Introduction to Electromagnetism
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Lecture – 71
Assignment - 3
Problem 6 – 10

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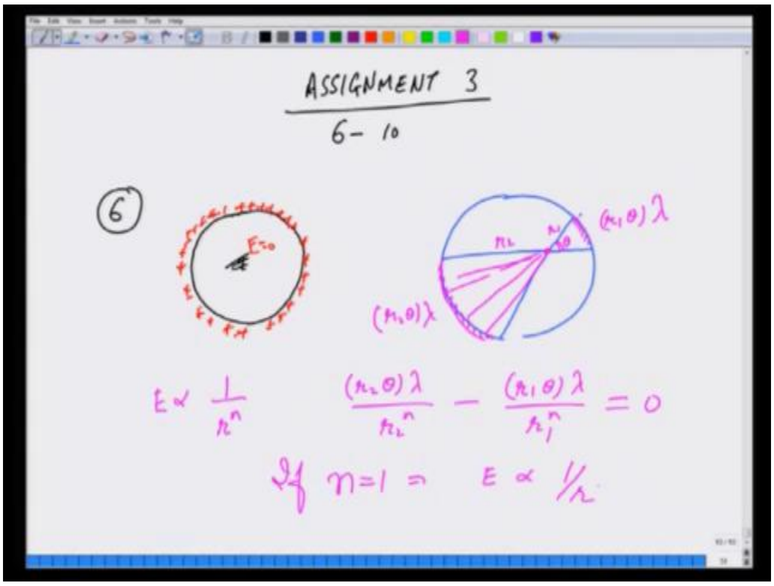

We have been solving assignment number 3 and we have solved problems up to number 5, so here we will be solving problems from 6 up to 10. The 6 problem says, suppose there is a two dimensional world, where it is seen that all the charge put on the conductor comes to it is surface. So, in two dimensional world, it will be like a disk and if you give a charge here, it is all coming to the surface, it get distributed on the surface.

Let me show that by red, let say this is positive charge, it all comes on the surface and since, there is a symmetry, a circular symmetry or cylindrical symmetry, all the charge will be uniformly distributed. Following the argument given in lecture 26 at 5 11 minutes find the r dependence of the electric field produced by the point charge in that world. Now, you see, if this is a conductor, field inside has to be 0.

So, argument we are going to say uses, suppose I look at the field at the point shown by this dot and make a cross section here shown by pink on the two sides. Now, if the field is 0 and the charge is uniformly distributed; the dependence of the electric field should be such that, the fields anywhere inside is 0. Let us say this point, which are at a distance r 2 form the bigger section and r 1 from the lower section, the field is 0, field is going to be 0, now all the distances are the same. So, field is going to be 0, if they cancel from the each other side.

If this angle is theta, then the charge on the smaller side is r 1 theta times, whatever, let us call it lambda is the charge density charge. Charge on the other side is going to be r 2 theta times lambda. Now, if the r dependence of electric field, let us say 1 over on r raise to n, then the field at this point in the middle is going to be r 2 theta times lambda over r 2 raise to n minus r 1 theta lambda r over r raise to n.

The only way this can become 0 is, if n equals to 1 and this implies E should go as 1 over r. Keep in mind that in the three dimensional world E goes as 1 over r square. Now, how do I realize this two dimensional world? It is very simple, suppose I take long cylinder, infinitely long cylinder in which the third dimension any way gets integrated out. Essentially, for electric field becomes a two dimensional world and I know indeed there are the field goes as 1 over r.

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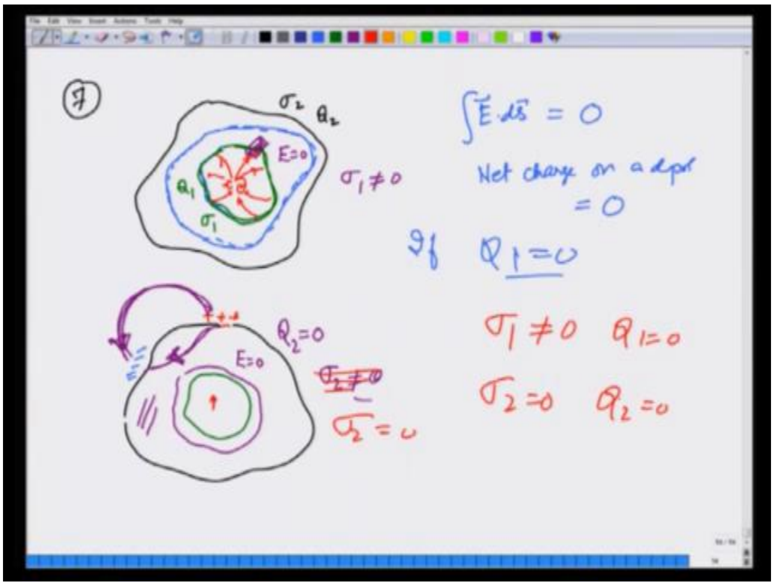

Next, I am going to solve problem number 7, which says there is a point dipole, which is put inside a cavity in a conductor. So, I have a conductor of arbitrary shape and make a cavity inside and put a dipole here, a point dipole. The surface charge density on the surface of the cavity, which I show by sigma 1 and outside it is sigma 2 and the net charge inside is Q 1, net charge outside is Q 2.

What is true about sigma 1, sigma 2, Q 1 and Q 2? So, let us look at point the field produced by this point dipole and I am going to remove this dipole, put a point here, where it is and the field lines are going to be something like this. Here, they will close on themselves, here they will come in and here they want to close on themselves. So, if I look at the surface out here and make a very small box, keep in mind that field inside the metal is 0.

The net flux through this small box is going to be non-zero and therefore, sigma 1 is non-zero. Certainly, sigma 1 is going to be on this side is going to be negative and on this side is going to be positive on the lower side, but it is non-zero. However, if I look at the Gaussian surface inside the metallic flux of this Gaussian surface is 0, because there is no field number 1.

Number 2, the net charge on a dipole is also 0, since the charge enclosed by this Gaussian surface is 0, dipole already has a net charge 0. So, that can only happen, if Q 1 is also 0. So, what is happening is that, inside this cavity green surface, surface of the cavity inside has a distribution of charge positive and negative charges; that means sigma 1 is non-zero. However, sigma 1 is such that the net charge integrates to 0.

What about outside? Now, you see if I look at it, so here is the inner cavity, here is a point dipole, field out here is 0, net charge enclosed here is 0. So, that outer side total charge Q 2 will also be 0, because the cavity is neutral. So, there is no charge taken on the inner surface, outer surface also has 0 charge. They cannot be any charge distribution inside the metal.

So, anyway in the bulk metal, there is no charge, charge comes only on the surface, so outside charge is 0. Can this 0 charge be distributed? That means, can I have sigma 2 not equal to 0. Suppose I did, so there was some positive charge somewhere and consequently this is net charge is 0, there has to be some negative charge somewhere else. And therefore, what will happen in this case is there will be a field line going from positive negative charge and that means, work done in taking a charge from positive negative charge will be non-zero.

However, a metallic surface is always a constant potential surface. Therefore, if we go from inside the metal or conductor work done is 0. So, it cannot happen that if I come from the outer side, there will be some work done and from the inner side work done will be 0, because it is the conservative field. The only way it will not happen is, if there is no separation of positive and negative charges and therefore, sigma 2 outside is also 0.

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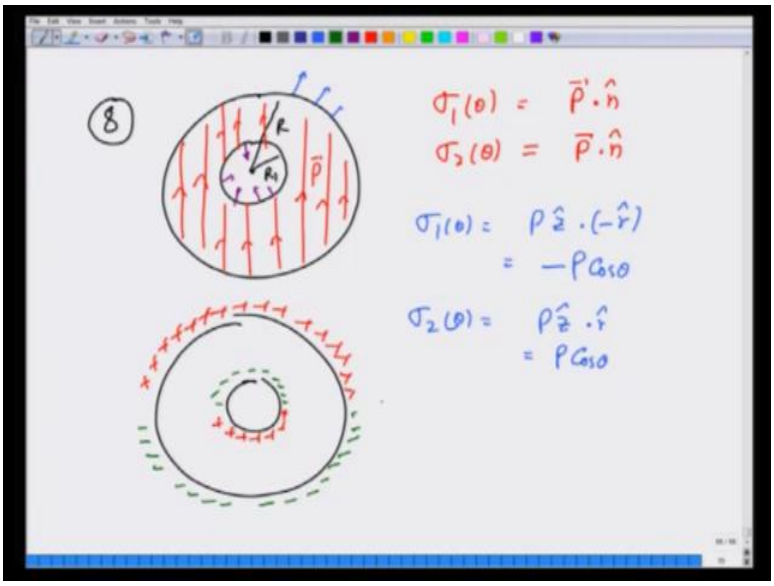

So, the answer in this case would be sigma 1 is non-zero; however, Q 1 is 0, sigma 2 is 0 and Q 2 is also 0. Next problem number 8, an insulating spherical shell of inner radius R 1 and outer radius R 2 carries a uniform polarization. So, we have a shell of inner radius R 1 and outer radius R 2 and it carries a polarization P. The bound surface charge sigma 1 theta on the inner surface and sigma 2 theta on outer surface I have to be found.

We know bound surface charges are nothing but, P dot the normal, P dot the normal. The normal on the surface is pointing inside, because normal is away from the volume and on the outer surface normal is pointing out. So, sigma 1 theta is going to be P, which is P z times normal which is pointing in the minus r direction. So, this is going to be minus P cosine of theta, sigma 2 theta on the other hand is going to P z dot r, because it is radically out which is going to be P cosine of theta.

So, the bound charge on this cavity is going to look like, what I shown in the figure. On the inner surface is negative on the theta 0 onwards and positive on the lower side, on the outer surface is the other way. P cosine theta is positive on the upper hemisphere and negative on the lower hemisphere with the dependence being given as P cosine theta.

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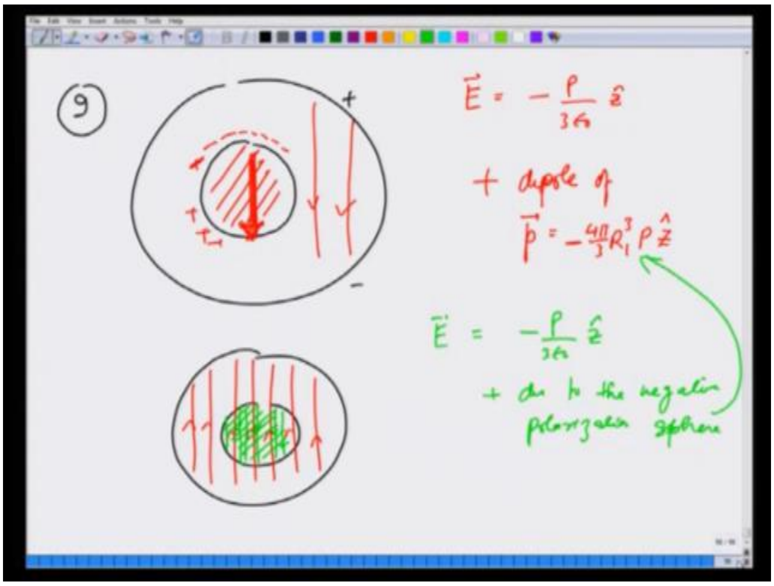

Next, we solve problem 9, which is asking for the electric field in the dielectric shell of problem 8. So, we all ready solve problem 8, where we found that the charges plus on this side and minus on this side, which cosine theta dependences. This charges I know gives me an electric field in the opposite direction E, which is going to be minus P over 3, Epsilon 0 z.

What about field due to the energy cavity, the inner cavity has a charged, which is negative on the upper side and positive on the lower. So, it has a dipole moment going down and this dipole moment will give a dipole field. So, additional field is going to be due to dipole of dipole moment P, which is 4 pi by 3 R 1 cubed P z in the opposite direction of z.

Another way of looking at it is, if I have this cavity which has this whole here, I can think of this as if it is a cavity with uniform polarization, which I will show by red and added to it is an additional polarization in the negative direction inside this cavity. So, that the two polarization is inside this spheres of radius R 1 cancel and there is nothing. So, electrical I can think of it like this and this, I know give me a field E uniform polarization gives ne a field minus P over 3 Epsilon 0 z plus of field due to this the negative polarization is sphere, which is preciously this filed, that is the answer.

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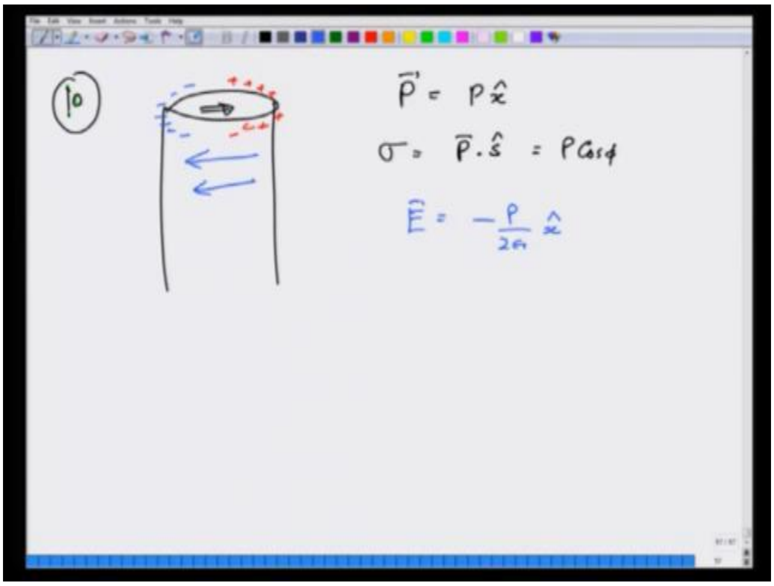

Finally, we solved question number 10, which is electric field inside in insulating cylinder, which are the polarization the x direction P is equal to P x will be. Now, this is a very simple, because P x means, there is a surface charge, there is no bulk charged, but the surface charge sigma is P dot S, because S is n, which is P cosine of phi. So, what we have in this is positive charge on the positive x axis and negative charge on the other side.

And therefore, I can see that field is going to be in the negative x direction and since, this is the two dimensional thing. So, E is going to be minus P over 2 Epsilon 0 x, there is the end of assignment 3.