Introduction to Electromagnetism
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Lecture – 74
Assignment 5
Problems 6 – 8

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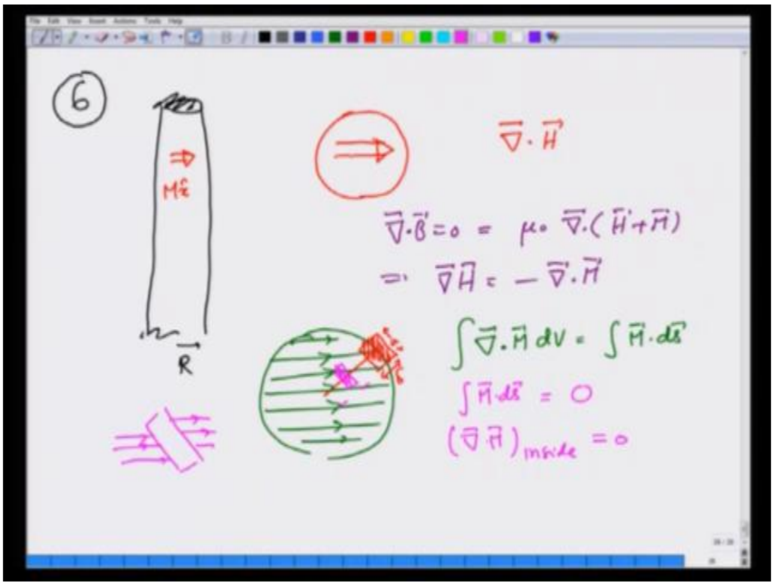

Continuing with solution of assignment number 5, I am now going to solve problem number 6, 7 and 8th. In problem number 6, it says consider a long cylinder of length L and radius R, R much less than L. So, we are really considering a very thin cylinder of length L and radius R. The purpose of taking it very long is so that we can ignore the Fringe effects; that means, we can essentially considered to be a an infinitely long cylinder and this has magnetization M in the x direction.

So, if I look at it from the top, this is how the magnetization looks. There is no z dependence and what we want to find is, what is divergence of H? I know that divergence of B is 0 and this is equal to mu 0, divergence of H plus M and therefore, divergence of H is equal to minus divergence of M. So, if I calculate divergence of M, I have my answer. Although, I can write the answer right away, I will show you how to systematically get it using Gausses law.

Now, if I look at it from the top as I said earlier, this is how M looks all over the place and best way to find divergence is using it is definition, which says that divergence of M integrated over a volume is going to be equal to the surface integral over the surface of this volume. If I want to find it at some angle phi, let me take a box here of very small length delta and area a.

Now, notice if this box is inside, if the box is inside and I calculate integral M dot d s. Since, this width, delta is very small that contributes to 0 the surface to this surface integral. On the other hand, look at this, M is coming in and M is going out. So, whatever flux is coming in is leaving. So, M dot d s is 0 and therefore, divergence of M inside is 0, which you would have expected, because this is a uniform magnetization.

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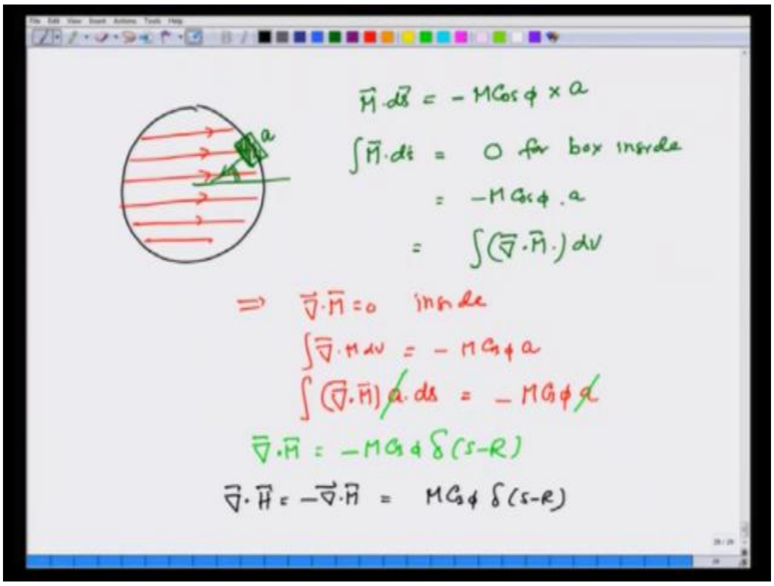

On the other hand, let us see what happens when I come to the surface. This is the magnetization, if I come to this surface and make this again this particular box of varying vanishing height, but area a. Notice that on the inner surface, the n is going out and therefore, M dot d s from this is going to be the component of M perpendicular to the surface, which is M cosine of phi. If I am at an angle phi from here and because, the normal to the surface is going in the opposite direction, it is going to be negative and there is no contribution from outside.

So, what we have found is that M dot d s, this should multiplied by the area is equal to 0 for box inside, completely inside the cylinder and is equal to minus M cosine of phi times the area. When one side of it is outside, one side of it is inside, of course, the width is the height of the box is nearly 0 and this should equal divergence of M times the volume d v.

So, this immediately implies that divergence of M is equal to 0 for inside, which is true and when the box is inside and outside, divergence of M d v gives me minus M cosine phi, even when the width or the height of the box is nearly 0. So, that means, there must be a delta function somewhere. So, I can immediately write that divergence of M times d v is a, times it is height d s.

Let us write is equal to or there is also minus M cosine phi times a, a cancels from both the sides and d s, when it goes across the surface is giving me 1, otherwise it gives me 0. And therefore, I can write that divergence of M must be equal to minus M cosine of phi delta of S minus R. And divergence of H being negative of that, divergence of H, which is equal to minus divergence of M should then be M cosine of phi delta S minus R and that is your answer.

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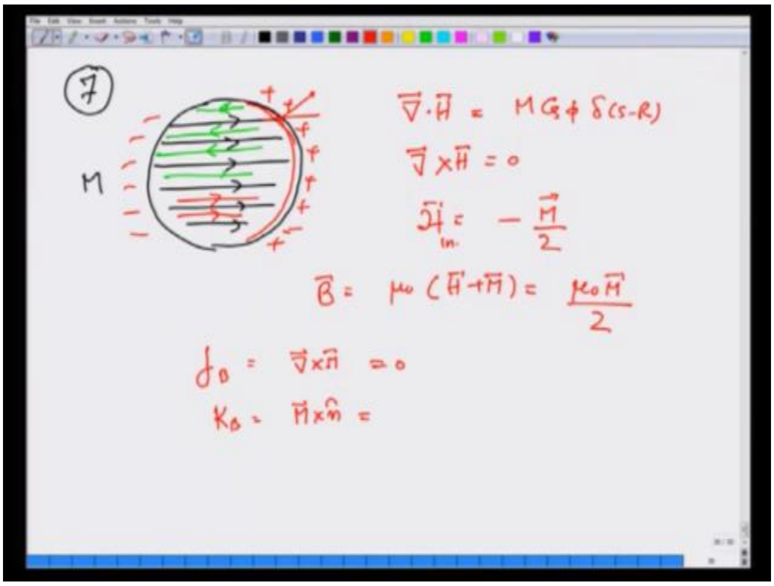

Next question number 7, it says the auxiliary field H and magnetic field B inside the magnetized cylinder of question number 6 is. So, again if I look at it from the top at the cylinder, I have the magnetization going in the x direction M. And we have already seen that divergence of H is equal to M cosine phi delta S minus R and therefore, since there is no current curl of H is 0. Therefore, H is going to be like the electric field of a charge distributed like sigma naught cosine phi on the surface and this immediately gives me.

So, this charge is positive on this side, negative on the other side and this immediately gives me that H should be equal to minus M over 2 or I can write the vector now minus m over 2. And therefore, H inside is minus M over 2 and B, which is equal to mu 0, H plus M is going to be mu 0 M by 2. So, H inside, this is H and M is in the same direction shown by red magnitude being M by 2 mu 0, M by 2. So, that is question number 7, solved.

I will let you think about, how would you solve this problem using the bound currents. Because, remember bound currents J B, which is equal to curl of M is 0 in this case and K B, which is M cross n is going to be equal to the surface current here, M is in this direction, n is going out. So, M cross n is going to be the long cylinder, I will let you think about solving this.

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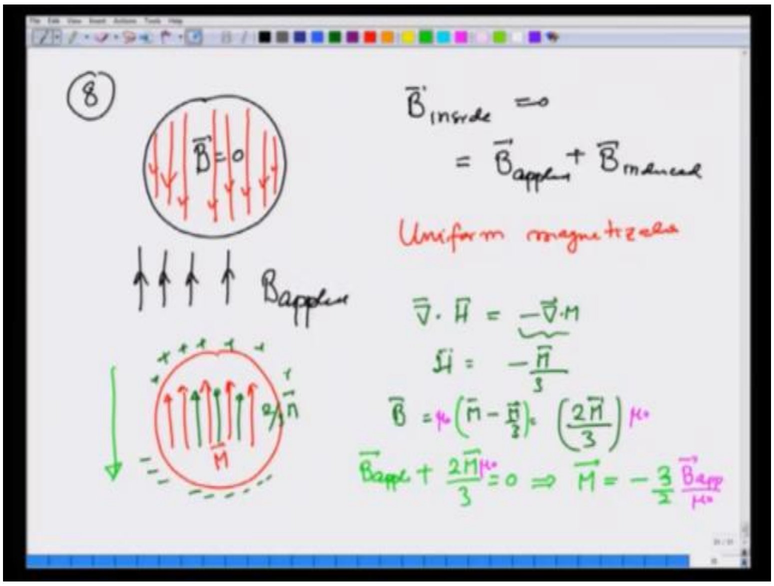

Let also shows the power of using H method sometimes. Question number 8, it concerns a material which is a perfect diamagnetic and that is a superconductor. What you are given is that, a superconductor no matter where you pour it, the field inside is always 0. So, if I take a sphere of superconductor and put it in an applied field B applied, we would like to know, what is M inside and what is the net H.

Now, I know B inside is 0 and this should be equal to B applied plus B induced due to whatever M is. Since, B applied is uniform, I also know then know that B induce has to be uniform and in the opposite direction. So, what we want now is that, I should have an M; that gives me an induced field in the opposite direction like this. We have already seen in one of the lectures earlier; that this is done by a uniform magnetization.

If I have a uniform magnetization M, let us say in the positive z direction. Then, so let me solve this problem, divergence of H, which is equal to minus divergence of M will give me for M going in the positive z direction. Some sort of a positive magnetic charge here, negative magnetic charge here and del cross H is 0. So, this is like the electric field due to a polarization and H in this case, then it is going to be minus M over 3 and therefore, the magnetic field is going to be M minus M by 3, which is 2 M by 3.

So, I have solved this problem of a hard spherical magnet, what the field should be inside, if it has magnetization M also height here. So, it gives me a magnetic field, I will write make it with green in this direction 2 3rd's M. Since, I want a magnetic field in the opposite direction, in the superconductor the M should be in the opposite direction. You can also see it by writing that B applied plus the field due to this magnetization, which is 2 3rd's M must be 0.

Because, field is 0 inside the superconductor and that immediately gives me that magnetization is minus 3 by 2, so I should have this is H. So, I should have written times mu 0 here, let me write it with pink, there is a mu 0 also. So, it should be mu 0 and so M is equal to 2 3rd 3 halves of B applied over mu 0; that is the M.

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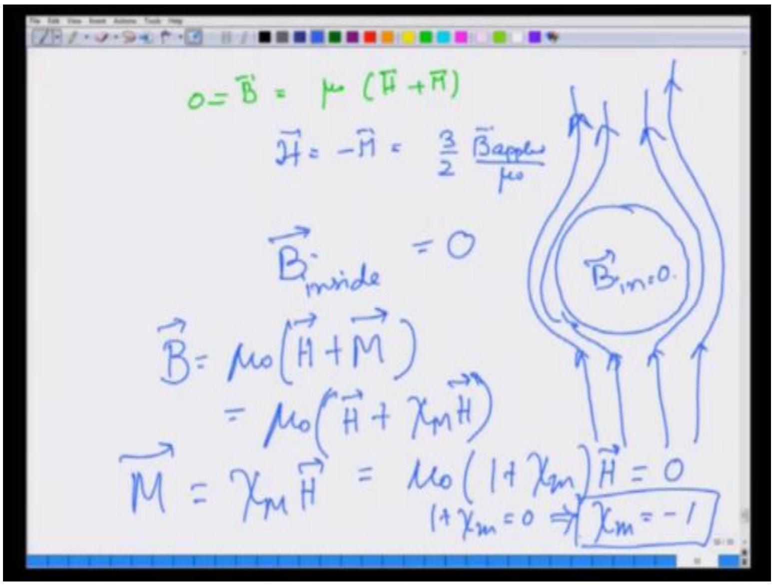

We have found M and I know that B is 0 and B is equal to mu 0 H plus M and B is 0, therefore, H is equal to minus M, which is going to be 3 halves B applied over mu 0 of this 3 halves. 1 H comes from B applied and 1 half of H comes from the M, which has been induced. Rest of the problems in this assignment number 9, 10 and 11 are going to be done by Miss, Horamita Das Gupta.

Student: So, in question number 8, we have solved for the auxiliary field H and the magnetization M inside the superconducting sphere. So, now, we are interested to know the value of magnetic susceptibility Chi M inside that superconductor. So, we know that magnetic field B inside a superconductor is 0, if I place a superconducting material in a magnetic field B, so the magnetic field lines behave like this. So, it will be like, it will not enter inside the dielectric.

So, if I place a superconducting material in a magnetic field, then the magnetic field will not enter inside the superconducting material and the field B inside is 0. So, I know B equal to mu 0, H plus M and equal to mu 0, H plus Chi M H, because M is Chi M H. So, I get B equal to mu naught 1 plus Chi M H and this is equal to 0. So, I get 1 plus Chi M equal to 0; that gives me the value of Chi M as minus 1. So, the value of magnetic susceptibility of a superconducting material is minus 1.

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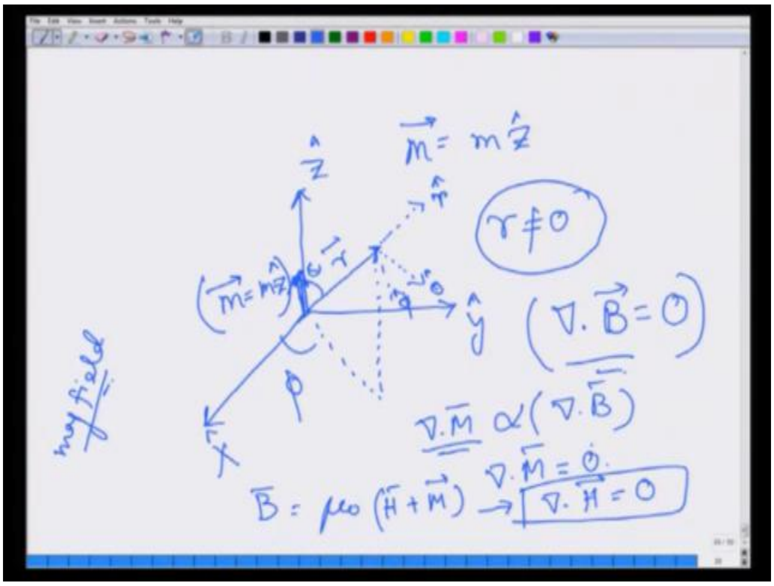

So, in problem number 10, we had to find out the auxiliary field H of a point dipole, which is of infinite extent at r not equal to 0. So, the dipole is placed at the origin and it is value of it is magnetic dipole is m equal to m z, let me draw a figure for this. So, this is Cartesian coordinate system, I am placing my magnetic dipole at the origin, the magnetic moment is m z.

So, it is directed along the z axis, this is a point dipole and I can also draw the spherical coordinate system here, this is my theta and this is my phi. So, this is the r vector, this is theta and this is phi. So, now, I will see, what is the value for the auxiliary field H at r not equal to 0? So, we know that magnetization is produced, because of the magnetic field. So, if magnetic field B has certain kind of dependence on the space, magnetization m will mimic that dependence.

So, suppose we know that divergence of B equal to 0 and this magnetic dipole is of infinite extent. So, we do not have a surface enclosed with it, as we know that divergence of B equal to 0 and divergence of M is proportional to divergence of B. So, divergence of M is also equal to 0, as B equal to mu naught H plus M, it follows that divergence of H is also 0.

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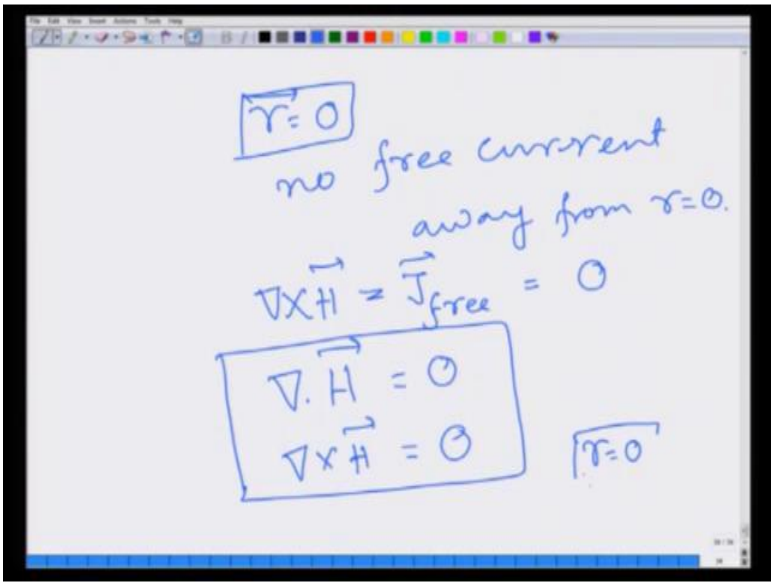

Now, in finding the curl of H, we know that the point dipole is placed at the origin r equal to 0. We can refer to the previous page ((Refer Time: 16:03)), where we have shown this diagram of the magnetic dipole and this is the coordinate system shown. So, now, it is placed at r equal to 0, so we do not have a free current. So, we do not have a free current, away from 0,

Student: Away from 0, away from r equal to 0. We can recall that curl of H is equal to J free. So, curl of H is equal to J free equal to 0. So, we have come to the conclusion that divergence of H equal to 0 and curl of H is equal to 0 for a point dipole placed at r equal to 0.

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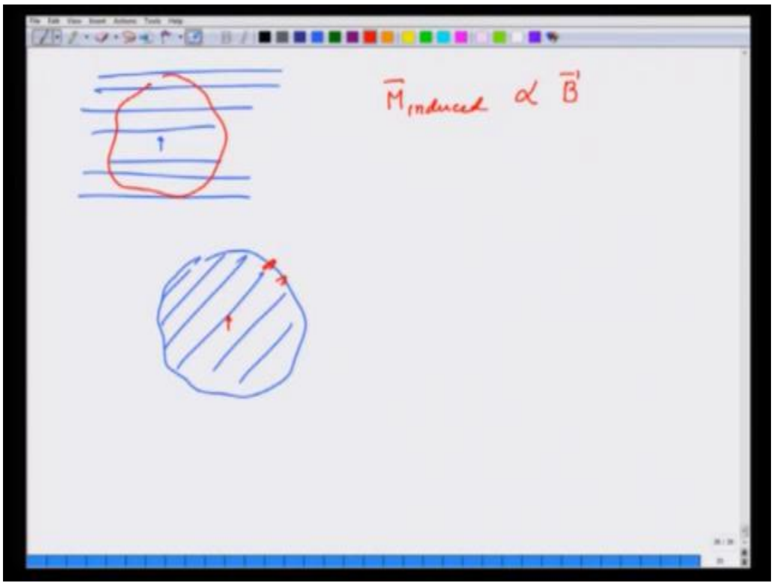

So, problem that you just saw the solution of in which we had a point dipole in an infinite medium and what was exploited was that M induced due to the magnetic field that was produced by this dipole is going to be proportional to B itself. And therefore, their functional dependence on r or space variables was the same. A point to be noted is because this is infinite extent material.

Suppose, we cut it, so suppose we change this question to that, we have a finite magnetic material or a sphere of magnetic material and I put a point dipole at the center. In this case, what would happen is that whatever M is produced, it will also have M dot n or M cross n at the surfaces and that will give rise to an additional H and the dependence will not be directly proportional to B; that will be slightly more complicated.

In this pay, particular case, where it was put in an infinite medium whatever arguments we were given are valid.

Student: In problem number 11, we will find out the auxiliary field H of a point dipole. So, we have two different ways of looking at the problem. In first way, we can make an analogy with the electrostatic case, we can see in the lecture videos, how we calculated the electric field of a dipole and here, we will make an analogy with that electric field and our auxiliary field H and find out the value of auxiliary field. So, let us proceed in this first way of doing this problem.

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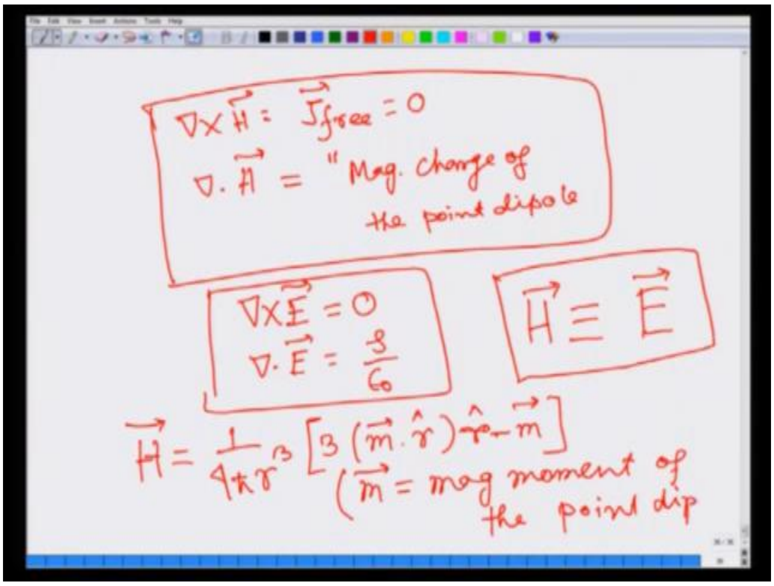

So, we know that curl of H is J free, which is 0. We have seen it in problem number 10 and divergence of H can be written as some magnetic charge of the dipole of the point dipole. Now, please recall that, curl of E is equal to 0, this E is the electric field of a point dipole and divergence of E is equal to rho by Epsilon naught, where rho is the charge density.

Now, if I can draw an analogy between this two, I will see that the auxiliary field of the point dipole is equivalent to the electric field of an electric dipole. So, this gives me a very simple way of looking at this problem and I do not have to go into the details of solving this, I can make an analogy. And I can just write that H is equal to 1 over 4 pi r cube 3 m dot r minus m, where m is the magnetic moment of the point dipole.

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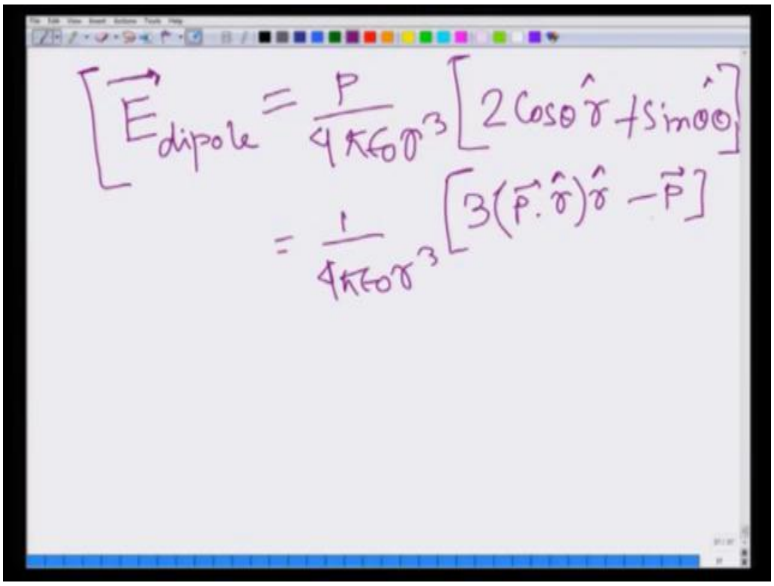

How can I get to this expression, because I know that for a point dipole, the electric field was P by 4 pi Epsilon naught r cube 2 cos theta r plus sine theta, theta. You can look at the lecture notes and you will get this expression and when you write it in a coordinate free form, you get the same form like 1 over 4 pi Epsilon naught r cube 3 P dot r, r minus P. So, when you relate it with the auxiliary field H, you can write down the expression for the auxiliary field. So, this is one way of looking at the problem.

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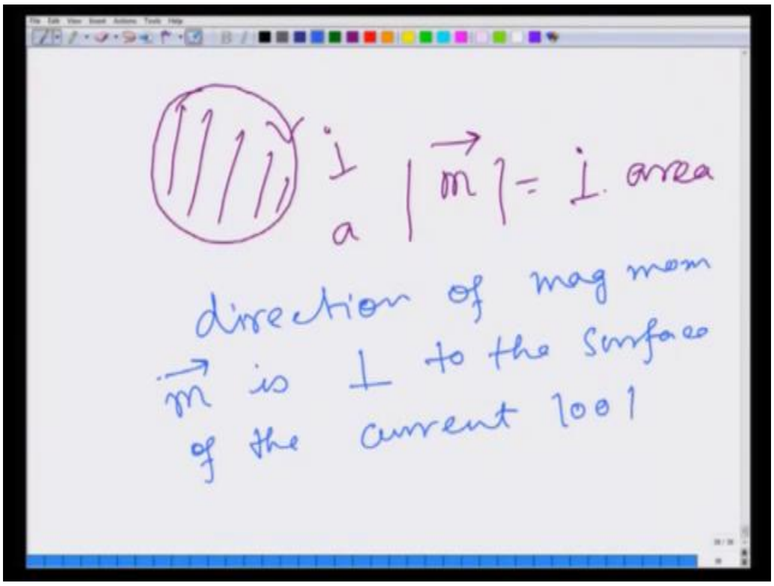

The other way of looking at the problem is purely magnetic. So, a magnetic dipole can be thought of a circular loop of current, say i and the area of this loop is say a. So, my magnetic moment m, the magnitude of the magnetic moment of this point dipole is current times area. So, now, I have to also know the direction. So, direction of the magnetic field is perpendicular, direction of magnetic moment m is perpendicular to the surface of the current loop.

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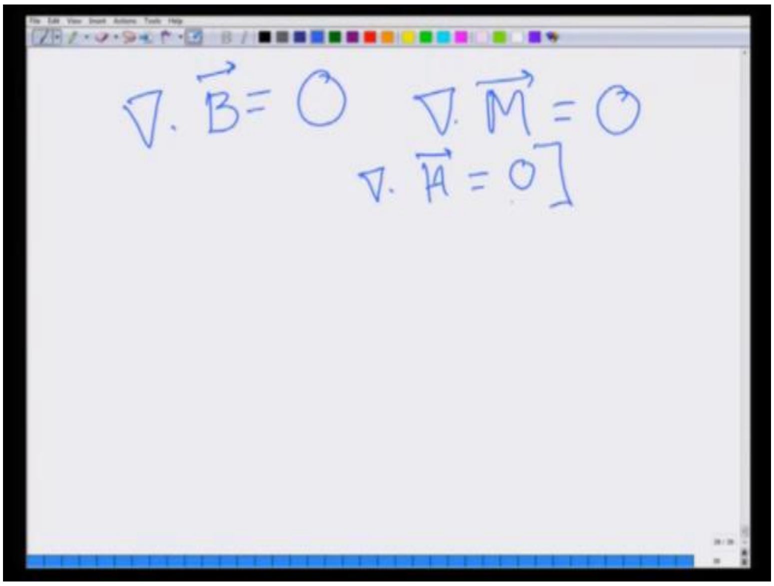

Now, we have seen that divergence of B equal to 0, divergence of M is also 0 and divergence of H is 0 in our previous solution.

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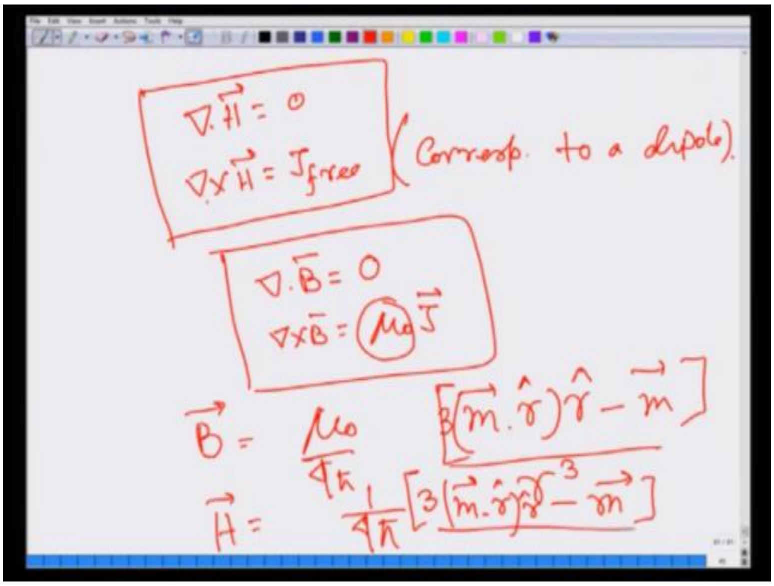

We know that divergence of H equal to 0 and curl of H is equal to J free corresponding to a dipole. Now, in case of a point dipole placed in free space, we know divergence of B equal to 0 and curl of B is equal to mu 0 J. Now, relate these two sets of equation, we can see that except this factor mu naught, these two equations are analogous. So, I can write, so we know B is equal to mu 0 by 4 pi 3 m dot r, r minus m by r cube as I can make an analogy between this two set of equations. I can directly write H except this mu naught. So, I will write H as 1 by 4 pi 3 is inside it. So, 1 by 4 pi 3 m dot r, r minus m by r cube.

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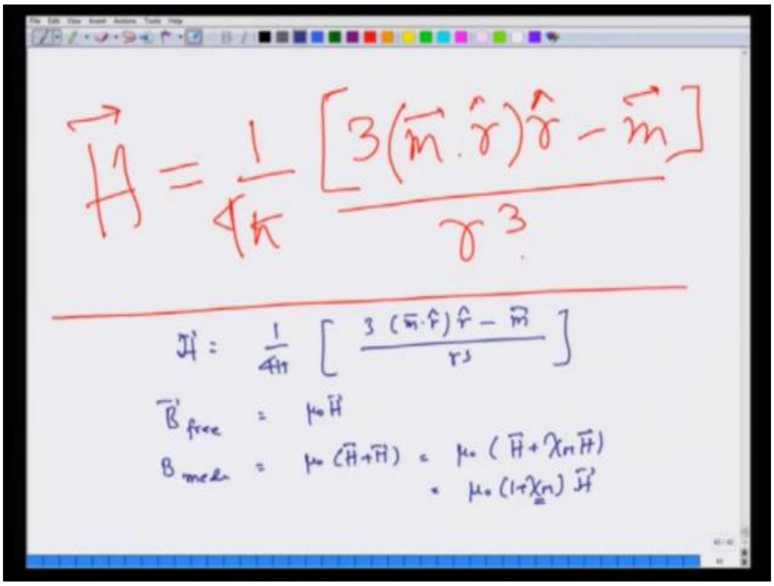

So, my final expression for the auxiliary field of a point dipole is 1 over 4 pi 3 m dot r, r minus m by r cube. Notice that this value of H is independent of whether the point dipole is in free space or it is an in infinite medium of magnetic susceptibility Chi M, because the equations give you that answer. The difference between free space and a medium would be when we calculate the magnetic field. So, H in both cases is 1 over 4 pi 3 m dot r, r minus m over r cubed as just derived, because of the analogy of the equations. B in free space, when the dipole is in free space, it will be just mu naught times H, there is no m. Whereas, B in a medium will be mu 0 H plus m, which is mu 0, this H, which is coming plus Chi m H; that will be mu 0, 1 plus Chi m times the H calculated. Depending on Chi m, if it is a paramagnetic material, it will be slightly larger than it will be in free space.