Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 07
Divergence of a Field

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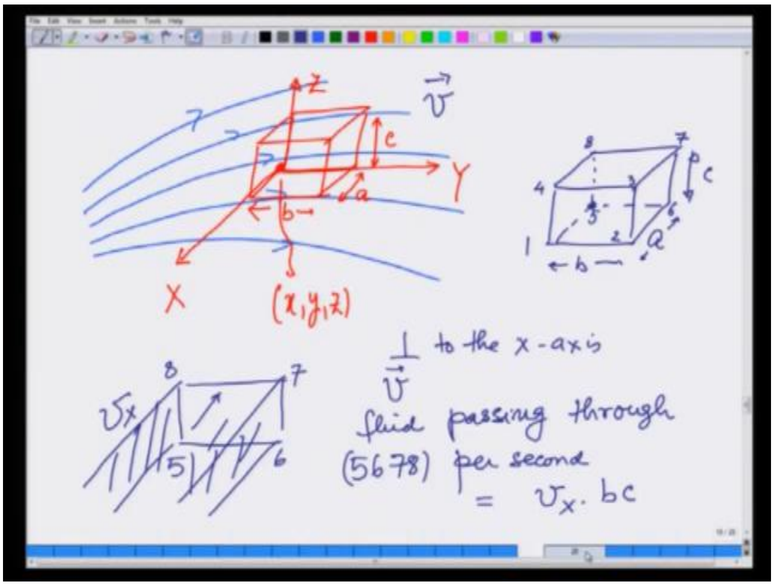

Let us now make it mathematically, how do we do that. Let us again go back to fluid flow, and I will make in this the rectangular small box. At some point x, y, z with x axis this way, y axis this way, z axis this way, and this is point x, y, z. Let the size of the box be a in the x direction, b in the y direction and c in the z direction. Let me make this box separately once more, this is the box for this side being a, this side being b and this side being c.

Let us calculate, because now water coming in and water going out it looks like, you know this is more coming in and going out. Let us really see what happens, let us look at the surface let me name it 1, 2, 3, 4, 5 in the backside 6, 7 and 8. Let us look at the point 5 is my x, y, z, let us look at surface 5, 6, 7, 8. 5, 6, 7, 8 is perpendicular to the x axis and if there is a velocity of fluid v we talking about velocity field.

If there is a velocity v, then fluid passing through 5, 6, 7, 8 per second will be really, if you make a big rectangle of length v x whatever the fluid is in this v x is going to pass through this surface, because in one second it will cover the length fields. So, fluid passing through this is going to be v x times the area, area is going to be b times c, this is a fluid passing through it.

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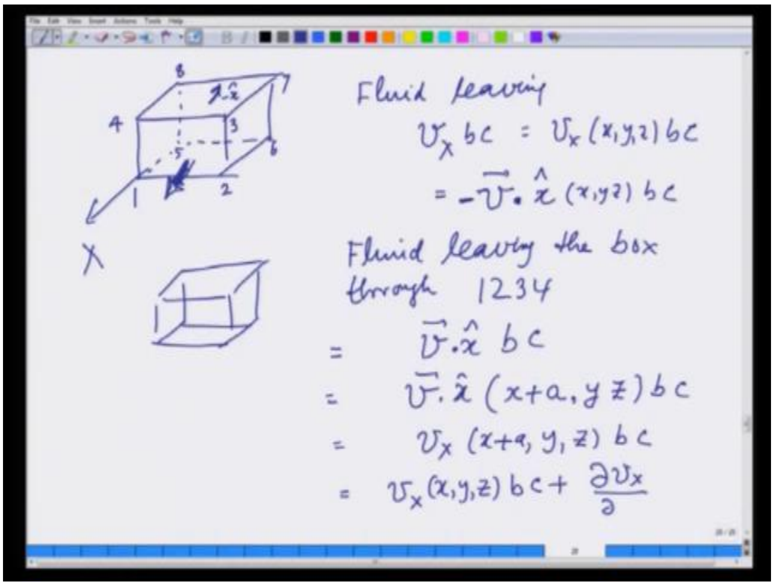

Let us make it more precise. This is the box and I am interested in what fluid is going out. So, for that I need the component which indicates a flow out and that is for 5, 6, 7, 8 the component in minus this is my x axis, in minus x direction is going to tell me whether fluid is leaving or coming in. So, fluid leaving, magnitude be already said is v x b c which is v x at x and y and z b c and which component the component the minus x direction.

So, I am going to write this as v dot x in the minus direction at x, y, z b c this is fluid leaving from 5, 6, 7, 8. Let us look at surface 1, 2, 3, 4. At 1, 2, 3, 4 fluid leaving would have that component in the x direction and therefore, fluid leaving the volume the box through 1, 2, 3, 4 is going to be equal to the component of v in the x direction coming out of the volume times b c again.

And this v though is going to be act x is now x plus a, remember the surface 5, 6, 7, 8 is at x this is going to be x plus a. But, all other the points they have the same, all the points have same y and z b c. So, which I can write as v x at x plus a y and z b c, which I can further write as v x x y z b c plus partial derivative of v x with respect to x b c.

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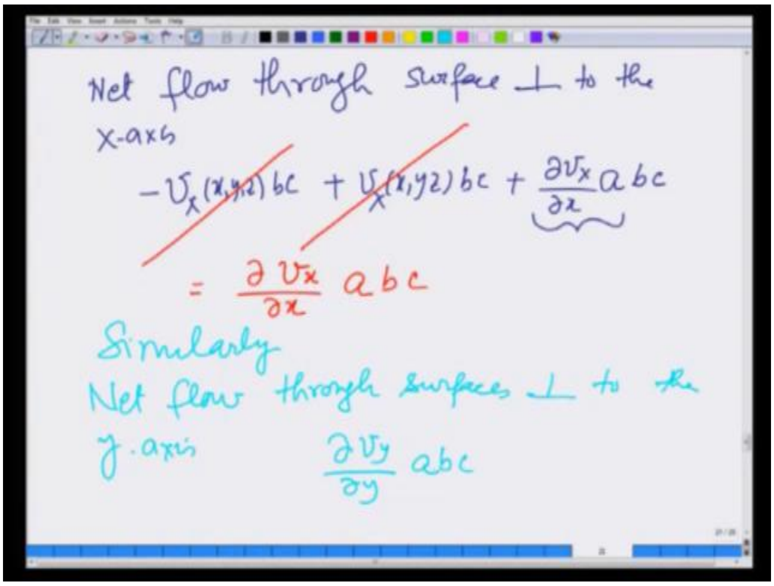

With the result that net flow through surfaces perpendicular to the x axis is going to be minus v x at x, y, z b c plus v x at x, y, z b c plus partial of v x y partial x a b and c, this is the change in v x through a, let me see if I do not that is early no I did not. So, there should be in a here, this cancels and we are left with partial v x by partial x a b c. In a similar manner, similarly you can show that net flow through surfaces perpendicular to the y axis is going to be partial v y over partial y a b c.

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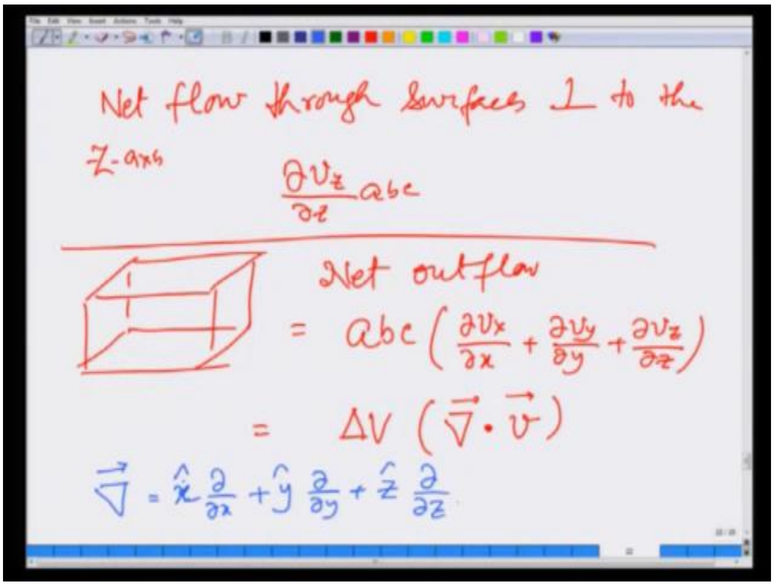

And net flow through surfaces perpendicular to the z axis is going to be partial v z over partially z a b c. So, over all through this box if I count all the surfaces in net flow is a b c is common partial v x by partial x plus partial v y by partial y plus partial v z the partial z we of course, assume that a b c is small. So, I kept only the first order and which is nothing but small volume of this box times what I said earlier, I am going to write it as a diversions of v where, symbolically this symbol is going to be x unit vector partial x plus y unit vector partial y plus z unit vector partial z.

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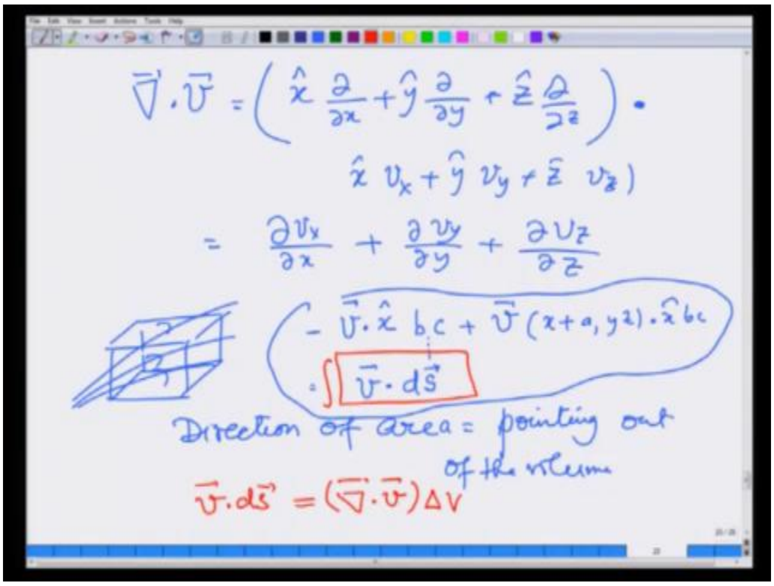

So, that when I write divergence of v it is partial x plus y partial y plus z partial z dotted with and I am going to take dot product first x v x plus y v y plus z v z, if I take dot product x dot x gives me 1. So, I left with this plus y dot y gives me again one d v by d y plus partial v z by partial z all other dot products gives me 0. So, what we found is that if I take a small box a very small volume and talk about this fluid flow through this, then the net flow with now I am going to write as v dot x and we had written remember b c plus v at x plus a y z dot x d c.

And other components is actually nothing but v dot d small area where the direction of area is equal to pointing out of the volume. So, what the quantity we have calculated is this, it actually calculated this quantity which is nothing but v dot d s integrated over the entire area, and what we have found is that v on this is small area d s is nothing but equal to divergence of v times small volume delta v.

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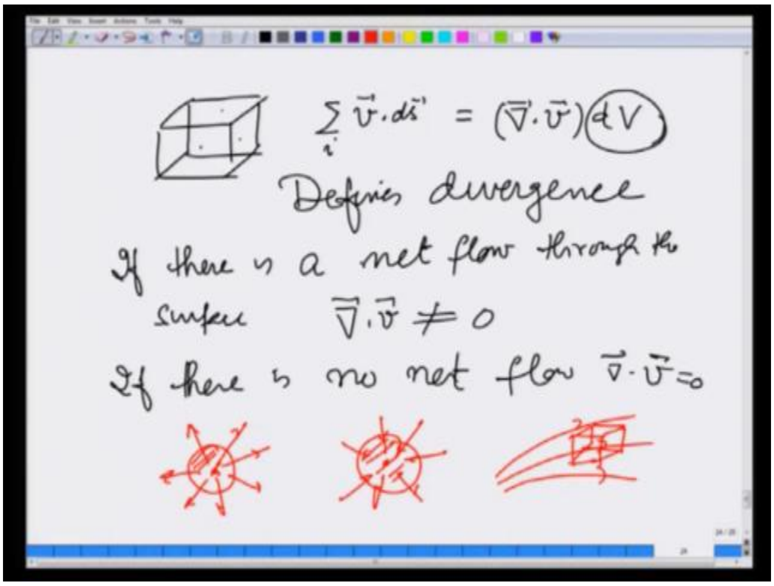

Let us rewrite it, if I rewrite it I have this box very small box and if I calculate v dot d s. Let me write it even better some over all the surfaces, surface 1, surface 2, surface 3, surface 4 this is nothing but equal to divergence of v dot d volume and this defines divergence consistent with the ideas I was talking about earlier. If there is a net flow outside, if there is a net flow through the surfaces, notice that this is positive quantity.

So, divergence of v not zero, if there is no net flow divergence of v is 0, as I said earlier if I take a point source of light, all the rates are diverging from it. Because, this is a net flow light from here, similarly if I source of light in which everything is coming in there is a net flow of light in this is also divergent or negative of divergent convergent. On the other hand, if I take volume here nothing accumulates then divergent is 0.

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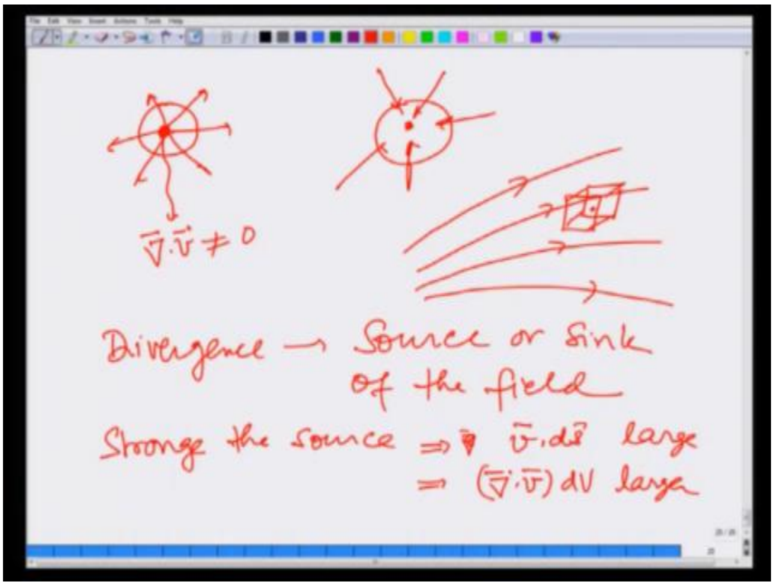

If better way of looking at it is would be a again in a fluid flow, supposed there is a point from which or a fountain water is going all around. If I make a spherical surface around it you see there is an net flow or water outside at this point no matter how small the surface, this point there will be some water going out. So, this point has divergence of v not zero, if I take a sink in which water is going into it form all around, then again divergence is not zero, but highly negative.

On the other hand, if you see a pipe although it is area may change, the net flow through any small volume is 0. So, divergent of any point is 0, so I hope this gives you a picture of what divergence is and divergence immediately you can see is related to source or sink of the field and stronger the source which also includes a sink, which is negative source, stronger to the source more the flow, stronger the source del v dot d s will be larger, stronger the source implies v dot d s larger and this implies divergence of v d v is larger.

So, now we have given a mathematical definition to any divergence of any vector field, it is a related to the strength of the source and larger the source more powerful it is more the divergence and it is going to be non zero only at points, where there is a source or a sink, because then there is a net out flow or net inflow. If net flow in flow in a through a small volume is 0 and at that point divergence is going to be 0.

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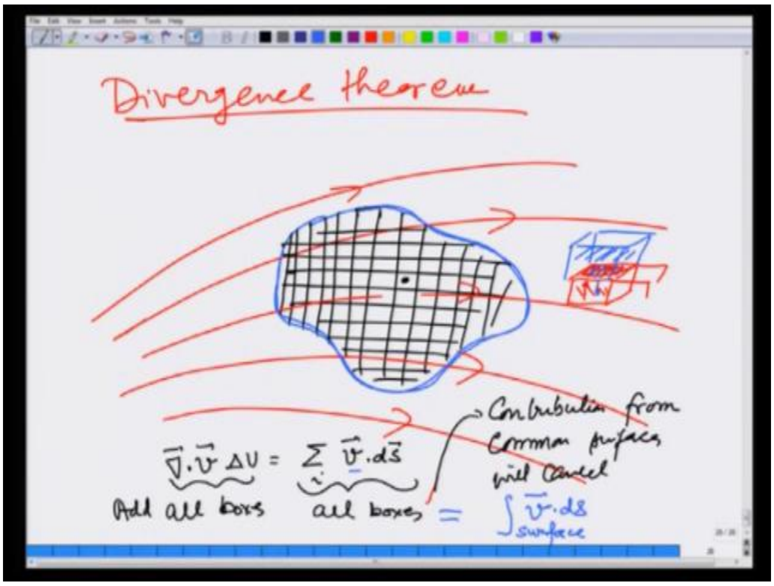

So, that is the definition of divergence with this definition of divergence there is related theorem and that is called divergence theorem. Take any vector field and now take a surface which is of arbitrary shape and encloses the volumes, this is the volume. In this volume I divide this volume into very small rectangular boxes very, very small I am making into two dimensions. But, assume these are boxes and at each point I apply to calculate the divergence.

So, that at each point let us say this point at this given point I have divergence of v times delta volume is equal to summation i v dot d s through each. So, now, if I add this over all boxes I have to add this or over all boxes, look at two particular boxes, if I take one box like this I live look at an adjacent box on this common surface v dot d s for the lower box would have d s going this way and for the upper box d s is going this way, there are opposite in science.

Therefore, v dot d s will give positive when I do the volume integration, here is supposed gives to positive in give negative for the volume integration here. So, the contribution on the surface from this common surface will cancels, not only this two boxes any two adjacent box will happen. So, that the right hand part when I add over all boxes contribution from common surfaces will cancels with the result that the only contribute remain will be only from outside surfaces.

Why because there is no nothing to cancel on the outside surface, so when I add over all the boxes I am going to left with integration v dot d s over the outside surface of the close volume and that is what the divergent theorem is.

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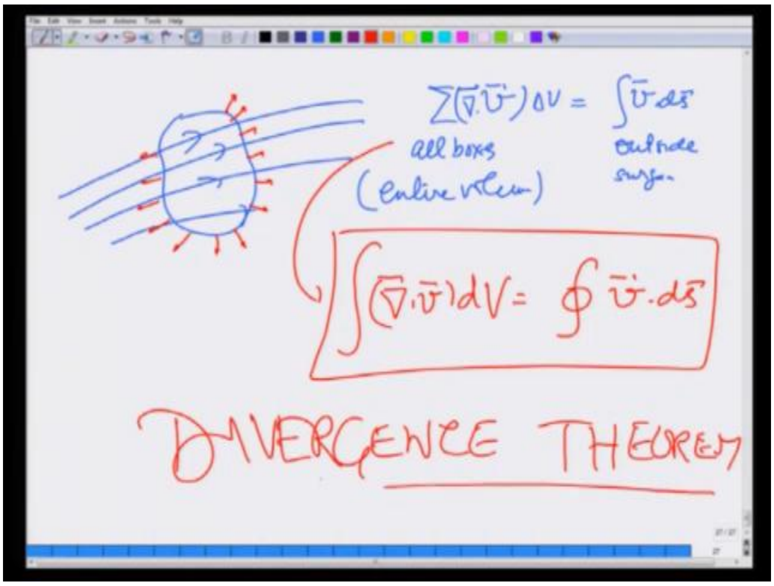

What it says, is that given a volume large volume and vector field through this I did this summation divergence of v delta v over all boxes. And that means, entire volume is going to be equal to v dot d s over the outside surfaces, because d s is all pointing a perpendicular pointing away from this surface. So, this I can right now as integration of divergence over the volume is going to be equal to integration over v dot d s, this is known as divergence theorem. In the next lecture I will talk about divergence of electric field due to electrostatic charges and then go on to describe the curl of the electric field.

Thank you.