3 I 2. So, we have got two linear equations in two variables; I 1 and I 2. So, we can solve these equations and so to solve this equation, let us multiply it by 10; so that, we can eliminate I 1 and this by 3. And solving, we get I 2 is equal to; I am writing the direct answer here. So, this will be minus 60 divided by 21 plus 10 R. Now, the potential V one is R I 2. So, which we can see now is minus sixty R divided by 21 plus 10 R.

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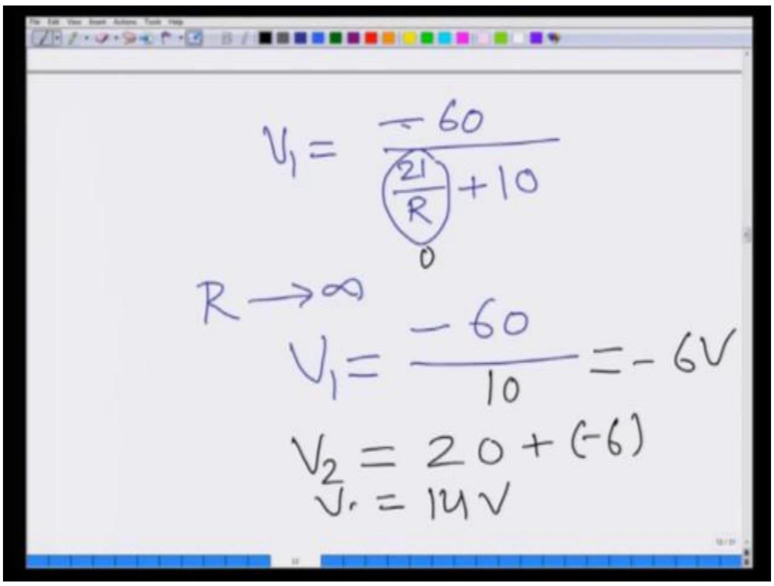

Let us divide by R. We get V 1 is equal to minus 60 divided by 21 divided by R plus 10. So, now for volta meter, let us take for volta meter R is very large. So, in that limit we get V 1 is equal to minus 60; this thing goes to 0, this goes to 0. So, we are left with 10 and, so we get minus 6 volt. Now, for v 2 in the circuit, now V 2 is actually 20 volt plus, this V 1, which is minus 6 volt. So, it becomes 14 volt.

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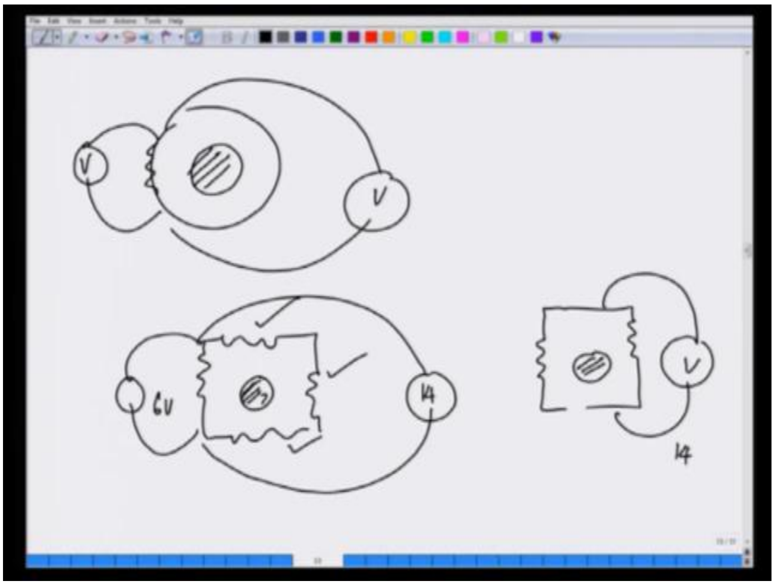

So, let me relate this problem to the demonstration that we showed you. And, there we had a circuit and a resistance. And, in between the circuit the flux was changing and we had once connected a volta meter on one side and once on the other side. And, we have obtained different voltages. We have showed the non-conservative nature of the field. And, that is precisely shows you what this problem solves. that if in the circuit where there are four different resistances and there is a solenoid that is creating the flux change, then voltage on one side comes out to be 6 volts and on the other side comes out to be 14 volts.

You solve it using twenty minus fourteen. But actually if you see the second circuit, it is precise; this I could have assumed this to be one circuit here and one resistance. All these three resistances together to be one resistance and, we are calculating the voltage here, which comes out to be 14 volts. It is again outside the field.

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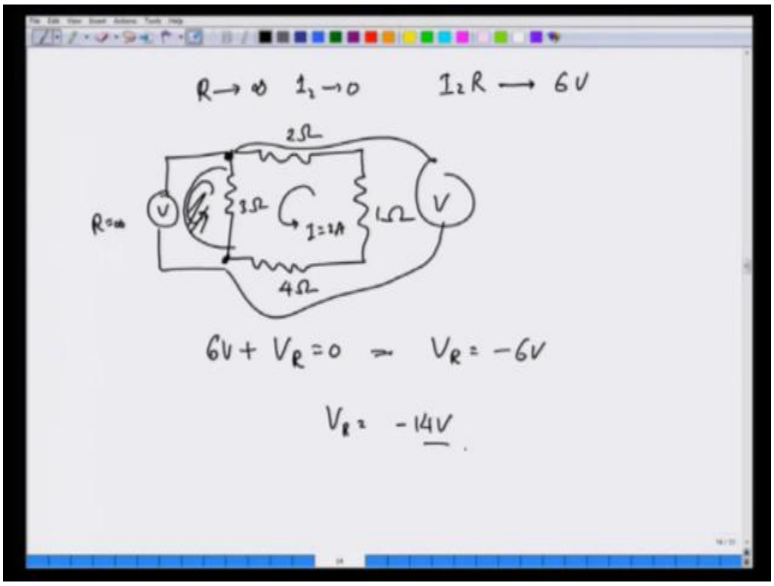

He did a very thorough job of taking resistance of the voltmeter and then taking the limit that R was tending to infinity and I two was tending to 0, yet I two times R converges to six volts. One could also look at it directly. If we took R to be infinity, right from the beginning we had 1 ohm, 2 ohms here, 3 ohms here and 4 ohms here.

Now, if I took this voltmeter and took R to be infinity right from the beginning, then the current in this loop is I equals 2 Amperes. And if I go around this path, you can see that the voltage drop here because the current is 6 volts plus V r will be 0; because there is no EMF being produced here. And, this gives you V r equals minus 6 volts. On the other hand if I go around the loop on the other side with the same analysis, you will get V r equals minus 14 volts; which is the same answer. But, what the solution earlier obtained showed you in proper limit because you may have questions while solving this. What you know; there may be some leaky current through the voltmeter. What you showed you very carefully by an analysis that I take the resistance of voltmeter to be R and as R goes to infinity, I 2 goes to 0. But, their product converges to its value; which we measure.

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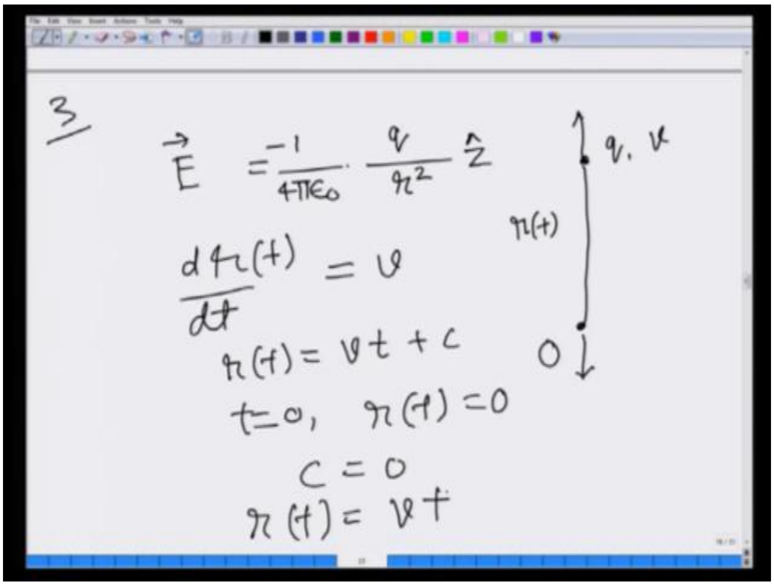

So, in the third problem there is a charge particle moving along z axis; positive z axis with constant speed V. It passes through the origin. So, let me; so, this is the charge particle. It is moving in positive direction. So, let us; this is the origin. So, let us; at any time it is. Okay. And, we want to find the displacement current density at the origin. So, when this charge will move, the electric field produced at the origin; because of this charge will change. So, that change in electric field will induce magnetic field. And, the responsible factor for the magnetic field is the displacement current. So, to find the displacement current let us assume that at any time the particle is at r, which is the function of t; because the particle is moving so, r t. Then, the field at origin; because of this charge, will be E. 1 divided by 4 pi epsilon naught q, which is the charge and its velocity is v. So, q divided by r square. And, the field; this is positive charge.

So, field produced by this charge will be in negative direction. So, let us put minus sign here. So, this is the electric field produced by this charge; where this charge is moving. So r, we are given that it is moving in the constant velocity; that is rate of change of r is v. Now, we can solve this equation. So, r t will be V times t plus a constant c. But, we are told that; it is given that the particle passes the origin at time t is equal to 0. So at t is equal to 0, r is 0. So, that gives me c is equal to 0.

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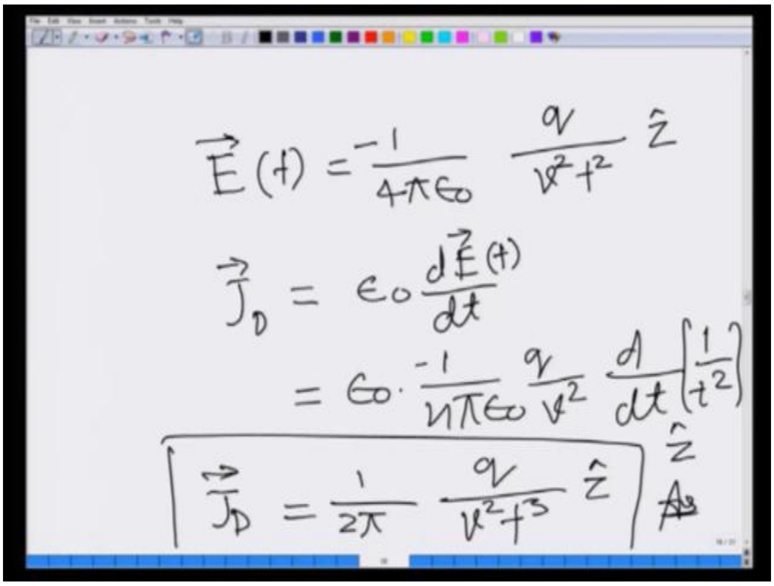

So, the electric field is actually E function of t is 1 divided by 4 pi epsilon naught, then q divided by v square plus t square in the negative direction. Now, displacement current density is actually given by epsilon naught d dt of E. So, that will give me 1 by 4 pi epsilon naught q by v square; v is constant. So, d d t of 1 by t square. So, we get; you derive it. And, you will get 1 divided by 2 pi q divided by v square t cube in the positive z direction.

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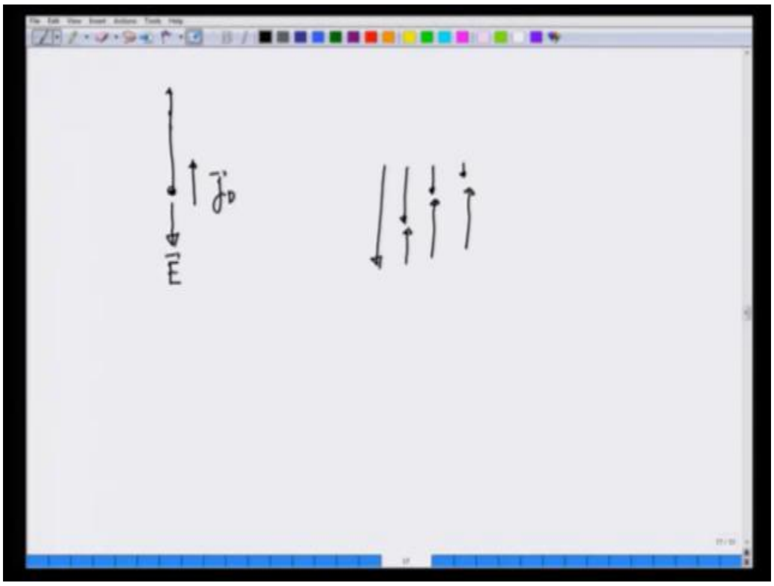

Notice that in this problem when the charge is moving at the origin, although the field pointing down and yet we have a displacement current, which is pointing in the positive z direction. Somehow, at the first glance it looks counterintuitive. But, not anymore when you realize that with the time, the field is actually decreasing in magnitude. As the rate of change of the electric field that is related to the displacement current, so you can see that the displacement current should be in the positive z direction;

Fourth problem, we have to calculate magnetic field because of a charged particle which is described in problem three and we want to find this magnetic field at a distance r from the origin in the x y plane. Now, one way is, think of solving this problem by using Biosavart law that charge is moving. So, it can be treated as a current. But, that is not the way of doing this problem because a moving charge cannot produce steady currents, while Biosavart law is used for a steady currents.

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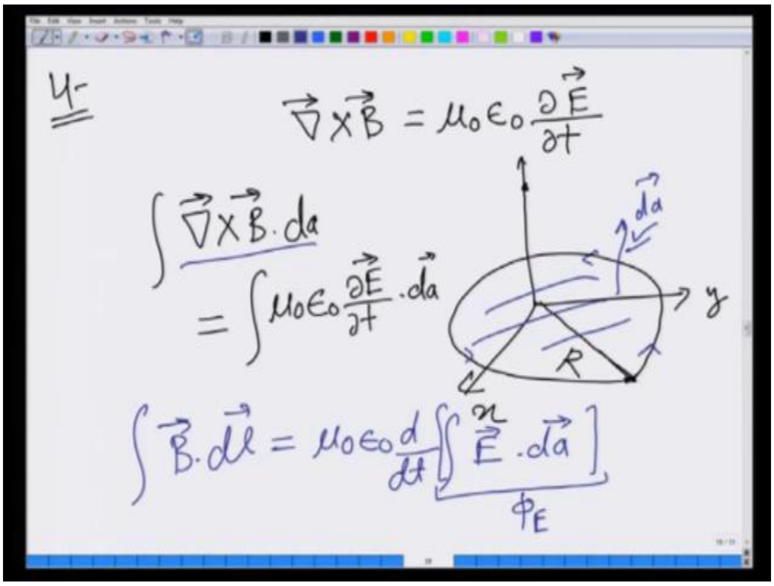

So, to solve this problem we have to go through the procedure of calculating, current density, displacement current density because of the changed electric field at a distance r from the origin. So, we know that magnetic; curl of magnetic field is mu naught epsilon naught double double t of E. There is j conduction also but, here there is no conduction currents. So, that is why that is ignored.

So, let us see the problem. So, here is the charge moving upwards. This is the x y plane and here is the point where we want to find the magnetic field. This is R. So, what we can do to find the magnetic field at this point? We can calculate the change in electric field bound by this path. So, let us integrate over the surface both sides. From the Stokes theorem, this thing can be written like B integrated over along the path bounding this surface. So, we can write this thing like d d t this surface. But, notice here that because I am going along the counter clockwise on the path. So, the direction of this d a is upwards as you can see here. And, I am using a total derivative here because this thing is now the function of t only. Now, we have to calculate. Now, we can see that this thing is actually flux bound by this surface.

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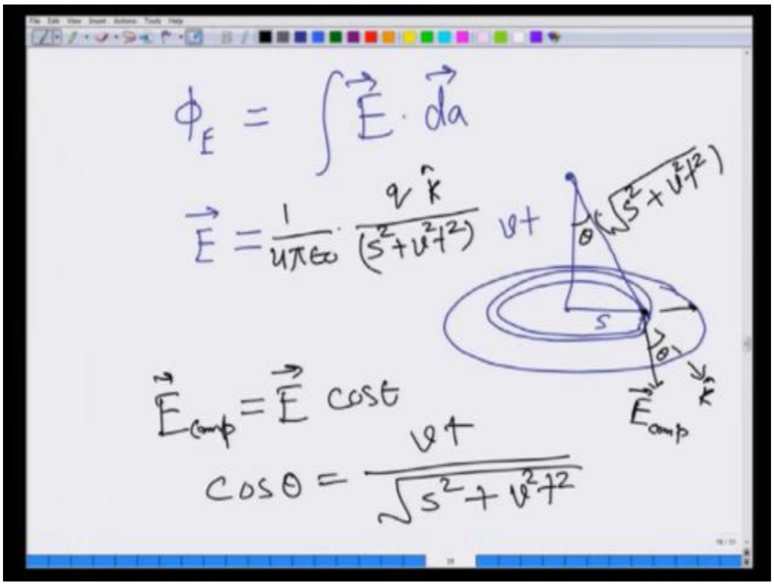

So, let us calculate this flux first. So, flux because of the electric field is E d a. Now E, electric field at point this is the surface over which we want to find the electric flux. So, this is the origin. Take some point or let us take a circular strip. So, at any point the electric field; let this is at s and this is the point charge. So, this; the distance of this point charge from the origin is v t because it is moving with constant velocity v as we have seen in the last problem. So, this distance will be square root s square plus v square t square. Now, electric field at this point will be 1 by 4 pi epsilon naught q divided by square of this distance. So, which is s square plus v square t square. And, it is in some direction. Let us call this direction k. Now, to calculate the flux we want this component only. You are not (Refer time: 28:56) this component.

Now, to calculate this component; so, let us call this angle theta. Now, you see this angle is equal to this angle. So, this component let us call it E component. So, this E component will be E cos theta; when cos theta you can see from here, it is v t divided by s square plus v square t square.

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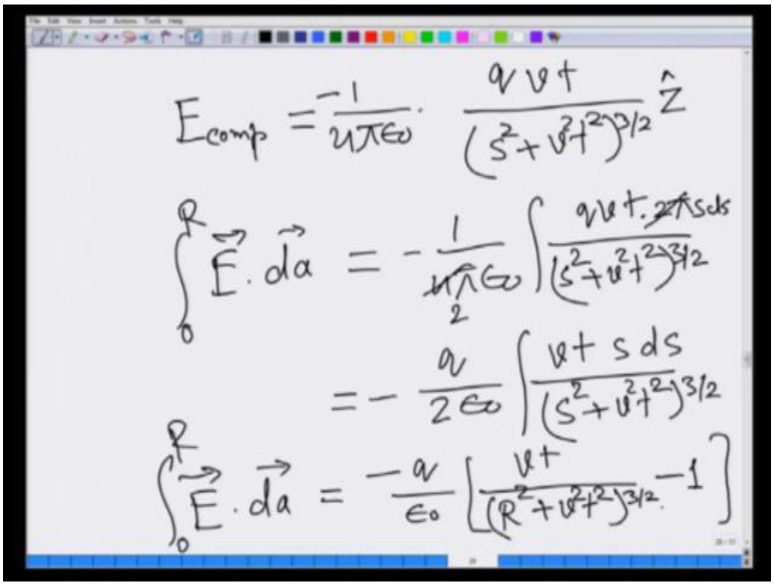

So, E component becomes 1 by 4 pi epsilon naught q v t divided by s square plus v square t square 3 by 2. And, you see this component is in the direction opposite to the z direction. So we can put z here, but one minus sign.

Now, E d a will be minus 1 by 4 pi epsilon naught integration over q v t s square v square t square 3 by 2 and the integration. This d a, we can integrate over this small parts, which we can write like two pi s ds. So, the integration gives q. Why so? This cancel this, here 2. So, 2 epsilon naught, and then we have v t s d s divided by s square plus v square t square three by two. This integration is straightforward. And, I am writing the result here. It is minus q by epsilon naught v t divided by; and, this integration is actually done from this 0 to R. So, 0 and here it is R. So, from 0 to r, we are doing the integration; 0 to R. 0 to R, we are doing the integration. So, we will get here R square plus v square t square 3 by 2 minus 1. So, we have got the flux now.

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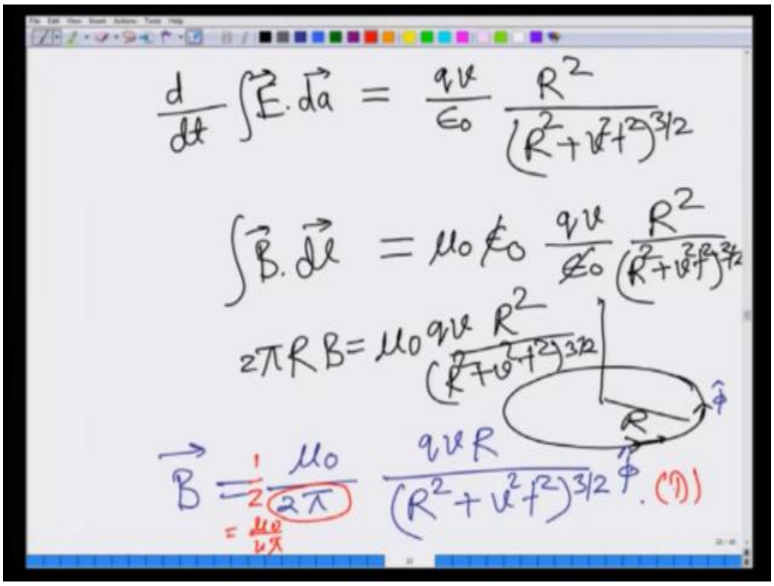

So, let us calculate the change in the flux with respect to time. So, d d t of E dot d a; which comes out q v by epsilon naught R square divided by R square plus v square t square three by two. Now, recall that we have B dot d l integrated over the path is equal to mu naught epsilon naught this change in flux, that is, q v. We have already calculated. So, R square plus v square t square 3 by 2.

So, this cancels this and, the integration along this path, which is the charge particle, since B is same at all the points. So, we can write it like this is R, 2 pi R B, magnitude of B, is equal to mu naught q v; and the this thing. So, B comes out to be mu naught divided by 2 pi q v R by R square plus v square t square 3 by 2. Now, the direction is this phi. So, the magnetic field becomes in phi direction. This should be 4 pi actually. So, let me see; we should keep this 2 here. This will carry forward there and, now it will become; final result will come with mu naught by 4 pi. So, the correct option is D.