Introduction to Electromagnetism
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Lecture – 69
Assignment – 2
Problem 5 – 11

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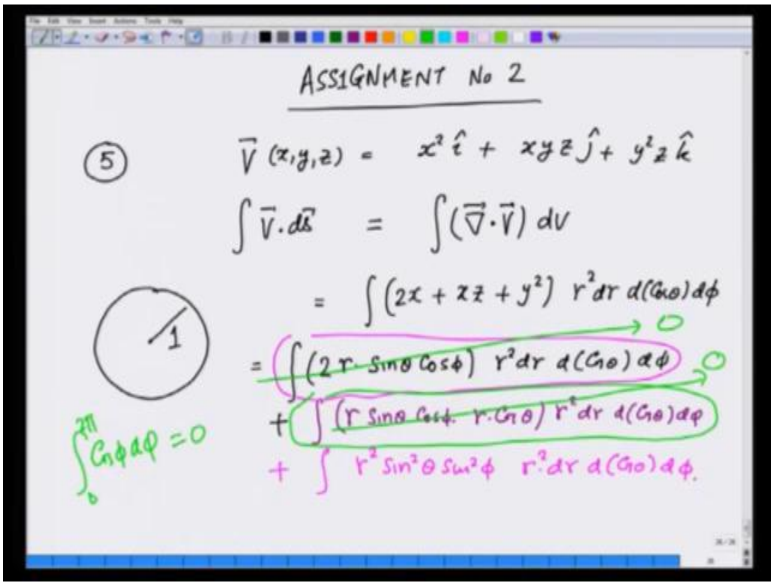

We solving assignment number 2, and this part I am going to solve the problems related to vector calculus and calculation of electrostatic potential. Problem number 5 of this assignments says that you want to calculate surface integral of the vector field V x, y and z which is equal to x square i plus x y z j plus y square z k. And you want to calculate it is surface integral v dot d s over a unit sphere; that means, this sphere has a radius of 1 centered at the origin and we want to use Divergence theorem and spherical polar co ordinates.

So, by Divergence thereon this v dot d s is going to be equal to divergence of v integrated over the volume of the sphere. Divergence of v now is easy to calculate, look at this, I will take the partial derivative of x component which gives me 2 x plus partial derivative of y component with respect to y which gives me x z plus partial derivative of z component with respect to z which gives me y square times d v.

D v in spherical co ordinate is nothing but r square d r d cosine of theta d phi, integral limits d phi are 0 to 2 pi d cosine theta minus 1 to 1 and r is from 0 to1, because we doing the volume integral over unit sphere. Now, x y and z are going to be express in terms of this spherical polar coordinates and therefore, this becomes 2 r sin theta cosine of phi which is integrated over r square d r d cosine theta d phi plus the second term, which is going to be x which is r sin theta cosine of phi z which is r cosine of theta.

And this is going to be integrated over r square d r d cos theta d phi plus the final term, which is y square which is r square sin squared theta sin square phi r square d r d cosine theta d phi. Now, notice that the first in the second term that is the term number, the first one which is I am circling now and the second term which I am circling in green. Both have cosine phi and if I integrate cosine phi over d phi over phi from 0 to 2 pi, so cosine phi d phi 0 to 2 pi, I am writing on the left gives me 0, and therefore both these terms contribute 0.

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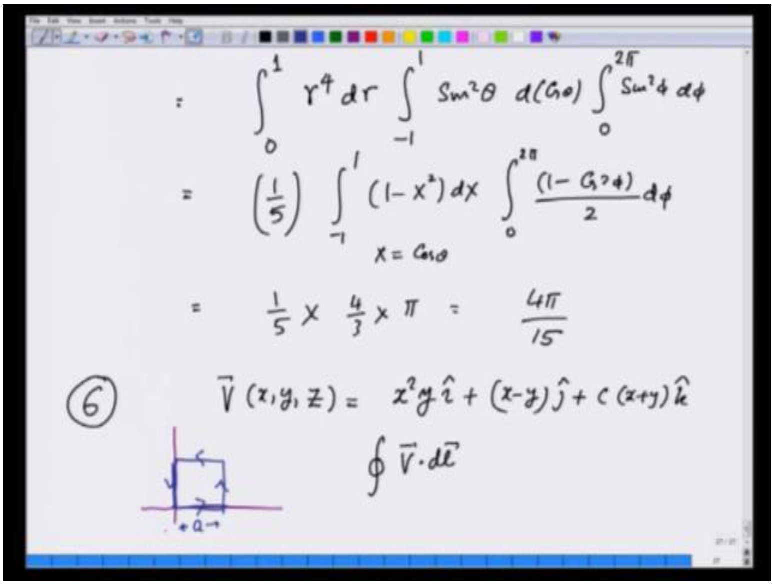

I am left with only the third term which is then equal to integration r raise to 4 d r 0 to 1 integration of sin squared theta d cosine theta minus 1 to 1 integration sin squared phi d phi 0 to 2 pi, which gives me 1 fifth. The first term gives me 1 fifth times sin squared theta minus 1 to 1 is nothing but 1 minus cosine squared theta which are like as x square d x, where x is cosine of theta times 0 to 2 pi 1 minus cosine 2 pi over 2 d phi.

This gives me 1 fifth times 2 minus 2 thirds which is 4 thirds times pi, so the answer become 4 pi over 15 and that is the answer for problem number 5. Let us see what problem number 6 says. Problem number 6 says, for a vector field V x, y and z is equal to x square y in the x direction plus x minus y in the j direction plus c x plus y in the k direction, where c is a constant. We want to calculate the line integral, integral v dot d l, where the curve that we are specifying is a square, let me show it with a slightly different color. So, this is a square with sides on the x and y axis and we have travelling counterclockwise on this and this square of side a, and we want to use Stokes's theorem.

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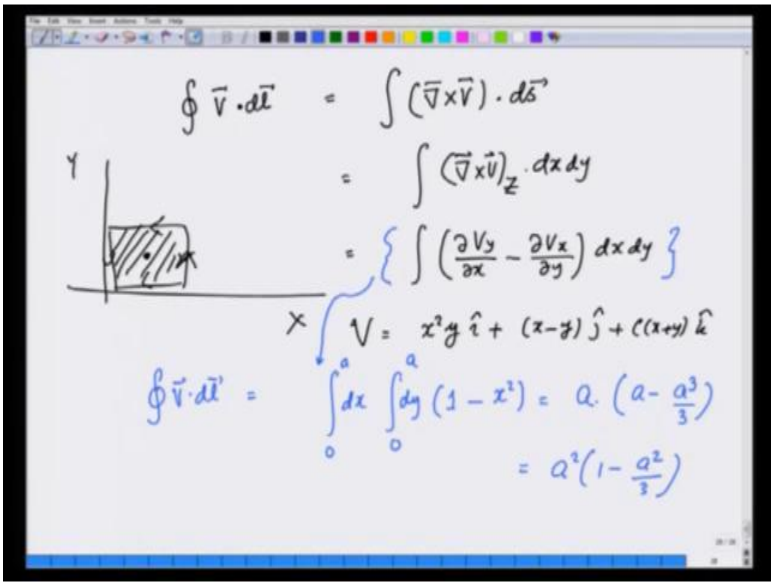

So, what is the Stokes's theorem says is that integral v dot d l is nothing but the surface over the same surface curl of v dot d s. Now, d s for this x and y surface, since I am traveling counterclockwise by right hand convention is coming out in direction k. Therefore, in this curl line I need to take only the z component, I can write this therefore, as curl of v, only z component times d x d y, that is the integral, notice that this is a scalar.

Z component of this can be written as d v y over d x minus d v x over d y d x d y where d x d y integrate the very over this square. Now, given the vector v which is equal to x square y i plus x minus y j plus c x plus y k. We have the integral v dot d l is equal to… I will now evaluate this term, the last term that we put in is equal to integral d x varies from 0 to a, y varies from 0 a, d v y over d x is going to be 1 minus d v x over d y is going to be x square d y.

The integrand depends only on x, and therefore this is going to be equal to a times 1 minus x squared d x is going to give me a minus a cubed over 3. So, the answer comes out to be a square 1 minus a square over 3.

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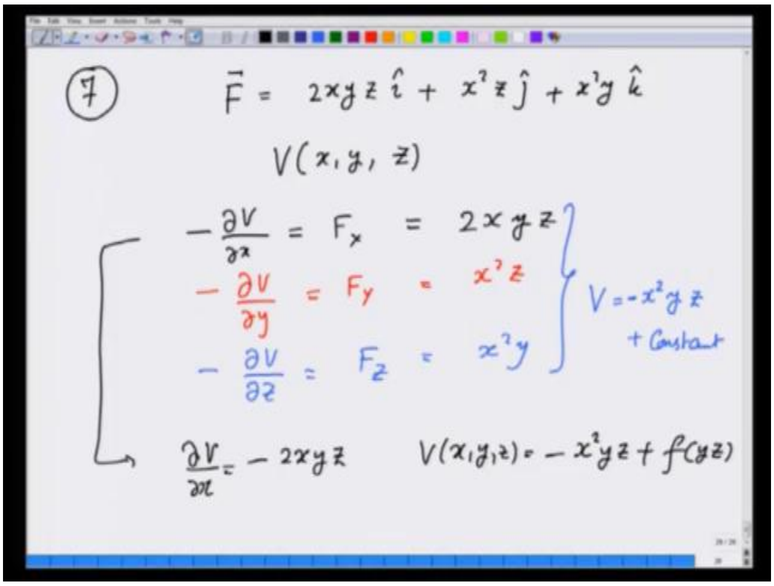

Next question number 7, question number 7 says a vector field is given as F is equal to 2 x y z in the i direction x direction plus x square z in the j direction plus x square y in the k direction and we wish to calculate the corresponding potential V x, y and z. Now, by definition I am going to have minus partial of v with respect to x equals F x which is 2 x y z. Similarly, partial v with respect to y is going to be F y which is x square z and partial v over partial z is going to be F z which is x square y.

If we just inspect it, you can immediately write that v should be equal to x square y z with the minus sign in front plus some constant, because potential is always defined up to a constant. But, I would like to do it more systematically, so that in future when you come across such problem, you will be able to handle it in a systematic manner. So, if I look at the first term partial v over partial x is equal to 2 minus 2 x y z and if I integrate it with respect to x, I am going to get v x y z is equal to minus x square y z plus, if function which is function of y and z only.

Why? Because, when I differentiate this entire function with respect to x, the first term gives me the correct answer minus 2 x y z and when I differentiate F which is the function of y and z only with respect to x, it gives me zero.

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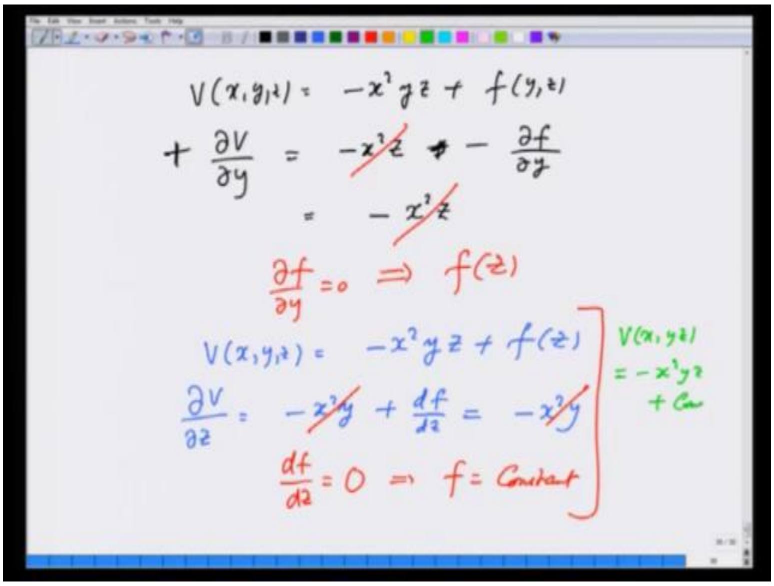

So, a possible form of V x, y, z is minus x square y z plus a function of y and z. Now, I know that minus partial v over partial y is minus x square z plus, minus d f over d y, but I am already given from the problem that this is equal to minus x squared z, it should be plus. D v by d y is already given to be minus x square z, I cancel these terms and I get d f d y is equal to 0. This can mean only one something that F is the function of z only.

And therefore, now we have merely down to V x, y, z therefore, should be equal to minus x square y z plus a function of z only. Now, let us take partial of v with respect to partial of z, which gives me minus x square y plus d f d z, but this is also from the given force field is equal to minus x square y. Again if I cancel terms, I get d f d z is equal to 0 which can one only mean that F is a constant, and therefore I have my answer which is V x, y, z is equal to minus x square y z plus a constant.

Given any force field, you can therefore do this kind of integration, first integrates x component, then y component and z component and instead of constant, you keep putting the functions of other variable, whose derivatives with respect to x y z gives me zero.

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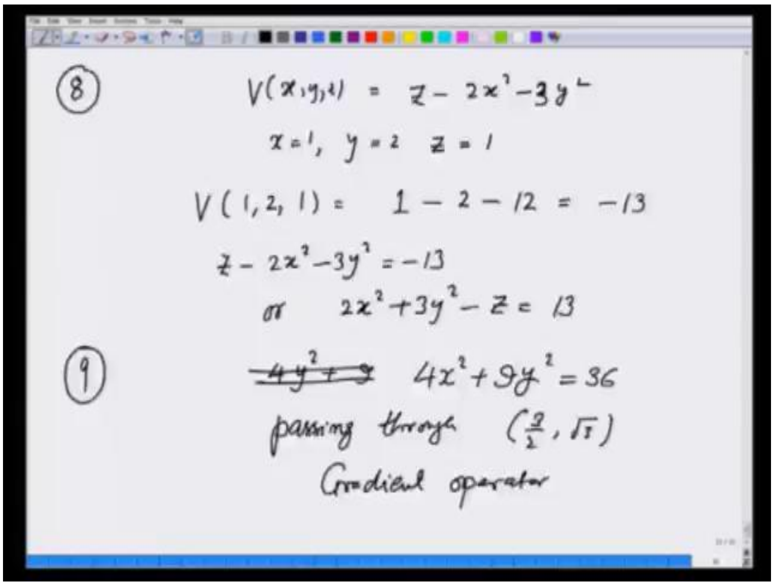

Next let us do problem number 8 which says for a given potential function, we given a potential function V x, y, z is equal to z minus 2 x square minus 3 y square. A constant potential surface passing through x equals 1, y equals 2 and z equals 1 will be described by. Now, if the potential surface is passing through these 3 points; that means, the potential value at that point is going to be equal to 1 minus 2 minus 3 times y square which is 12, which comes out to be minus 13.

And therefore, the surface will be described by z minus 2 x square minus 3 y squared is equal to minus 13 or 2 x square plus 3 y square minus z equals 13, that is the answer. Next let us do question number 9 which says, using the gradient operator one finds that the unit vector normal to the curve define by. We are defining a curve 4 y square plus 9, this is 4 x square plus 9 y square equals 36 and passing through 3 by 2 and root 3, what will be the unit vector normal to this curve. We want to use gradient operator.

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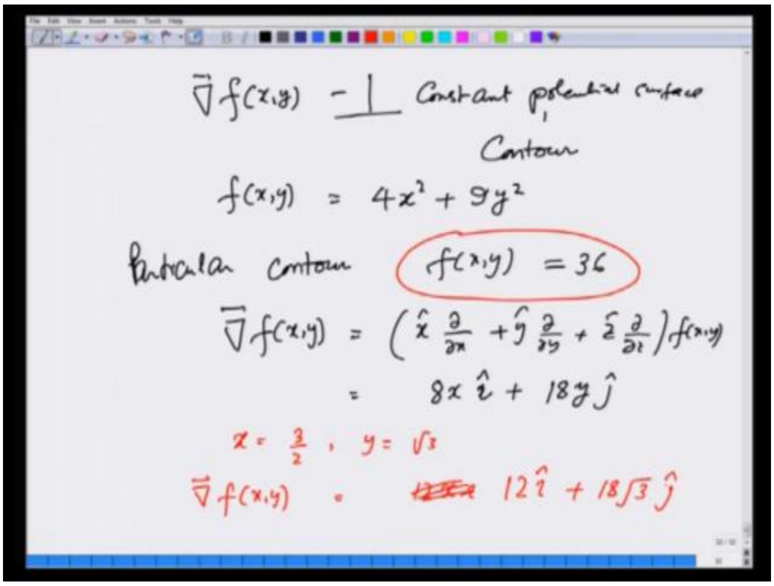

Recall from the lectures on gradient operator that, gradient operator acting on a function of x y gives me a vector which is perpendicular to the constant potential surface or contour, this we proved. Now, here I can take this f x y to be 4 x square plus 9 y square, it will have different contours. Depending on, if I make f x y equals constant and the particular contour we are talking about here is f x y equals 36, but that is the matter.

Now, if I take the gradient of f x y which is equal to x d by d x plus y d by d y plus z d by d z on f x y will give me 8 x in the x direction plus 18 y in the y direction and this is going to be perpendicular to the contour. Where do I want to calculate it? I want to calculate it on this particular surface which is f x y equals 36 and the point at which I want to calculate is x equals 3 by 2 and y equals root 3. And therefore, grad f x y at that point is nothing but 12 x I put in. So, this will be 12 i plus 18 root 3 j and all we want to do now is make it a unit vector.

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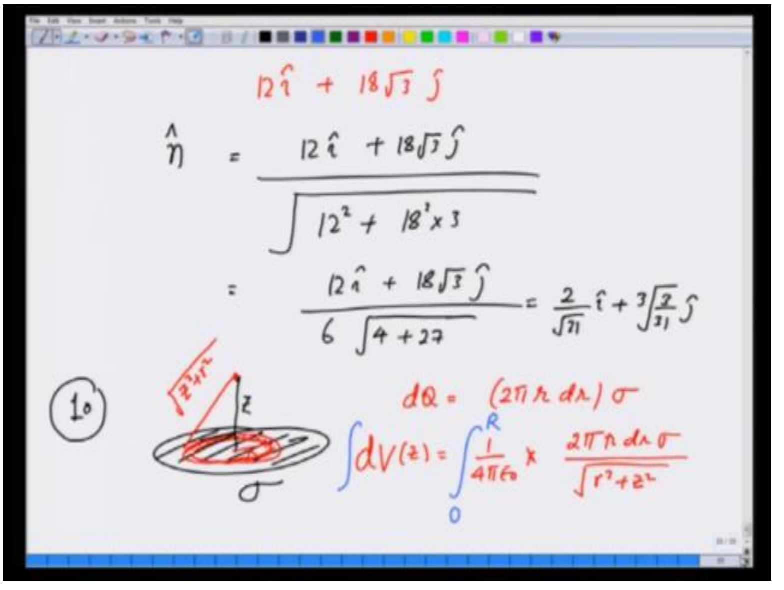

So, this vector which is given as 12 i plus 18 root 3 j unit vector, let me denote this by n is going to be 12 i plus 18 root 3 j divided by square root of 12 square plus 18 root 3 squared, so which is 18 squared times 3. This I can write as 6, I can take out, so 12 i plus 18 root 3 j divided by 6. Inside I will be left with square root of 4 plus 27, so which is 2 over root 31 i plus 3 root 3 over 31 j, that is the unit vector perpendicular to the surface f x y equals 4 x square plus 9 x square equals 36.

The direction of force is given from lower value of the control to the higher value of the control. Next problem number 10, ask the electrostatic potential on the axis of uniformly charged disc. So, we have a uniformly charged disc, let us say in the x y plane and it has a surface charge density sigma and we want to calculate the potential at height z from it. Notice, in this case that we calculate the potential therefore, I do not really have to worry about vector components as we did in the problem in the first assignment, where we calculating the force and therefore, calculation of electrostatic potential becomes slightly easier.

Now, what I will do for this is, take a ring at a distance r. Take a ring at a distance r, calculate the charge on this. The charge on this is going to be 2 pi r d r, that is the area of the ring times sigma and this, all the charge at disc on this ring r at a distance of a square root of z square plus r square from the point z. And therefore, the potential v at z is going to be 1 over 4 pi Epsilon 0, in fact I should write it d v very small potential due to the small ring is going to be d q which is 2 pi r d r sigma over square root of r square plus z square and the total potential is going to be integral of this, integral of this from r equal to 0 to capital R.

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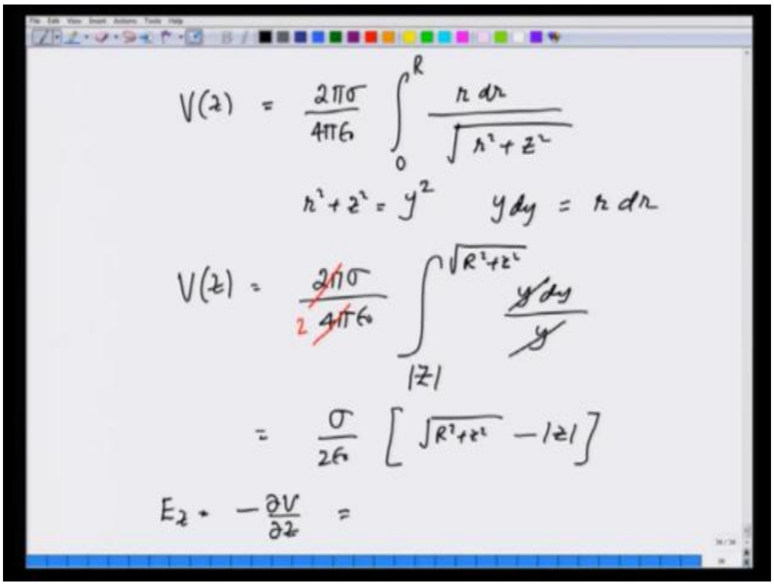

And therefore, the potential is given as V z is equal to 2 pi over 4 pi Epsilon 0 there is a sigma integral 0 to 2 R r d r over a square root of r square plus z square. Taking r square plus z square to be y, r square plus z square to be equal to y square and therefore, y d y becomes r d r and then I get V z equals 2 sigma over 4 pi Epsilon 0 integral modulus of z to R square plus z square y d y over y, y cancel and I get this equal to this 2 pi also cancels here and I get equal to answer equal to sigma over 2 Epsilon 0 inside square root of R square plus z square minus mode of z.

So, you see this calculation of potential is slightly easier, because is the quantity which is scalar. I could, if I want to calculate the z components of the electric field which is going to be the only component, I will take E z on the axis to be equal to minus d v by d z and I can get my answer.

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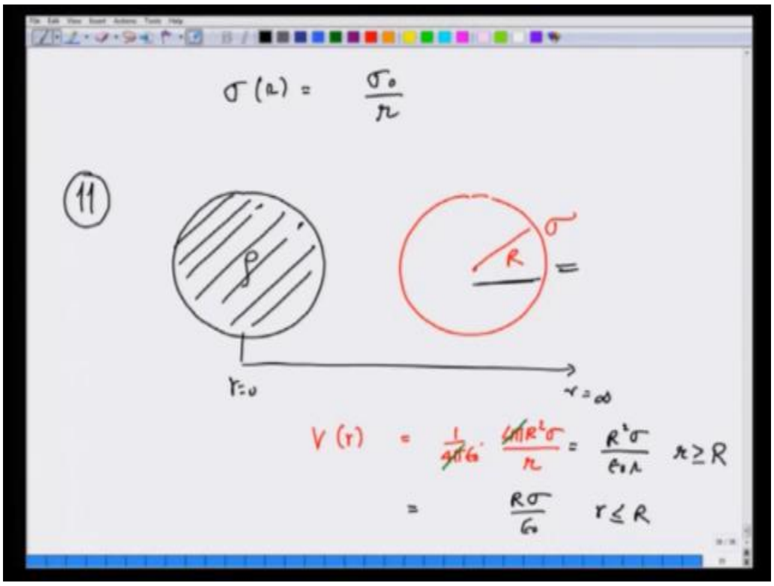

I can also take different sigma's; for example, in the previous assignment we had a sigma which is r dependent as sigma naught over r. I could also calculate potential for that, in fact I leave it for you as an exercise. You calculate potential due to the sigma r, calculate v on the axis, z axis and then differentiate it to get the electric field, and then compare that the answer obtained in assignment number 1.

Finally, the last problem in this assignment is using the electrostatic potential derived for a charge shell; get the result for uniformly charged sphere of radius r for the potential. So, what we have is a sphere which is uniformly charged with density rho and we want to calculate the potential from r equals 0 all the way up to r equals infinity using the result for a charge shell which I derived in the lecture.

If there is a charge shell of radius r carrying surface charge density sigma, then we have obtained in the lecture that the potential V r is equal to 1 over 4 pi Epsilon 0 4 pi R square sigma over r, which I can simply write as by cancelling this 4 pi as R square sigma over Epsilon naught r for r greater than or equal to capital R. So, outside it goes as 1 over r, on the inside it becomes equal to constant which is equal to the value at the surface.

So, inside it becomes R sigma over Epsilon 0 for r less than equal to R. Notice that at r equals capital R, the two answers match. So, we have this result for the surface shell.

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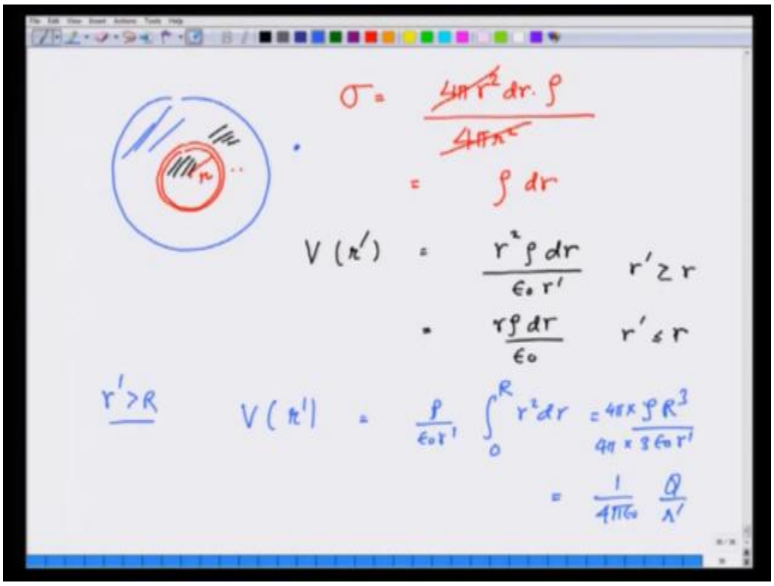

And we want to use it to find the potential for uniformly charged sphere. For the uniformly charged sphere, I take a shell inside of radius small r and if I think of it as a shell and thickness is d r, then the sigma on this is going to be charged per unit area. The volume of the shell is 4 pi r square d r, the charge on this is going to be a time sigma, times rho and if I divide by the area of the shell, this going to be sigma.

So, sigma is nothing but 4 pi r square cancels, rho d r and this shell gives rise to a potential outside which has 1 over r dependence and inside it is constant. So, if I write the potential due to the shell is going to be for r prime let us calculate it as r prime is going to be r square which is r square d r over Epsilon 0 r prime for r prime greater than or equal to r. And this is going to be equal to rho d r over Epsilon naught times r for r prime less than equal to r.

So, I found the potential inside, I found the potential outside. Now, all I have to do is integrate over d r to get values at different points. Let us for do, first to do the calculation for r prime greater than R that is I want to find the potential outside. For outside, if r prime is greater than R prime, all the points are lying outside, all the shells considered here and therefore, the potential at r prime greater than R is going to be…

The only expression that will work here is the first one, because all r primes are greater than the biggest shell I can take. And therefore, this is going to be 1 over Epsilon 0 r prime integration r square d r rho anyway comes out 0 to r, which is going to be rho R cubed over 3 Epsilon 0 r prime. I multiplied by 4 pi the bottom, 4 pi the top, 4 pi rho R cube by 3 the charge. So, this becomes 1 over 4 pi Epsilon 0 Q by r prime which is a known result for charge outside. For any spherical charge, it behaves like a point charge. What about the potential inside?

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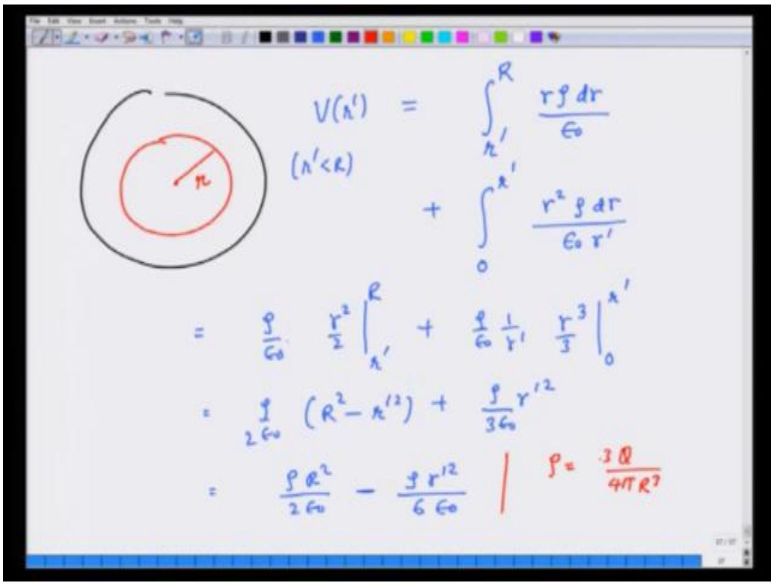

Now, for the potential inside if I am calculating the potential inside, if I take a shell at r, I want to calculate the potential V r prime, all the shells outside. Now, we are taking r prime to be less than R, all the shells outside will be giving a constant potentials. So, this is going to be all the potentials, all the shells outside from r prime to R, R going to give me a constant potential, which is equal to rho d r r over Epsilon 0 plus the shells from inside 0 to r prime.

R going to give a potential which is like a potential of a point charge or Q over r which is going to be nothing but r square rho d r over Epsilon 0 r prime. So, this is going to be the final answer, which I can write as rho over Epsilon 0 r square by 2 from r prime to R plus rho over Epsilon 0 1 over r prime r cubed over 3 from 0 to r prime. So, this answer is rho over Epsilon 0 to Epsilon, 2 Epsilon 0 R square minus r prime square plus rho over 3 Epsilon 0 r prime square, which comes out to be rho R square over 2 Epsilon 0 minus rho r prime square over 6 Epsilon 0.

Taking rho equals Q over 4 pi R cubed divide by 3 which will go up, I can write this answer as…

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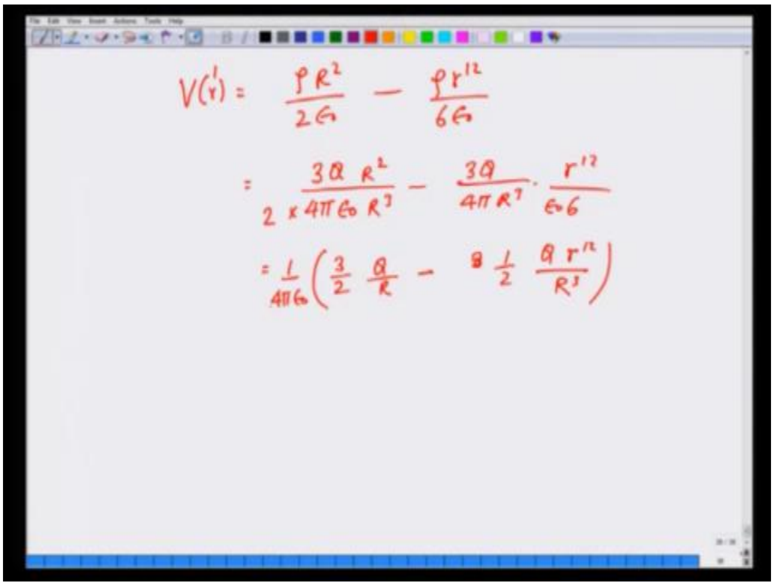

So, we have potential V r which is rho R square over 2 Epsilon 0 minus rho r prime square over 6 Epsilon 0 and I am putting for rho 3 Q over 4 pi Epsilon 0 R square over r cubed minus 3 Q over 4 pi R cubed r prime square. This is at r prime, r prime square over 6. So, this comes out to be 3 by 2 Q over R, there is a 2 also 2, 3 by 2 Q over R minus, there is an Epsilon 0, 3 1 half Q r prime square over R cubed 1 over 4 pi Epsilon 0 and that is the answer.