Introduction to Electromagnetism

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Module - 08
Lecture – 75
Assignment - 05
Problem 1-5

And this session and the next one, we will be solving assignment number five and assignment number six. You have seen me solving assignment number one through four so far. And, what we are going to do in these two sessions is that the problem solving will be done by the teaching assistants, who have been involved in this course right from the beginning. They are regular Ph.D. students at IIT Kanpur, Physics department. And, they have been responsible for correcting the assignments, generating their solutions. And, so today they will be sharing the solutions that I have done with you. Assignment number five will be solved by Mr. Rabeeth Singh and Miss Parmita Dass Gupta. So, here is Rabeeth sigh.

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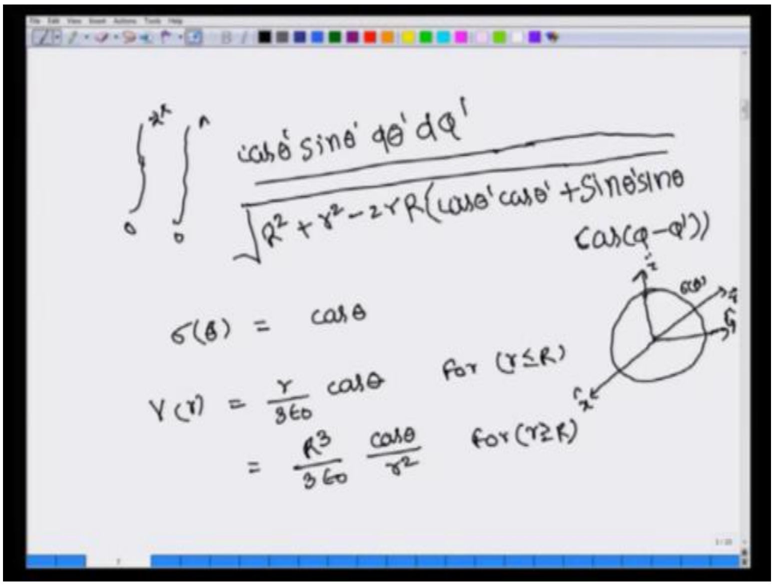

In the first question, you have to calculate this integral cos theta prime sin theta prime d theta prime d phi prime over R square plus small r square minus two r capital R cos theta prime cos theta plus sin theta prime sin theta cos phi minus phi prime .

So, what? Use the high symmetry density of a spherical shell to calculate this integral. So, first what we will do? We will calculate the potential due to a high symmetric density for spherical shell. We know, if we have given a density sigma theta equal to cos theta. So for this density, we have seen in the lectures, the potential. So, we have a spherical shell at which this density sigma is included. And, we have seen in the lectures that the potential due to this spherical shell at point vector r is given by this expression, which is r over three epsilon naught cos theta; for r is smaller than capital R. And for r is greater than capital R, this potential is given by R cube over three epsilon naught cos theta over r square. So, we will use this answer to calculate this integral.

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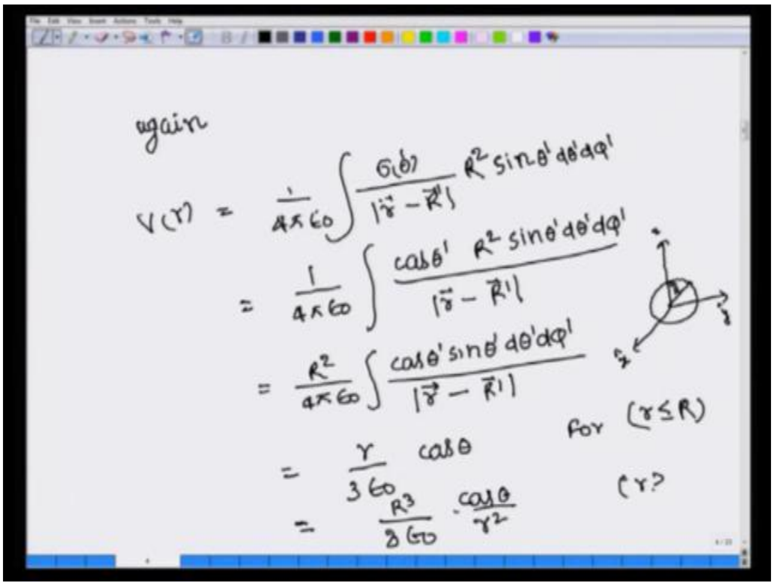

So, again V r equal to one over four pi epsilon naught sigma naught theta over vector r minus vector capital R d theta d phi. So, if sigma theta is equal to cos theta, we can write this answer. So, if the given density sigma theta prime is cos theta prime, so we can write this cos theta prime. And for the spherical system, this surface element is given by R square sin theta prime and d theta prime d phi prime. So, if the given density is cos theta prime, you can write this as R square sin theta prime d theta prime d phi prime over vector r minus vector R prime; where vector R prime refers to the point on the spherical shell.

So, we can write this as R square over four pi epsilon naught cos theta prime sin theta prime d theta prime d phi prime upon vector r minus vector R prime. So, if we compare the two answers, so I can see this potential is equal to; because we know the answer for this potential. So, we can compare this with those answers, which is given by r over three epsilon naught cos theta; for r is smaller than capital R. and, equal to R cube over three epsilon naught cos theta over r square; for r is greater than equal to capital R.

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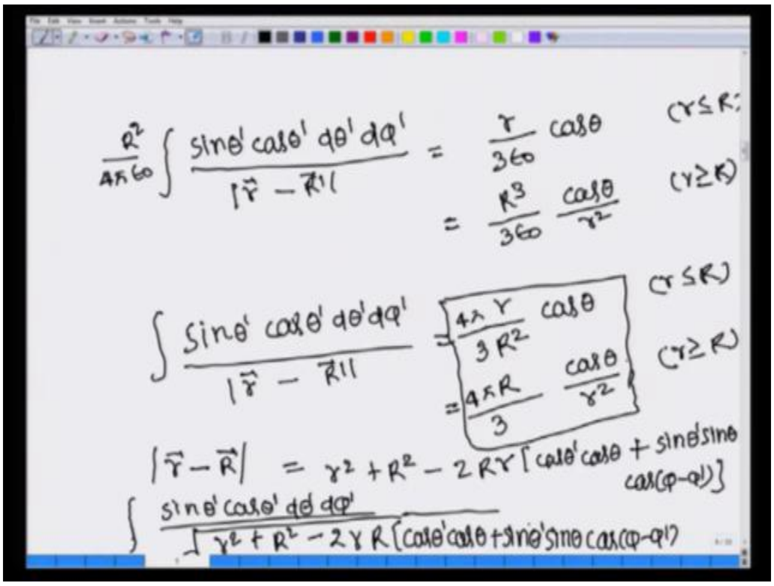

So, we can see that; we can write this as cos theta prime sin theta prime d theta prime d phi prime over vector r minus vector R prime one over four pi epsilon naught R square equal to r over 3 epsilon naught cos theta; for r is smaller than equal to capital R. And, equal to R cube over three epsilon naught cos theta over r square; for r is greater than equal to capital R.

So, you can write this as sin theta prime cos theta prime d theta prime d phi prime over vector r minus vector R prime equal to; so you can see, this can be written as r over capital R square into 4 pi cos theta; for r is smaller than capital R. And, equal to 4 pi R over 3 cos theta over r square; for r is greater than equal to capital R. So here in a spherical coordinate, vector r minus vector capital R can be written equal to, sorry mod of vector, mod of vector r minus vector capital R can be written equal to r square plus capital R square minus 2 capital R small r cos theta prime cos theta plus sin theta prime sin theta cos phi minus phi prime. So, if we write the value of mod r minus vector R, then we can see the integral sin theta prime cos theta prime d theta prime d phi prime over r square plus capital R square minus 2 r capital R cos theta prime cos theta plus sin theta prime sin theta cos phi minus phi prime. So, which is nothing but the integral we have to calculate. So, this is the answer for the first question.

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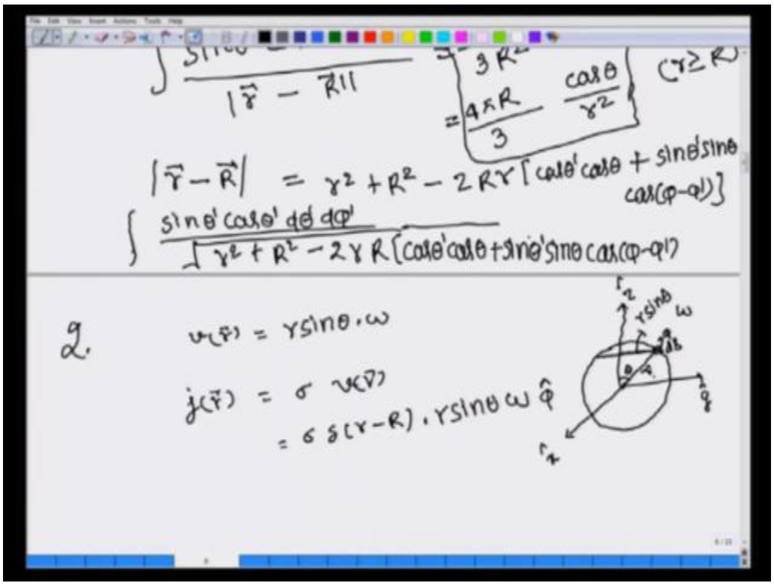

So, our next question is question number two. Question number two: we have a uniformly charged spherical shell, so this is rotating around the z axis. So, if we take a surface element on this spherical shell, we can see that every element is moving around the z axis with the angular velocity omega. So, the velocity of the surface element d s is given by; so, this angle is; so, if vector r next the angle theta with the z axis, you can see that this is the radius of the circle at which this surface element d s is moving. So we can see, this is the circle at which d s is moving. So, the radius of the circle is given by r sin theta. So, you can see V r is given by r sin theta into omega; which is nothing but the radius into the angular velocity. So current density, which is j r is given by sigma, means whatever density is there, sigma into V r. But, this is the surface density.

So, if we write the volume charge density, volume current density, we will write this as delta r minus vector R into v r. So, we can see this surface element moving along the phi direction. So, the r direction of j r would be along the phi capitalism. So, this is the answer for the second question.

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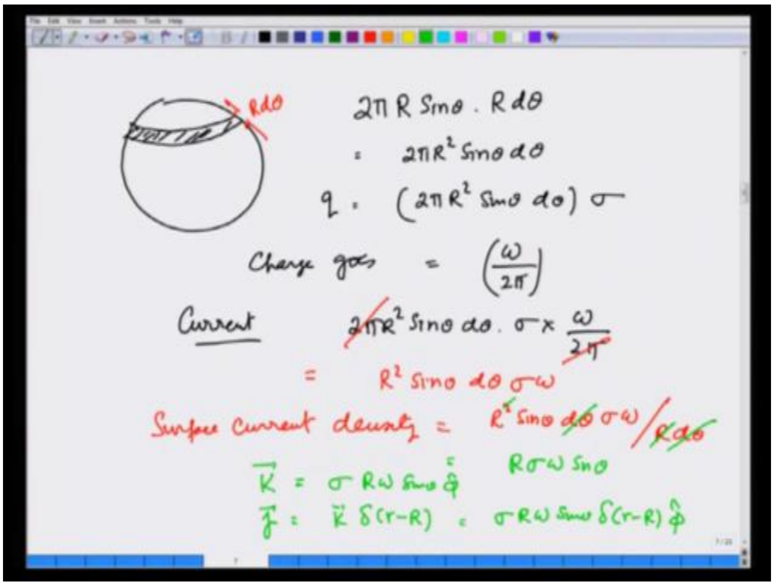

Another way of looking at the answer for the same problem would be; if I take this spherical shell and consider a band here, this band has an area which is two pi R sin theta times R d theta. This is the total area of the band. So, this is the 2 pi R square sin theta d theta. The charge on this band, net charge q is 2 pi R square sin theta d theta times sigma. And, this charge is going around. It goes around in per second; per second it goes. The charge goes around per second omega over 2 pi times.

So, the current that this band is giving is nothing but two pi R square sin theta d theta times sigma times omega over two pi. Let us do some cancellations 2 pi cancels and you get R square sin theta d theta sigma omega; that is the current going at the surface. So, the surface current density will be this current divided by the length out here, which I am showing by the red on this sphere, which is R d theta. So, this will be R square sin theta d theta sigma omega divided by R d theta. Let us again do some cancellations; d theta cancels, R cancels and you get R sigma omega sin theta. So, the surface current density K is sigma R omega sin theta in phi direction.

And, if I express this as a volume current density, j would be equal to k times. It is at r minus R. So, this becomes sigma R omega sin theta delta r minus R phi, which is indeed the same answer as earlier. So, what we try to do is give you two different ways of looking at the same problem.

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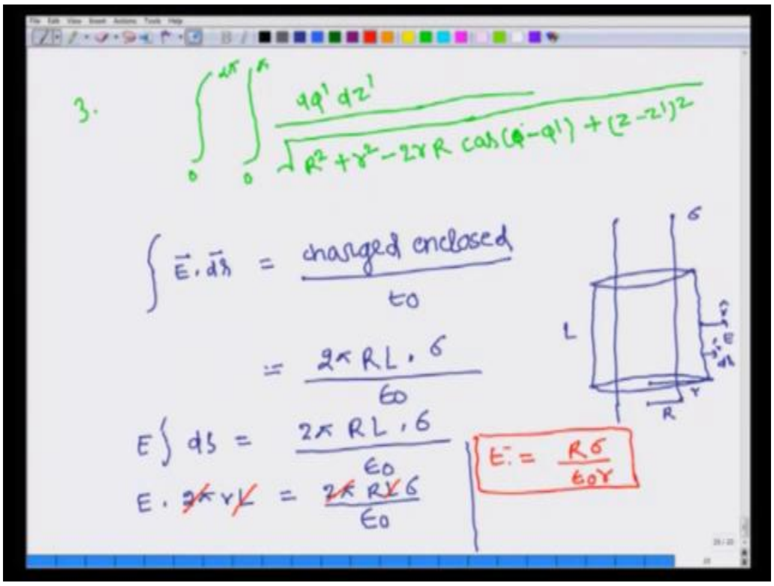

So, now we are going to solve a problem number three. So, in problem three we have to calculate the integral d phi prime d z prime over vector R square plus this small r square minus 2 small r capital R cos theta cos phi minus phi prime plus z minus z prime. This is square of z minus z prime. So, we have to calculate this integral. So, what we will do again? So, we have a cylinder. I bring a uniform density sigma. So, first I will make a Gaussian surface, which is cylindrical; because here is the phi symmetry. So, we can see electric field is perpendicular at every point on the surface, which is along the r k direction.

So, and surface element is also in the same direction, which is R capital. So according to the Gausses law, we can write. And, length of this surface is capital L, the radius of the cylindrical cell is capital R. And, the radius of Gaussian surface is smaller. So, we can see; we will use the Gausses law, which is E dot d s equal to; so charged enclosed. Gausses law is equal to charged enclosed in the Gaussian surface upon epsilon naught.

So, we can see charged enclosed would be equal to the surface area of the cylindrical shell, which is given by 2 pi capital R into L into charge density upon epsilon naught. So, because E is constant at every point, we can write this as d s equal to 2 pi capital R into capital L into sigma over epsilon naught. So, d s is the surface area of the Gaussian surface. So, which is 2 pi small r in to L equal to 2 pi capital R into L sigma over epsilon naught. So, the electric field E is equal to; because L 2 pi would be cancelled out. So, electric field is given by R sigma over epsilon naught in to r.

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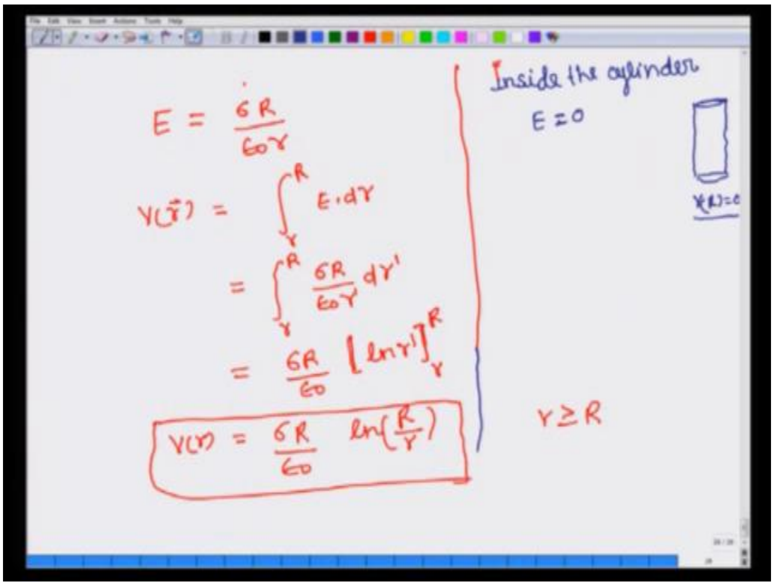

So, now because we know the electric field at point r is sigma capital R over epsilon naught into r. So, the potential v r is given by r to capital R E dot d r; where r is the reference point. So, what I will do? I will take the reference point at the surface of the cylindrical shell. So, this would be equal to E sigma capital R over epsilon naught into r d r, which is equal to sigma R over epsilon naught l n r prime r to capital R. So, this is given by capital R over epsilon naught l n capital R over small r. So, this is the potential. So, this is the potential due to this cylindrical shell, cylindrical cylinder, having the uniform density sigma.

So, I will use this result to calculate for r is greater than capital R and inside the cylinder, E equal to 0o. So inside the cylinder, potential is constant and which is would be equal to whatever potential would be on the surface of the cylindrical shell. So if we take potential at the reference point 0, let it be r equal to 0. So, we have v r equal to 0 inside the cylinder and outside the cylinder, v r equal to this point.

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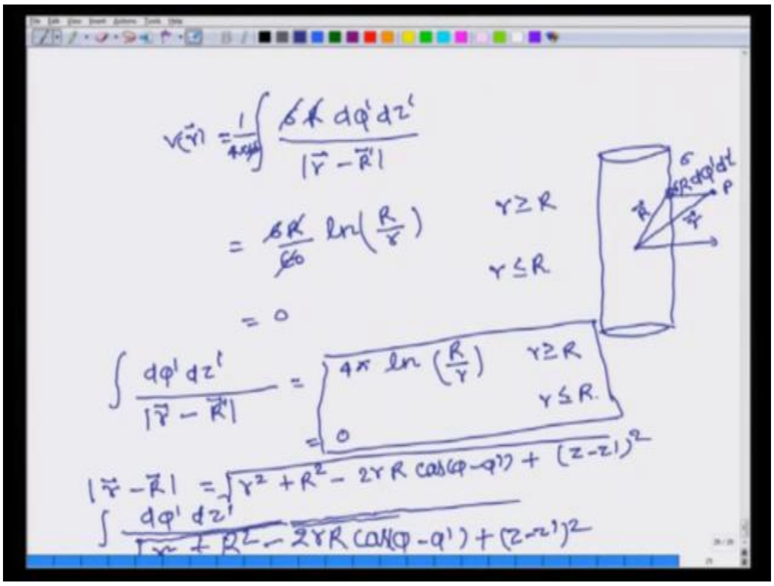

So again if I write, if I again; if you have given a cylinder shell having a uniform charge density sigma, so potential at point p and this is the charge element, which is given by R d phi prime d z prime into sigma.

So, this is the charge element. Due to that, you have to calculate the potential at point p. So, this is vector r. So, we can see the potential d b. So, the potential at point p will be the sum of the potentials due to the all the surface elements, which is given by the integral v r equal to sigma R d phi prime d z prime over vector r minus vector R; which is the distance from the charge element to the point at which you have to calculate the potentials.

So, we know the answer of this problem, which we have calculated. So, this v r is given by; this is sigma R over epsilon naught ln R over small r; for r is greater than capital R and, equal to 0; for r is smaller than capital R. So if we see this integral, you can; sigma r would be cancelled out and there is a one more vector, that is, 1 over 4 pi epsilon naught.

So, epsilon naught would be cancelled out. So, we can write d phi prime d z prime vector r minus vector capital R. You see this is r prime equal to 4 pi l n vector R over small r; for r is greater than equal to capital R. And, equal to 0; for r is smaller than capital R. So if we see this integral, we can write mod of vector capital R; which is r square plus capital R square minus two r capital R cos phi minus phi prime plus z minus z prime whole square.

So, if I put the value of vector r minus capital R, mod of vector r minus vector capital R, so that is d phi prime d z prime over r square plus capital R square minus 2 r capital R cos phi prime phi minus phi prime plus z minus z prime whole square. So, this is nothing but the integral which we have to calculate. So, this is the answer for problem number 3.

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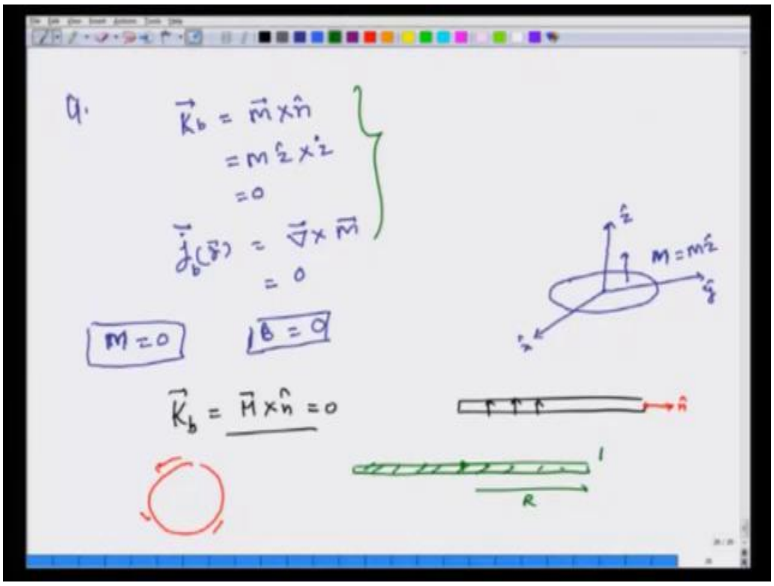

So in the assignment, there is a factor; which is two phi was not there. So that is a just typing error. In question number four, we have a disc; which has the uniform magnetization along the z axis. So, magnetization is along the z direction; which is this. So if I calculate the bound current, which is sigma B; which is given by M cross n cap. So, here if I write vector M, which is along the j direction. And, we can see the surface element is also along the z direction. So, sigma B is equal to 0. So, this is our K b.

So, and the volume charge density, which is j b is given by del cross M because we have a uniform magnetization. So, this is also g, outside the disc, M equal to zero; because magnetization is only on the disc. And because there is no free current and no bound current, so we can write B is also equal to 0. So, this is the answer for problem number 4.

The couple of the comments on the solution and different way of looking at it. What you told is that K b; which is M cross n is 0; which in effect, it is on the upper and lower surface. But, you may raise a question. And, the question would be if I have this and M is in this z direction, on the side surface there is a n so that, you take M cross n. You do see here current on the periphery of the disk.

So if I look at from the top, this current may be going like this. However, what we have done this problem is that this is a thin large disk in which the extent of the radius of the disc is much larger than the thickness. And therefore, if you think may be the surface current gives rise to some magnetic field, yes, it does. But, it is very small. Effectively zero because large R would give you a very small field at the center. And therefore, what was done was correct. But, if you have a question that there may be peripheral current, you are right. But that current contributes nothing to the magnetic field. You can also look at the same problem from the point of view of H or the auxiliary field of that you are talking about.

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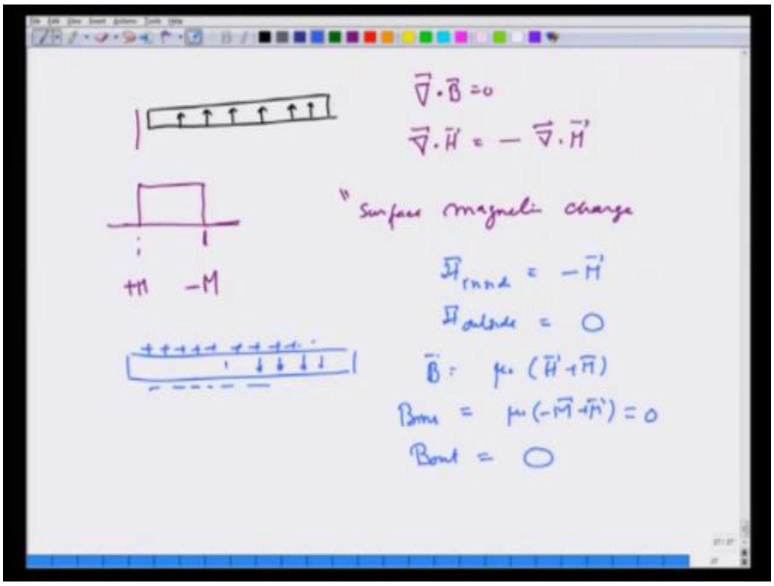

Now, there is a M like this. You recall that we have done in the lecture that divergence of B is zero. And therefore, divergence of H is equal to minus divergence of M. What that gives; you see if you look at across M, it is finite and then suddenly goes down.

So, M is like a delta function or it gives rise to a surface magnetic charge if you like. Surface magnetic charge, you can see out here divergence of M is going to be minus M. And on the other side, divergence of M is going to be plus M; because as you go across M is increasing, then M decreases. And therefore, when I take minus of divergence of M it is as if on this disk for calculation of H, there is a positive magnetic charge here and negative magnetic charge here. And, the kind of field is given rise to would be like the electric field. So, H will be in the other direction here.

So, H inside will be equal to minus M and H outside will be equal to 0. I also know that B is equal to mu 0 H plus M. And therefore, B inside will be mu 0 minus M plus M, which is 0 and B outside, since H is 0 will be 0 and M is 0 there. So, this is same. As the problem was done earlier, both give you the same answer. We have given you two ways of looking at the same problem, either through the bound currents or by electric analysis. This also answers the question number two to question number five.