Introduction to Electromagnetism
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Module - 8
Lecture - 72
Assignment 4
Problems 1-5

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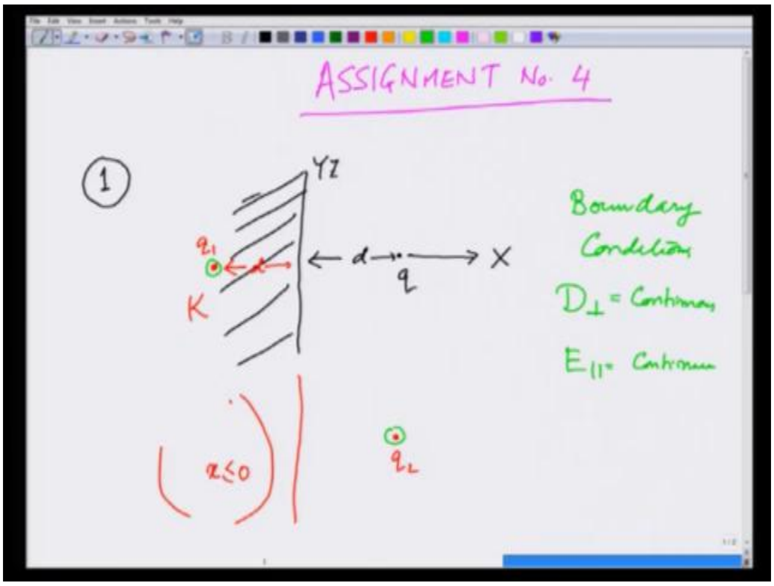

We will be solving assignment 4 today. It concerns dielectrics and some magnetostatics problem. So, the first problem in this is if a point charge this relates with image problem for dielectrics. So, question number 1 - a point charge q is in front of a dielectric semi infinite slab; semi infinite means it is all the way going up to minus infinity on this side, and there is a charge q in front of it at a distance d, the dielectric constant of this material is k, take the plane to be y z plane. So, this plane - the black plane this is going to be taken as y z planes. So, that this direction is the x direction. To calculate the potential and electric field in the region x greater than 0. So, if you want to calculate the potential and electric field in this region, we place an image charge q 1 at x equals minus d. So, we are placing an image charge q 1 at a distance d on the opposite side. So, this is charge q 1. So, that notice that the Poisson equation is satisfied in the x equal greater than 0 region, and to calculate potential and field in the other region in this region, we put a charge q 2 here, and this is to calculate potential and field in this region. So, that there is no charge in the region of interest to satisfy the required by boundary conditions what do we need?

So, let us see what we want to do now is this is a dielectric medium. So, therefore, there are 2 boundary conditions; boundary conditions are going to be that D perpendicular is continuous and E parallel is continuous, and notice to satisfy these 2 boundary conditions we have introduced 2 unknowns - q 1 and q 2.

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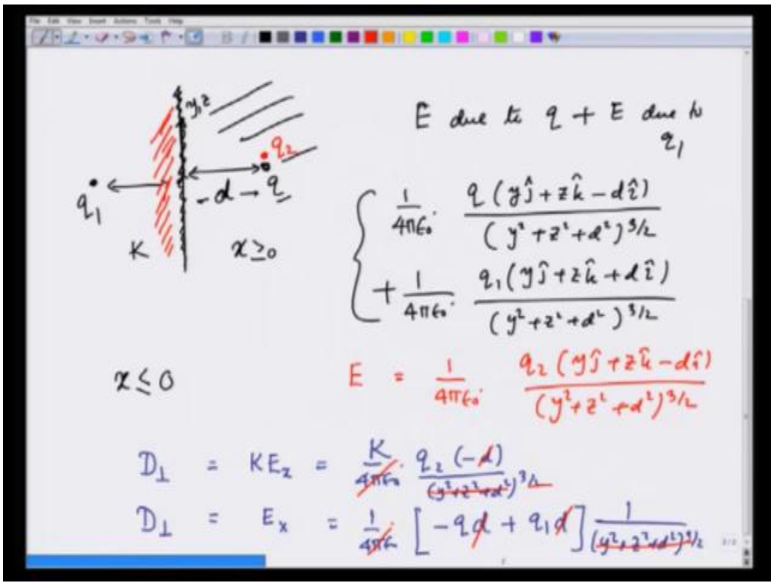

So, now if we take this dielectric, there is a charge q at a distance d from the surface and charge q 1 on the other side; this side has dielectric constant k, then the electric field due to these 2 charges, and this electric field will be only in the region x greater than or equal to 0, it will be valid only in this region is going to be field E due to q plus field due to q 1. And notice the boundary conditions we are only interested in field and D at this interface. So, we just calculate it here, we will calculated at a distance y and z, x anyway is 0 at this point. So, electric field due to q out here is going to be 1 over 4 pi epsilon 0 q, we are looking at point y j plus z k and due to q is at x at at D on the x axis. So, it is going to be D i divided by y square plus z square plus d square raise to 3 by 2; that is the field due to charge q plus field due to charge q 1 is going to be q 1 times y j plus z k plus d i, because this is at minus d divided by y square plus z square plus D square raise to 3 by 2; that is the field in the region x greater than 0.

How about for region x less than or equal to 0 in this case we calculate the charge as if there is a charge q 2 on the right hand side. So, in this case the field for regions x less than 0. So, on this side, but still on the surface is going to be 1 over 4 pi epsilon 0 q 2 y j plus z k minus d i divided by y square plus z square plus d square raise to 3 by 2, and this is field which is valid for regions for x less than or equal to 0. Now we got to apply the boundary conditions. The boundary conditions tell me that D perpendicular is the same. So, D perpendicular for this surface is going to be D in the x direction for x less than 0, D is k times E x which is equal to 1 over 4 pi epsilon 0 q 2 times minus d over that distance square y square plus z square plus d square raise to 3 by 2 times k.

And D perpendicular from x greater than or equal to 0 side is going to be only E x which is equal to 1 over 4 pi epsilon 0 in the brackets minus q d plus q 1 d 1 over that distance y square plus z square plus D square raise to 3 by 2, and these 2 should be equal. If I make them equal lets cancel certain terms 1 over 4 pi epsilon 0 cancels. So, does d…

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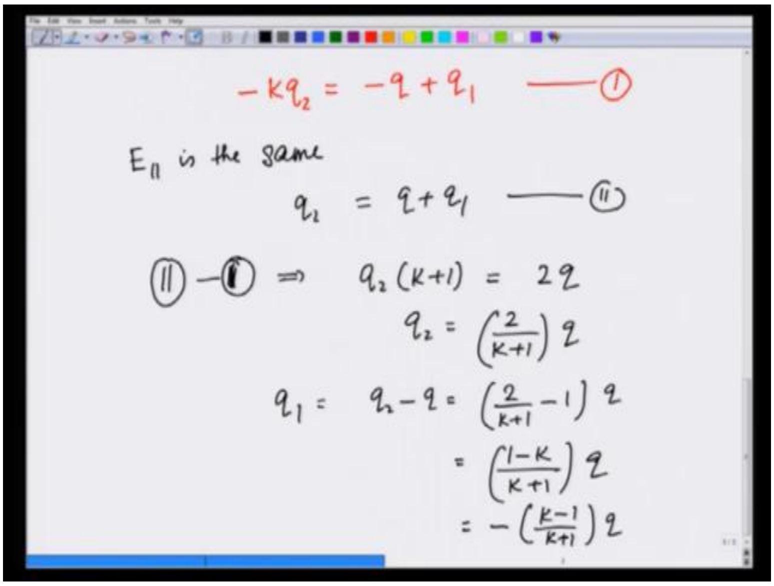

So, does the distance, and we are left with. If I make D perpendicular the same on 2 sides, I am left with minus k q 2 is equal to minus q plus q 1 that is my equation 1. And the other equation is that E parallel is the same and that immediately gives me q plus q 1 is equal to q 2, this is my equation 2. Now we can very easily solve these 2 equations, if I subtract number 1 from number 2, this gives me q 2 k plus 1 is equal to 2 q or q 2 is equal to 2 over k plus 1 q. And from this I can calculate q 1 which is going to be equal to q 2 minus q which is 2 over k plus 1 minus 1 q which is equal to 1 minus k over k plus 1 q or minus k minus 1 over k plus 1 q notice that if k equals 1; that means, the dielectric medium is not there then q 2 comes out to be q and q 1 comes out to be 0 as must be the case.

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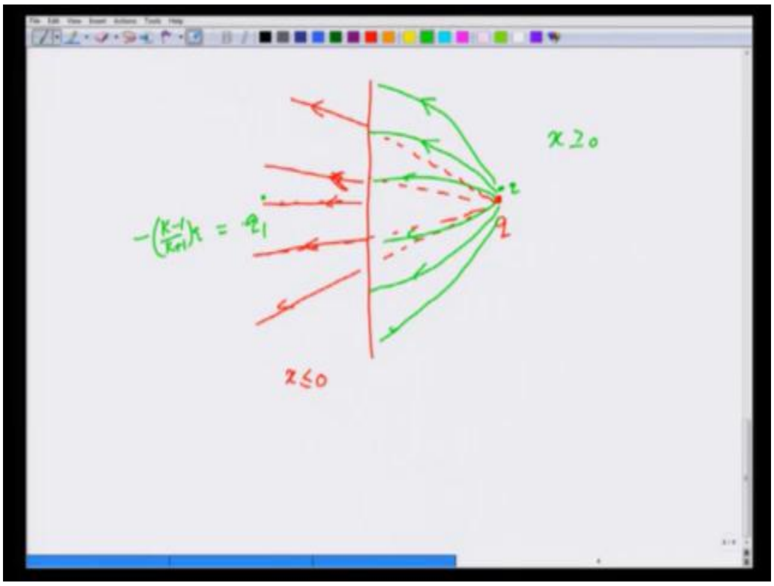

How about the electric field? Now if I want to plot the electric field here is q. So, electric field in the region for x less than or equal to 0 is going to be as if the charge is sitting at the same point where q is, but it is slightly less 2 over k plus 1. So, the field lines are going to look like straight lines emanating from the position of q. This is how the field lines are going to look in the region x less than 0 for x greater than 0, that is this region. The charge is as if there is a q sitting here, and there is a minus q sitting here minus or q 1 which is equal to minus k minus 1 over k plus 1 q. If k is close to 1 it is a very small charge, but what it is going to give is little curvature to the electric field, and therefore the electric field lines on this side are going to look like this, because they are going to turn towards charge the negative charge. So, this is how the field lines like look in problem 1.

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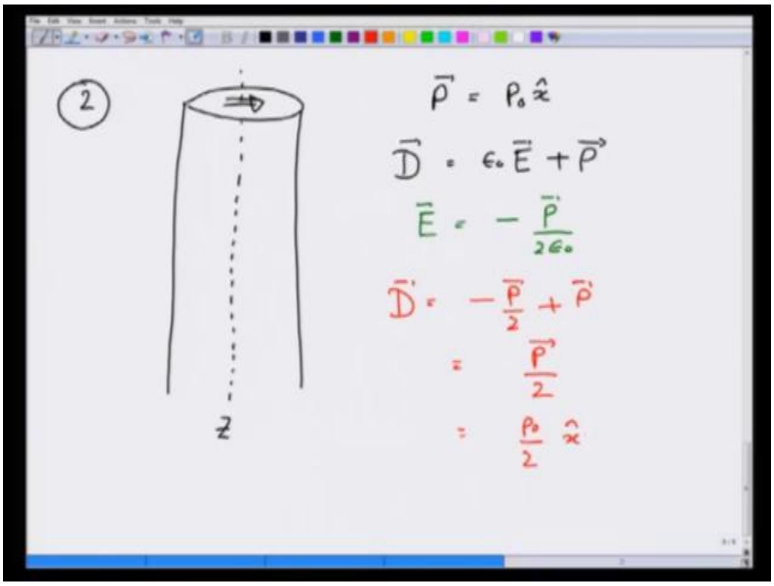

Next we solve problem 2 which says a solid infinitely long cylinder has axis along the z axis. So, here is a solid infinitely long cylinder on, if there is axis on the z axis and it carries a polarization p which is equal to p naught in the x direction, and the displacement vector inside the dielectric is by definition the displacement vector is epsilon 0 E plus p where E is the electric field. We have already calculated electric field many times an electric field due to this polarization is given as minus p over 2 epsilon 0, and therefore the answer for D is going to be minus p over 2 plus p which is equal to p over 2, that is our answer which is equal to p 0 over 2 x.

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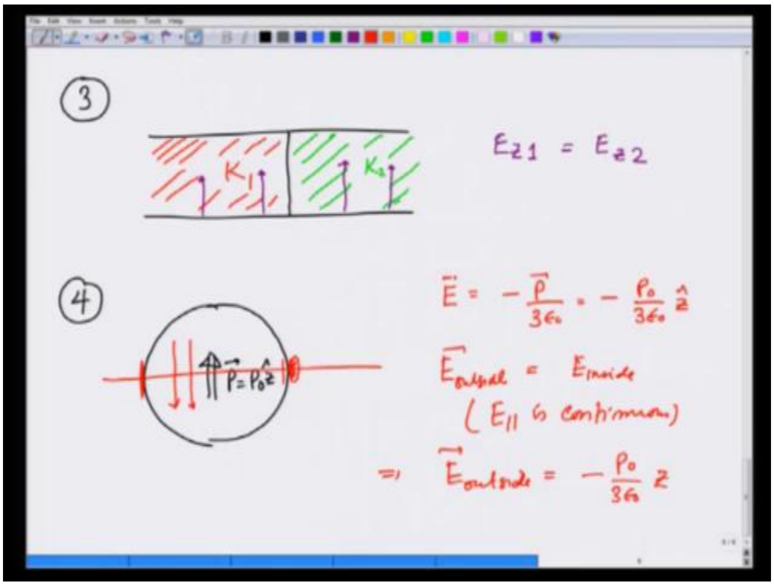

Next problem number 3 - a parallel plate capacitor has placed perpendicular to the z axis. So, these are huge plates. So, that we can ignore the fringe effects and it carries 2 dielectric slabs with dielectric constant k 1 on one side, and dielectric constant k 2 on the other. Ignoring the fringe effects what are the boundary conditions at the dielectric interface. If we ignore the fringe effects, the electric field when the capacitor is charge is going to be perpendicular to the plates. And therefore, the boundary condition is going to be that E z on side 1 is going to be E z on side 2, that is the boundary condition.

Let us solve question number 4 now, a sphere carries a polarization p equals p naught z. So, here is the sphere, it carries a polarization p equals p naught z, electric field just outside this equator in the plane perpendicular to z is… So, we want to know the electric field just outside its equator. So, that means, we want to know the electric field at this point shown by red. I can either calculate it explicitly or use the boundary conditions, I know for this sphere the electric field inside is in the opposite direction, and electric field inside is given as E equals minus p over 3 epsilon 0, which is in this case minus p 0 over 3 epsilon 0 z. And this electric field is going to be continuous across the boundary here, because at this point equator this is the tangential electric field. So, E outside is going to be same as E inside and this comes from E parallel is continuous, and this immediately gives you that E outside is also going to be equal to minus p 0 over 3 epsilon 0 z. Let us also obtain this answer by explicit calculation.

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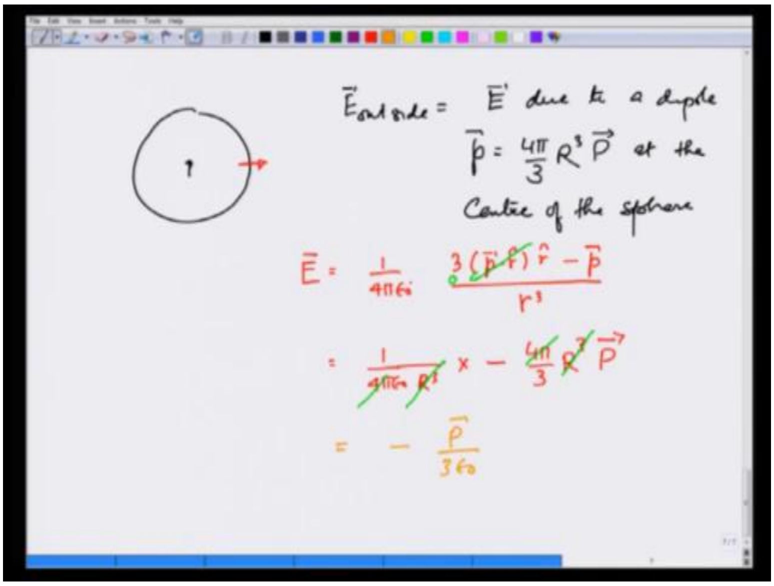

In the explicit calculation, we know that field outside is E due to a dipole p equals 4 pi by 3 r cubed p at the centre of this sphere. So, outside the field is going to be as if there is the point dipole p sitting at the centre, and electric field due to this I know is 1 over 4 pi epsilon 0 3 p dot r r unit vector minus p divided by r cubed. In this case r unit vector is in the direction perpendicular to z, therefore p dot r term is 0, the electric field therefore is going to be 1 over 4 pi epsilon 0 r value is r cube times minus 4 by pi by 3 r cubed p; that is a polarization. Now let us cancel terms r cube cancels 4 pi cancels, and I am left with minus p over 3 epsilon 0 indeed the answer we had obtained using the boundary condition.

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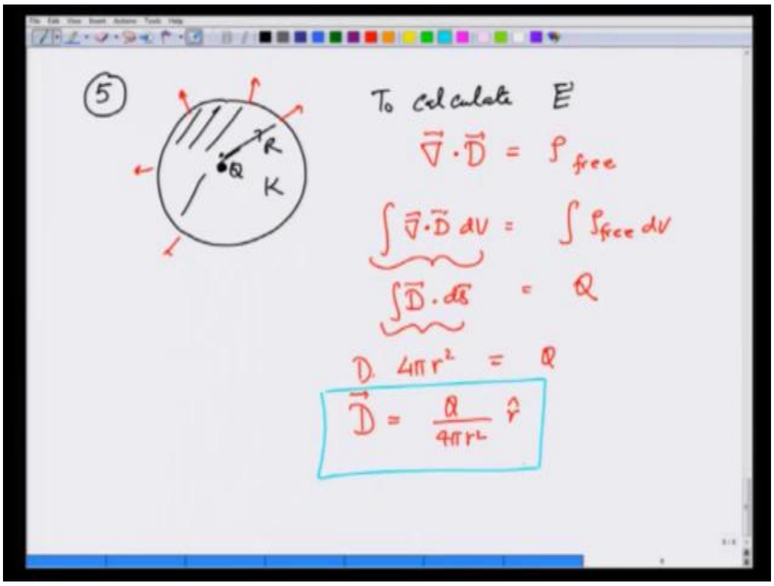

In the next problem we are asked if a point charge q is kept at the centre of a dielectric sphere of radius R. So, this is a dielectric sphere of dielectric constant k, and we have a charge q at the centre of it. What is the potential? To calculate the potential, I will first calculate the electric field now to calculate E, we are going to go by the Gauss law in presence of dielectrics, which is del dot D is equal to rho free. In this case rho free arises only from this point charge sitting at the centre.

And therefore, if I do the volume integral of divergence of D d v which will be integral rho free v v, this by divergence theorem is integral D dot d s which is equal to rho free d v which is the total charge Q and this gives me, since by symmetry there’s a spherical symmetry D is going to be only radial. So, D is all radial. So, this D dot d s gives me D times 4 pi r square is equal to Q or D magnitude is q over 4 pi r square, if you take the vector it is going to be in the radial direction, that is my D.

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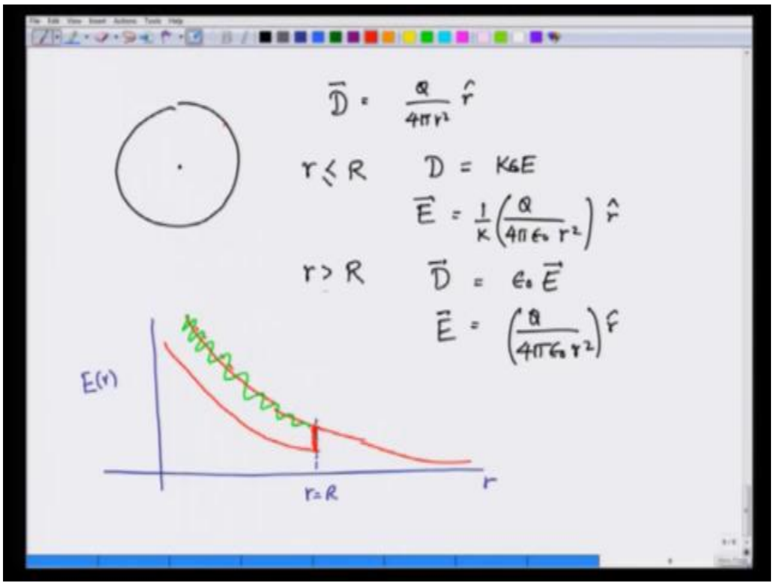

And from this I can calculate the electric field here is my sphere, and we have already calculated D is q over 4 pi r square r for r less than equal to R, D is equal to k times E. And therefore, electric field E is going to be Q, there is an epsilon 0 over 4 pi epsilon 0 r square 1 over k in the radial direction. And for r greater than R, D is nothing but epsilon 0 E, even the vector sign is phi. So, E is going to q over 4 pi epsilon 0 r square in the radial direction, notice that E is discontinuous across the boundary. So, let us see that if I have to plot E r with r, here is r equals r, if there was no dielectric the field would have gone like this. However, because of the dielectric the inner part is slightly less it gets reduced by a factor of k, and therefore it is less by a factor of k, this is how the electric field goes the discontinuity comes, because of the surface charge on the surface.

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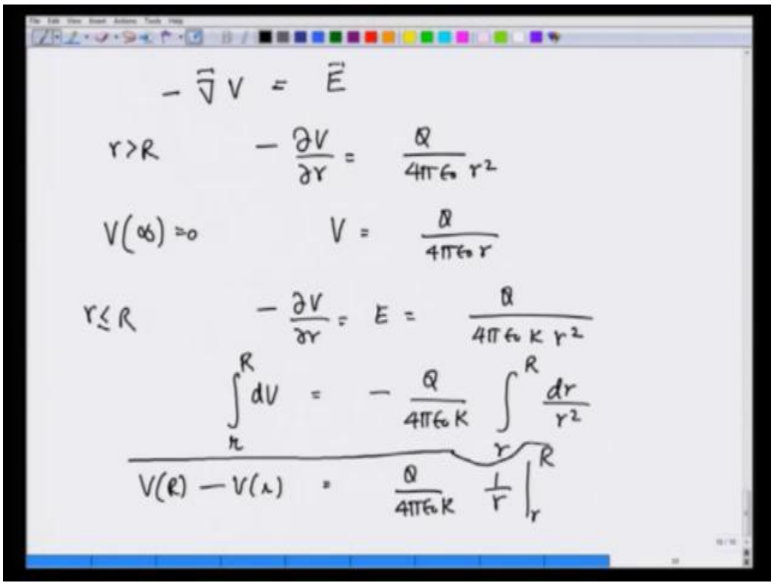

Now, to calculate the potential, I know grad of v with the minus sign is equal to E. So for r greater than R, I have minus d v by d r is equal to q over 4 pi epsilon 0 r square, and taking v at infinity to be 0, this is easy to integrate you have done it many, many times for point charges v comes out to be q over 4 pi epsilon 0 r for r less than equal to R, we have again minus d v over d r equals E which now is q over 4 pi epsilon 0 k r square. So, if I integrate d v from to capital R, its going to be minus q over 4 pi epsilon 0 k integral D r over r square from r to capital R, and this gives me q over 4 pi epsilon 0 k 1 over r r capital R is equal to v at r minus v at r.

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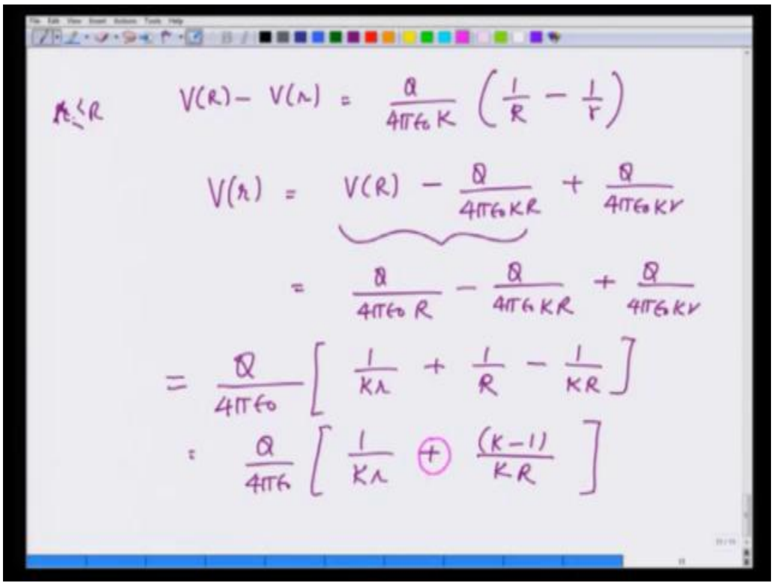

So, I get v at R minus v at r for r less than equal to r is equal to q over 4 pi epsilon 0 k 1 over r minus 1 over r. And therefore, v of r is equal to v of capital R minus q over 4 pi epsilon 0 k r plus q over 4 pi epsilon 0 k r. This can be easily calculated, because I know v of r with respect to infinity is equal to Q over 4 pi epsilon 0 r minus Q over 4 pi epsilon 0 k r plus q over 4 pi epsilon 0 k r which can be then written as q over 4 pi epsilon 0 can be taken out, I have 1 over k r plus 1 over r minus 1 over k R. And that is your answer. Let us simplify it further q over 4 pi epsilon 0 1 over k r plus k minus 1 over k R. notice that if r small r equals capital R, this term cancels and you get 1 over r which should be right. I think in the problem that I gave you, I had in the assignment this sign is given incorrectly. So, correct that now.