Introduction to Electromagnetism
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Lecture – 73
Assignment - 4
Problems 6 – 10

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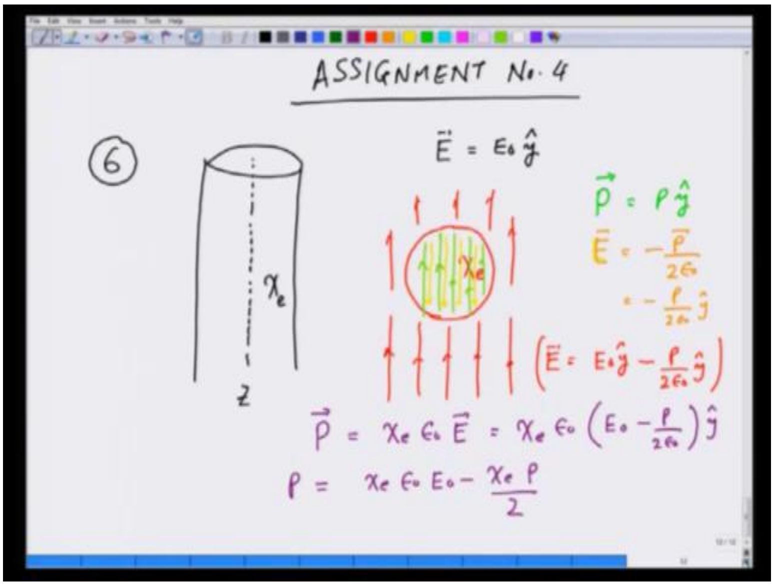

We have been solving assignment number 4, and we have solved problems 1 to 5, so we now solve problem number 6, which says consider a long cylinder with it is axis on the z axis. So, we have a long cylinder, so that fringe effects are gone with it is axis on the z axis. It is made of a dielectric material of electric susceptibility chi e, if it is put in a uniform electric field E equals E 0 y, find the electric field polarization and displacement in the cylinder.

So, if I look at it from the top, if I look at this cylinder from the top, what is happening is, here is a material chi e and we have put it in a uniform field going in the y direction. Now, from experience we know that this is going to generate a uniform polarization inside and what does this uniform polarization do, it creates intern an electric field in the opposite direction. It creates an electric field which I am showing by yellow in the opposite direction.

So, that the net field inside is a combination of the applied field and the field which has been created. So, let us say that the polarization created is p in the same direction it is E, so this is p in the y direction. This intern will create an electric field E which is minus p over 2 Epsilon 0 or minus p over 2 Epsilon 0 in the y direction. So, that the net field inside E is going to be E applied which is E 0 y minus p over 2 Epsilon 0 y.

Now, I can find p using the constitutive relationship which says that p inside, since everything is in y direction, I will not bother to write the vector sign on top or let us write it for the time being. P inside is equal to chi e Epsilon 0 E at that point, so this is going to be equal to chi e Epsilon 0 E 0 minus p over 2 Epsilon 0 in the y direction. Since, everything is in the y direction I can therefore, write p is equal to chi e Epsilon 0 E 0 minus chi e p over 2.

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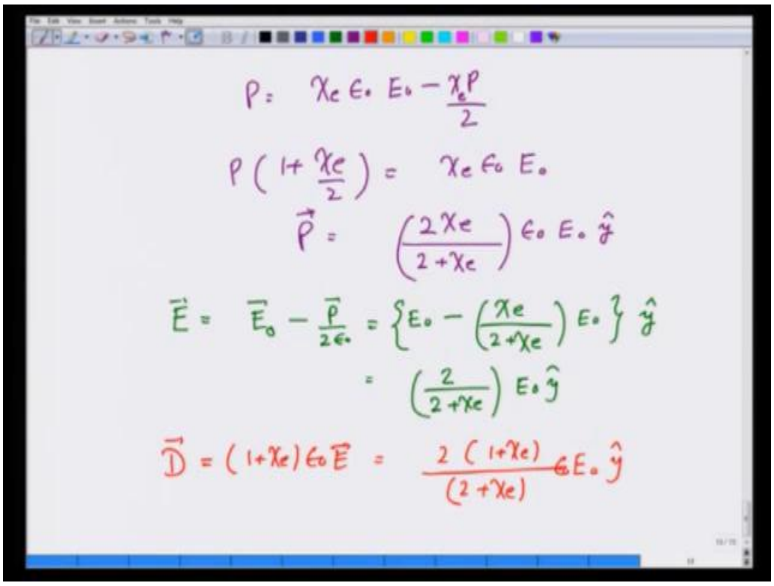

So, the equation now I have is p is equal to chi e Epsilon 0 E 0 minus chi e p over 2 and that gives me p 1 plus chi e over 2 is equal to chi e Epsilon 0 E 0 or p is equal to 2 chi e over 2 plus chi e Epsilon 0 E 0. Notice, if chi e is 0, then there is no p. So, now I can put the vector sign p is going to be in the y direction. The electric field E inside is nothing but, E applied which is E 0 minus p over 2 Epsilon 0, so this is going to be E 0 minus chi e over 2 plus chi e E 0 in the y direction.

So, this comes out to be 2 over 2 plus chi e E 0 in the y direction. What about the displacement? Displacement is nothing but either I can directly write 1 plus chi e Epsilon

0 E or I can use the relationship Epsilon 0 E plus p, which is all the same thing. So, this is going to be 2 times 1 plus chi e over 2 plus chi e times E 0 Epsilon 0 in the y direction.

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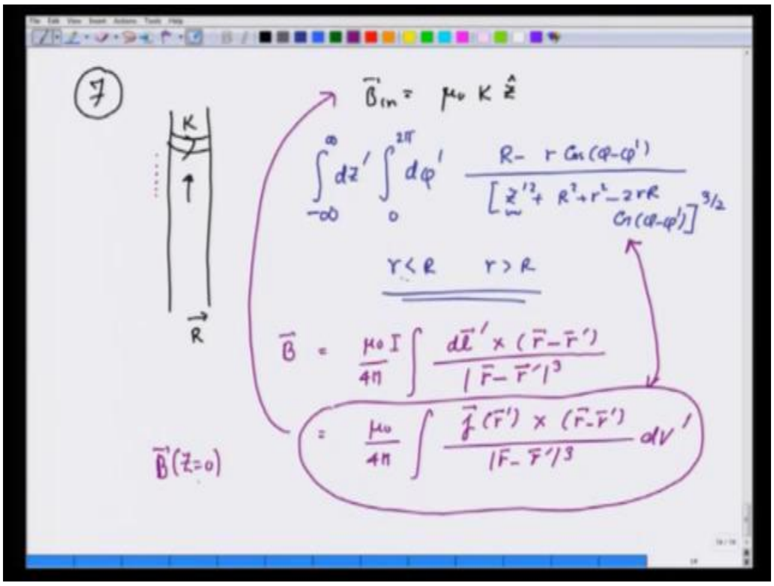

Next question number 7. This is actually making use of an known relationship to find an integral. So, we are looking at the field of a long solenoid and use that to calculate a particular integral. So, consider a long solenoid, it has a field inside B in is equal to mu 0 k z, where k is the surface current. If R is the radius of the solenoid, then on the basis of this fact and Bio Savart law, the value of the integral.

So, we want to use this to find the value of integral d z prime minus infinity to infinity integral d phi prime 0 to 2 pi and I have R minus r cosine of phi minus phi prime divided by z prime square plus R square plus r square minus 2 r R cosine of phi minus phi prime raise to 3 by 2. There is a typographical error in the assignment that we gave, instead of z prime we have written z out there, so correct that please. This value is for both r less than R or r greater than R, we want to find this.

Now, according to Bio Savart law B is nothing but, integral of mu 0 over 4 pi integral, if there is a current I. I can even write I d l prime cross r minus r prime over modulus r minus r prime cubed. This is also equal to mu 0 over 4 pi integral j r prime cross r minus r prime over r minus r prime cubed d volume prime. If we calculate this integral written here and show that it comes to this form that is, what the integral value you want to calculate, then we can relate.

We can find the value of this integral relating this to the value calculated through amperes prime, so let us do that now. What I will do in this problem is, since this is translationally invariant in the z direction, no matter at what value of z I calculate the magnetic field is going to be the same. So, I will calculate magnetic field at z equal to 0, which makes my life a little easy.

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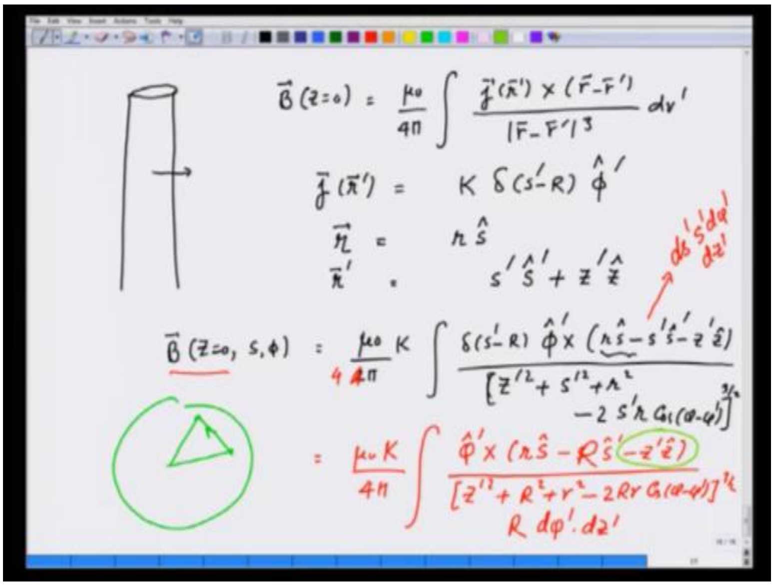

So, let us see we are looking at this solenoid and we are trying to find d for this at z equal to 0 at some r which is equal to mu 0 over 4 pi integral j r prime cross r minus r prime over r minus r prime cubed d v prime, let us see what the current density is. The current flows only on the surface and therefore, I can write current density j r prime which is in the phi prime direction.

So, this is going to be k delta I will use cylindrical coordinates s minus R or s prime minus R, where s is the cylindrical coordinate radial direction delta s prime minus R and this is going in phi prime direction, r vector we are taking z equal to be zero. So, this is nothing but r s, r prime vector is going to be s prime in the s prime direction plus z prime in the z direction. And therefore, I can write B at z equal to 0 and some s and phi is equal to mu 0 over 2 pi k comes out, integral delta s prime minus R phi prime unit vector cross r, which is r s minus s prime s prime unit vector minus z prime z unit vector divided by…

Now, the distance is going to be z prime square plus this r s unit vector minus s prime unit vector distance, which is s prime square plus r square minus 2 s prime r cosine of phi minus phi prime raise to 3 by 2. This is to see the distance that I just used, if you look at s and s prime and we are actually looking at a distance here. Take an x y, x and y components and you get this answer. So, I can do the s prime integral write over here and write this as mu 0, how did this 2 pi come, this is 4 pi.

Mu 0 k over 4 pi integral phi prime cross r s minus s prime will now will come R s prime unit vector minus z prime z divided by z prime square plus R square plus r square minus 2 R r cosine phi minus phi prime raise to 3 by 2 and I am left with only d phi prime R times d z prime integral, the volume integral. I should have written that here also, because the volume integral is nothing but, d s prime s prime d phi prime d z prime, d s prime we have already integrated over, so that s prime gets replaced by r.

Now, I know that final B is in the z direction, phi prime cross s is in the same x y plane is going to give me a component in the z direction. Phi prime cross s prime, again it is going to give me a component in the z direction. However, phi prime cross z is going to give me a component in the x y plane and that should integrate to 0, because final B on the left hand side, it is in the z direction. So, the last term this does not contribute to the integral at all. We can immediately see that if I write this integral with z prime, I know that the answer is going to be 0, because on the left hand side there is no component in the x y plane.

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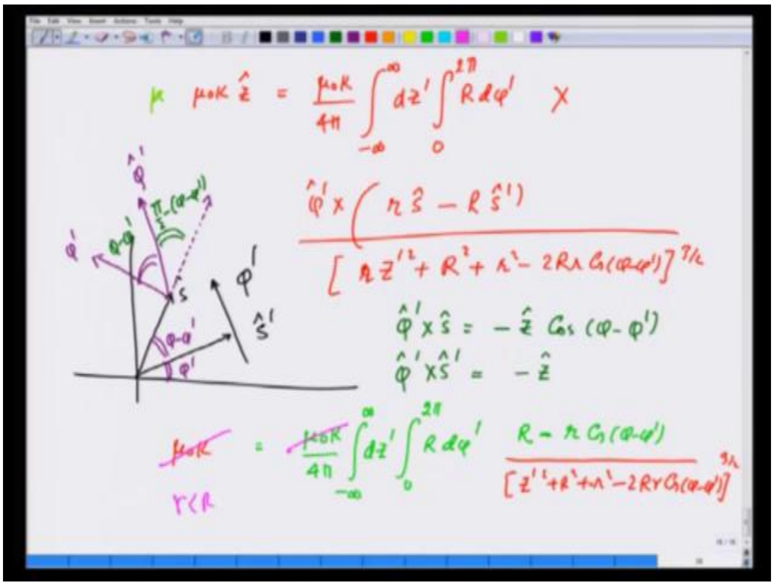

So, I am left with B which is mu 0. Let me write mu 0 k in the z direction is equal to mu 0 k over 4 pi integral d z prime from minus infinity to infinity, integral R d phi prime from 0 to 2 pi times phi prime cross r s minus R s prime divided by z prime square plus R square plus r square minus 2 R r cosine of phi minus phi prime raise to 3 by 2. Let us see what phi prime cross s is. So, let us just look on this x y plane, let us say this is s prime, this is phi prime and this is s.

This angle is phi prime and this angle between s and s prime is phi minus phi prime. So, at this point this is s, this is phi prime, the angle between s and s prime unit vector is phi minus phi prime, and therefore angle between phi prime unit vector and phi unit vector is also out here. Let me show it in a different color is phi minus phi prime and therefore, this angle is pi by 2 minus phi minus phi prime. And now, you can see that unit vector phi prime cross s is going to be in the z direction going into…

So, minus z direction minus z direction sine of pi by 2 minus phi minus phi prime, so that is going to be cosine of phi minus phi prime and phi prime cross s prime is equal to phi prime cross s prime. You can see is phi prime cross s prime is minus z, and therefore the right hand side in the above becomes is equal to mu 0 k over 4 pi integral minus infinity to infinity d z prime 0 to 2 pi R d phi prime. Inside I get R minus r cosine of phi minus phi prime in the z direction.

So, I can now removes z unit vector and write mu naught k on the left hand side divided by z prime square plus R square plus r square minus 2 R r cosine of phi minus phi prime raise to 3 by 2. Rest of the things are simple, I can cancel mu naught k on the two sides and this is only for r less than R. For r greater than R there is no field, so it is got to be 0.

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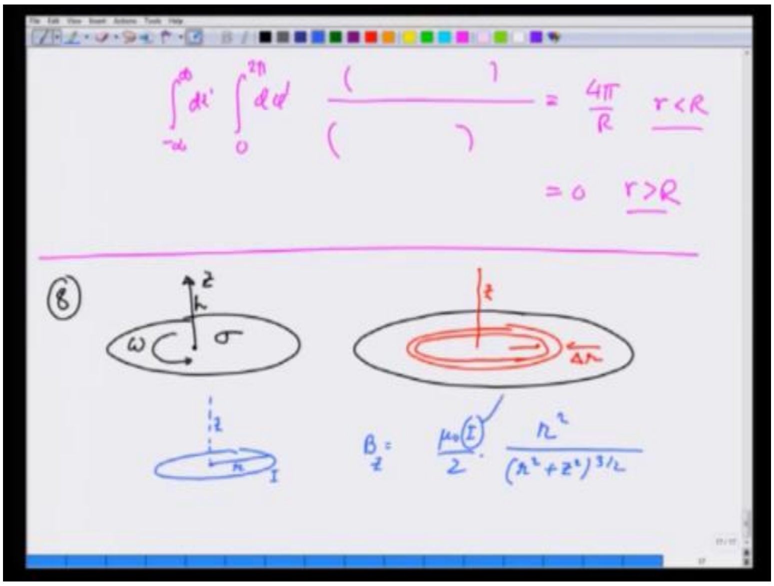

And therefore, the answer I have is therefore, that integral minus infinity to infinity d z prime integral d phi prime 0 to 2 pi. All that, that I have written is going to be 4 pi over R for r less than R and 0 for r greater than R and that is the answer. Next question number 8, a disc with it is center at the origin. Therefore, this is the z axis carries the uniform surface charge density sigma and is rotating with angular speed omega.

Then, the magnetic field on the z axis at height h is going to be… What I can do this problem is, divide this disc into small rings of width delta r. Calculate the magnetic field delta B due to each ring and then add it up, so I will calculate the field due to this ring at height z. Now, I know if I have a ring of radius r with current I, then at height z the magnetic field due to this is nothing but, mu 0 I over 2 r square over r square plus z square raise to 3 by 2 and it is in the z direction. In the present case, this I is going to be corresponding to the I due to this ray, so let us do that now.

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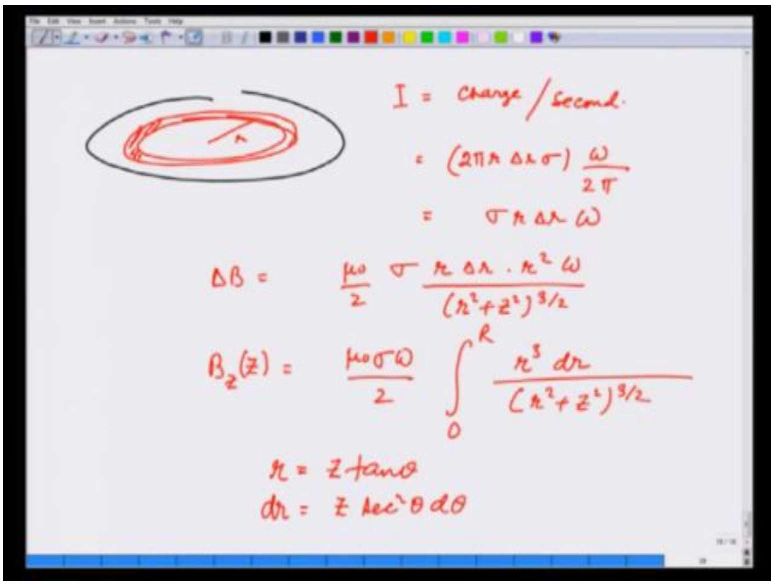

So, if I look at this disc, take a ring of thickness delta r radius r, then the current I is the charge passing per unit time. The charge in this ring of thick width delta r is going to be it is area 2 pi r delta r times sigma and per second, this charge passes omega over 2 pi times on the frequency times. So, this is equal to sigma r delta r, this is the current in this ring, and therefore that small delta B due to this is going to be mu 0 over 2 sigma r delta r times r square over r square plus z square raise to 3 by 2.

And total B which is in the z direction at height z therefore, is going to be mu 0 or there is an omega, there is an omega mu 0 sigma omega divided by 2 integral 0 to R r cubed d r over r square plus z square raise to 3 by 2. And this integral is very easy to calculate, you can calculate many different ways, but let see I can write r equals z tangent of theta or I could also do some other substitution, then d r becomes z secant square theta.

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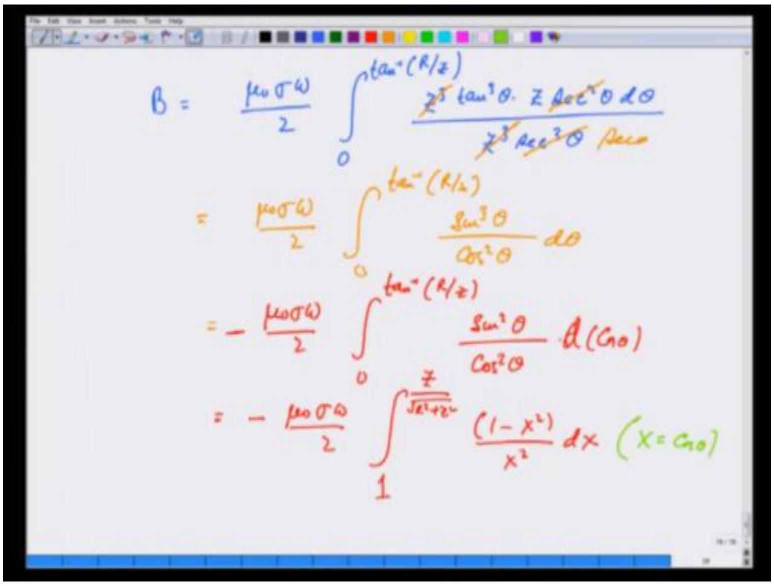

Then I have B equals mu 0 sigma omega over 2 integral theta equal to 0 to tan inverse R over z r cubed will become z cubed tan cubed theta, this is z secant square theta d theta divided by z cubed theta. Let us cancel a few terms that z cube cancels, secant square theta cancels gives me sec theta and this therefore, this becomes mu 0 sigma omega over 2 0 to tan inverse R over z sin cube theta over cosine square theta d theta, this is an easy integral to calculate.

I can write this as equal to mu 0 sigma omega over 2 integral 0 to tan inverse R over z sine square theta over cosine square theta, sine theta d theta which I will write as d of cosine theta with the minus sign in front. Now, we take cosine square theta to be x square, so this becomes minus mu 0 sigma omega over 2. At theta equals 0 cosine theta is 1, at theta equals tan inverse R by z cosine theta is z over square root of r square plus z square, I have 1 minus x square over x square d x, where x is cosine of theta.

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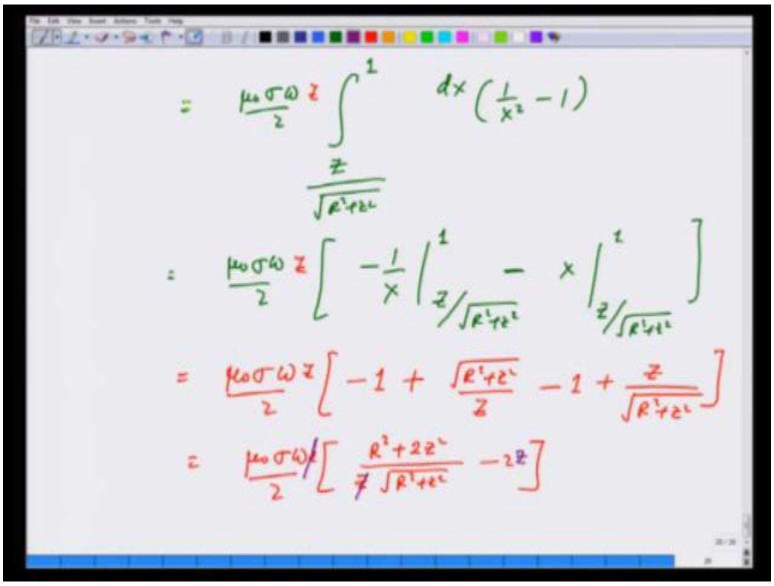

So, this is equal to mu 0 sigma omega by 2 z over square root of R square plus z square to 1 d x 1 over x square minus 1. I have change the minus sign and change the limits, so which is mu 0 sigma omega by 2. Inside, I have minus 1 over x going from z to square root of R square plus z square 1 minus x z over square root of R square plus z square to 1. And this gives me mu 0 sigma omega by 2 minus 1 minus, minus plus square root of R square plus z square over z minus 1 minus, minus plus z over square root of R square plus z square.

So, that the answer becomes mu 0 sigma omega by 2, R square plus 2 z square over square root of z R square plus z square minus 2 and there is a z outside also and let us see there was a z here. So, this z remains here, this z remains here, this z remains here, z remains here, this z remains here, this z comes here, z comes here and this is your answer. So, this z then cancels, I can cancel this with this and this is 2 z and that is my answer.

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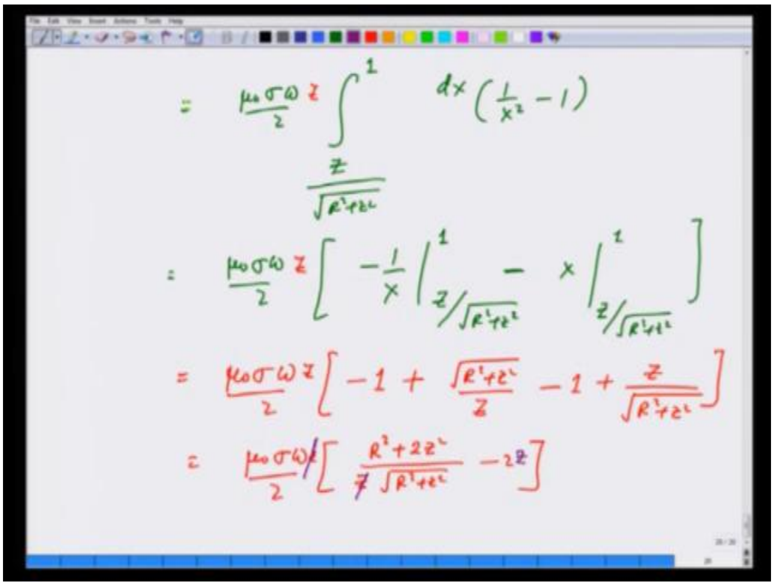

Next, problem number 9; consider a thin solenoid of length L. We have a thin solenoid of length L and radius a, L is much, much, much greater than a. So, that when I turn it into a torrid a ring shape, then this radius R is much, much, much greater than a. It is bent into a ring, so that it becomes a torrid which is kept at center at the origin. So, this is the origin and it is axis being the z axis, so z axis is coming out of the screen and the magnetic field in it is in the phi direction.

So, this has a magnetic field going in the phi direction. The solenoid carries a current I, so it carries a current I and has N turns. So, it carries a current I and has N turns, so that the number of turns per unit length is N over L. Taking into account the similarity of the equations of, we want to take into account the similarity of equations of curl of B is equal to mu naught j and curl of A is equal to B. Find the vector potential for this torrid on the z axis at height z.

So, let us see I am going to map this torrid with this B field going around in the phi direction. Make this similar to a ring with current I going around in the phi direction and the corresponding B. Since, the underlying equations are the same, the equation for B is going to be the same as the equation for A with mu naught j and B playing similar roles.

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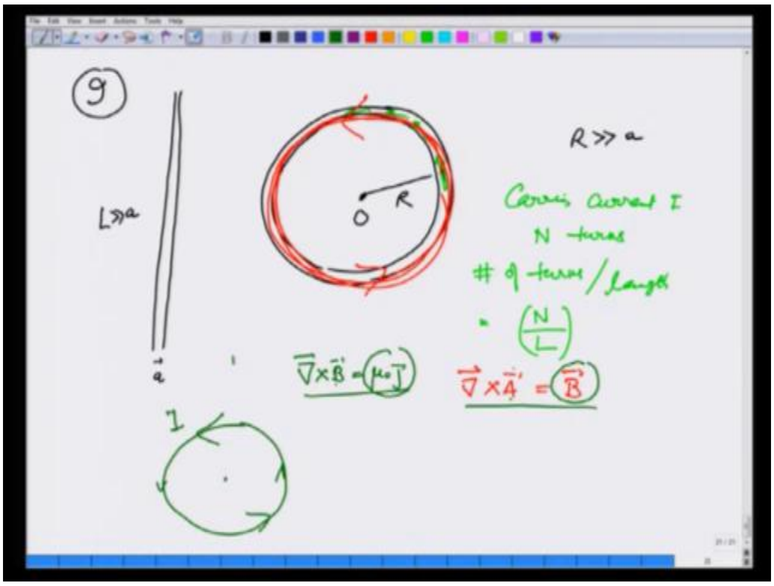

So, let us see now if I have a ring which carries current I, now if I solve for B, del cross B equals mu naught J. Then, B on the z axis is in the z direction and it is given as we have already seen it just now, mu naught I divided by 2 radius being R square R, R square plus z square raise to 3 by 2. What is I? I is nothing but, J dot d a. Now, let us look at the similar situation, for this torrid in which there is this B field going around and I have del cross A equals B.

So, I can write that A should be in the z direction, so the function of z on the axis and it should be there is mu 0 I, I will be replaced by now B dot d a which is the flux. So, flux through the ring divided by 2 times R square over R square plus z square raise to 3 by 2. What is the flux? Now, rest is all keeping the values in, so flux is nothing but, pi a square which is the cross section of the solenoid times the B field there, which is pi a square B is nothing but, N over L times I.

So, this becomes pi a square N I over L, R square is going to be L over 2 pi square divided by L over 2 pi square plus z square raise to 3 by 2 and rest is all algebra and you can calculate your answer. I must add here that, I think the answers that we have given in the assignment are missing a factor of pi square or something, please check that. So, let us just work it out and see what it comes out to be. It comes out to be pi a square N I L square over 4 pi square L times L square plus 4 pi square z square raise to 3 by 2 times 8 pi cubed.

And there is a factor of 2, there is a factor of 2 here, so this 8 cancels pi square cancels with this and I get a pi square, this L cancels with L. So, the final answer that we get here is pi square a square N I L over L square plus 4 pi square z square raise to 3 by 2. And I think then therefore, I am missing a factor of pi square in the answers that we gave in the assignment.

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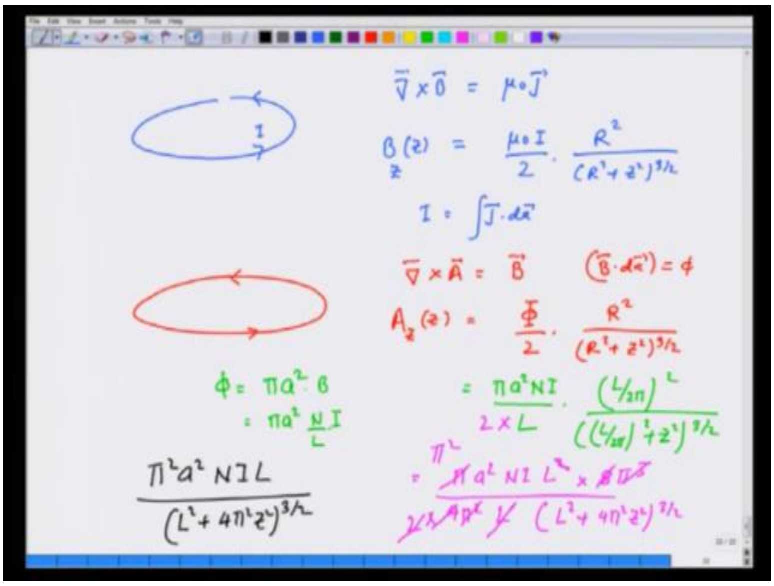

Finally, the last problem which says that magnetic vector potential of on the z axis of this disc, which is rotating with omega and carries a charge sigma will be… We want to use the equation del square A equals minus mu naught J. In this case, we have already figured out J is going to be only the surface current k which is nothing but, sigma 2 pi r delta r which is the current being carried by this ring at distance r from the center divided by delta r times omega over 2 pi is going to be J is in the or k which is in the phi direction.

So, this is going to be this 2 pi cancels, delta r cancels, you will get sigma r omega in phi direction. Now, using the Poisson’s equation del square A equals minus mu naught J and therefore, del square A x is minus mu naught J x and del square A y is minus mu naught J y. You can see that A should have, should be in the direction of phi itself, because J is in the direction of phi. But, on the z axis there is no uniquely well defined phi and therefore, A on the z axis is going to be 0 and that is your answer. So, this completes assignment 4 for us.