Introduction to Electromagnetism
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Lecture - 68
Assignment 2
Problems 1-4

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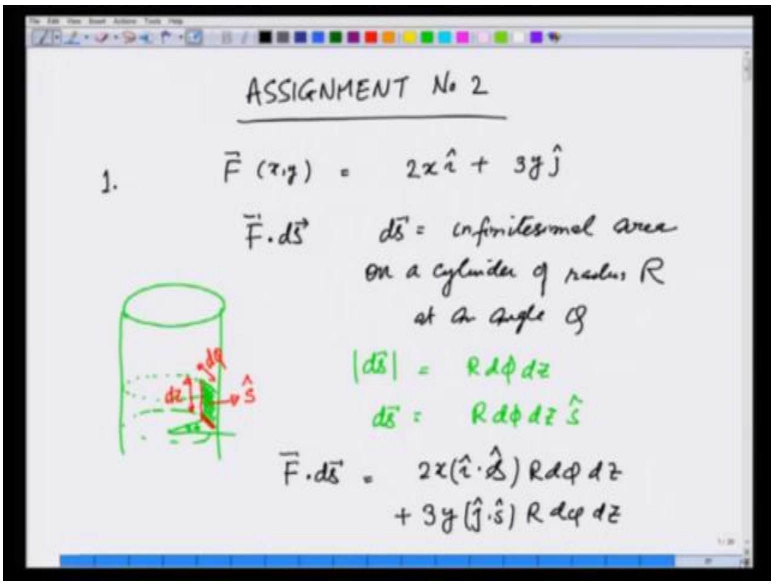

In this tutorial and next we will be solving home assignment number 2, which is on basic electrostatics and vector calculus. Problem number one of this assignment says for a given field F, which is a function of x and y is equal to 2 x i plus 3 y j. The dot product F dot ds, where ds represent and infinitesimal area a on a cylinder of radius R. So, we wish to find the F dot ds product. So, this is infinitesimal area on cylinder of radius R at an angel phi, and we are using cylindrical coordinates. So, if I make a picture of the cylinder we are trying to look, there is a force filed F and we are trying to find the dot product of F dot ds, where this small area is at an angel phi at radius R.

So, you can see in cylindrical coordinate this area is actually going to point in the direction of s - unit vector in cylindrical coordinates, its height is d z or delta z. This angel is d phi, therefore this area magnitude. So, if I write ds magnitude is going to be R d phi which is this distance, let me show it by red the curve distance of the curved surface, and the height is d z. So, magnitude is going to be R d phi d z, and the direction is d in the x direction. So, ds area is R d phi d z in the s direction. And therefore, when I calculate F dot ds is going to be 2 x i dot s R d phi d z plus 3 y j dot s unit vector R d phi d z.

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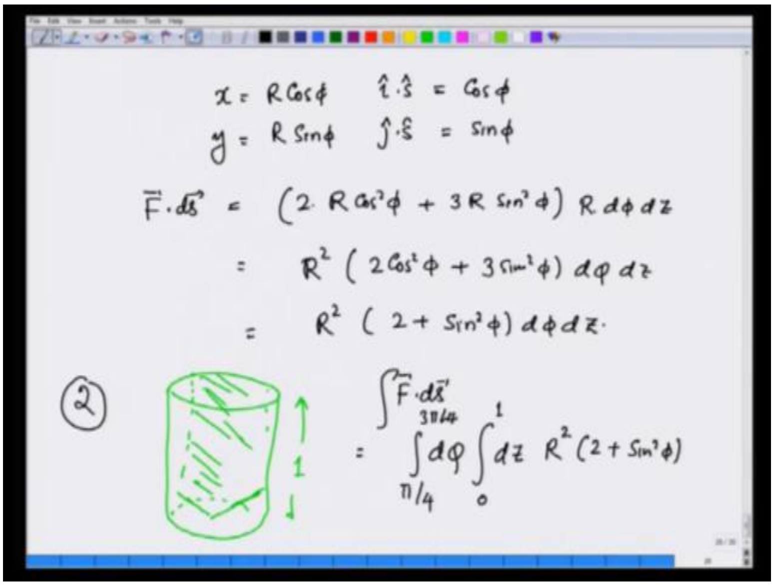

Now, I know in cylindrical coordinates on the surface x is going to be R cosine of phi i dot s unit vector is cosine of phi, y is R sin of phi j dot s is sin of phi. A nd therefore, F dot d s is going to be equal 2 R cosine square 5 plus 3 R sin square phi and then we have from outside R d phi d z which becomes equal to R square 2 cos square phi plus 3 sin square phi d phi d z which by using the identity that cosine square phi plus sin square phi is 1, I can also write as R square 2 plus sin square phi d phi d z, and that is the answer. For question number 2 - the answer, the question is the flux of the field in problem 1 over a quarter of a cylindrical surface extending from pi equals to pi by 4 to phi equals 3 pi by 4 is...

So, what we are now asking for, let me again make this cylinder and we are taking the length of the cylinder to be 1, over this length of 1, we want to integrate from phi equals pi by 4 up to phi equals 3 pi by 4. So, in the somewhere like this over this cylindrical surface which is on the back side, we wish to calculate the flux passing through this cylindrical surface. So, what we need to do is integrate F dot d s over the surface which is going to be equal to integral d phi is formed pi by 4 to 3 pi by 4 d z is of length 1. So, it is from whatever length let us say 0 to 1, because there is no z dependence it does not matter, what they initial and final points are R square 2 plus sin square phi. Now, z integral since there is no z dependence gives me 1.

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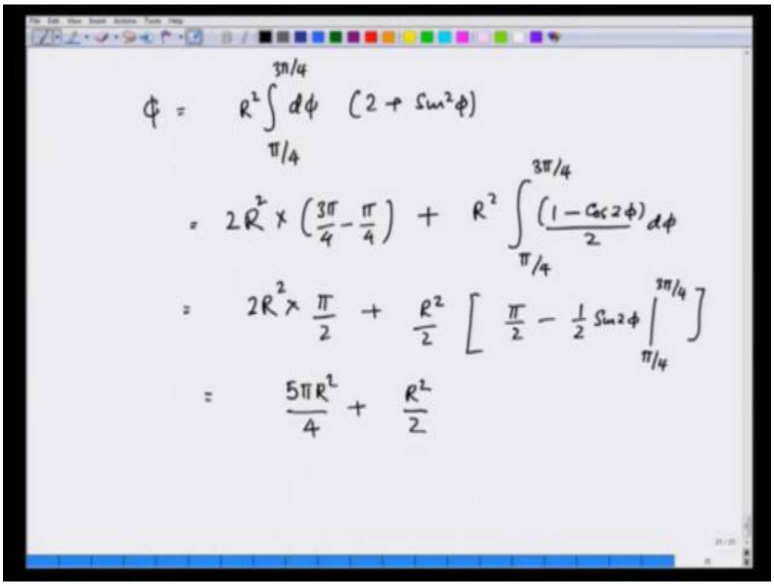

And therefore, I get this flux is equal to integral d phi from pi by 4 to 3 pi by 4 R square comes out 2 plus sin squared phi which is equal to lot 2 R square times 3 pi by 4 minus pi by 4, that is the first term plus R square integration 1 minus cosine of 2 phi over 2 d phi from pi by 4 2 3 pi by 4, which is equal to 2 R, sorry this is R square; R square times pi by 2 plus R square by 2 inside I am going to get pi by 2 to from the first term, minus 1 over 2 sin 2 phi pi by 4 and 3 pi by 4. And this integral you can easily do and the final answer then comes out to be 5 pi R square by 4 plus R square by 2. The integral that is quite simple to do, so I leave them for you.

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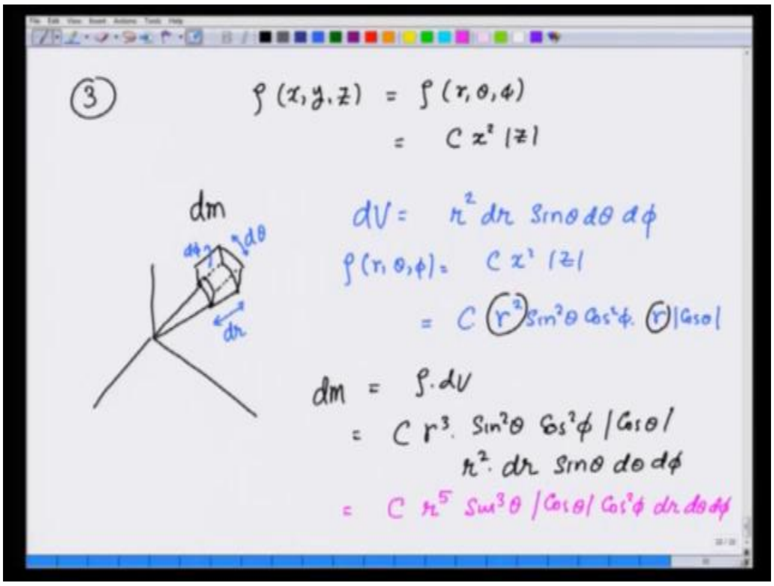

Question number 3 is if the density of the matter distributed in a part of space is given as. So, let me write density which is a function of x, y and z; I could also write it as density R theta and phi is given as C x square modulus of z. What it means is in the lower plane or z negative value or positive value its positive. And x it depends on x square, so density all positive. It is given as C x square mod z, where C is a constant the mass of in infinite dissimilar volume element in spherical polar co ordinates is given as.

So, we want to fine mass of a small volume element. Let me again make a picture, you recall that the volume element in spherical polar coordinates is going to be between theta and d theta, and it is going to look like this, where stands the distance this is small d R, you moving along d theta and when you move in the x y axis, the angel is d phi. So, the volume element is given as d v is equals to r square d r sin theta d theta d phi, the density in terms of R theta and phi is given as C x square mod z, which I am going to change in spherical polar co ordinates. So, this still becomes C r square sin square theta cosine square phi and then z is nothing but r; r is again or only positive numbers. So, cosine theta would be positive and negative, so I will put it more mode cosine of theta.

And therefore, this small mass is going to be equal to rho times d v which is C r cubed this term here, and this term here become r cubed times sin square theta cos square phi mod cosine of theta, and then you have r squares d R sin theta d theta d phi d phi, which I can then finally write as C r raise to 5 sin cube theta mode cosine of theta cosine square phi d R d theta the d phi, that is the mask element.

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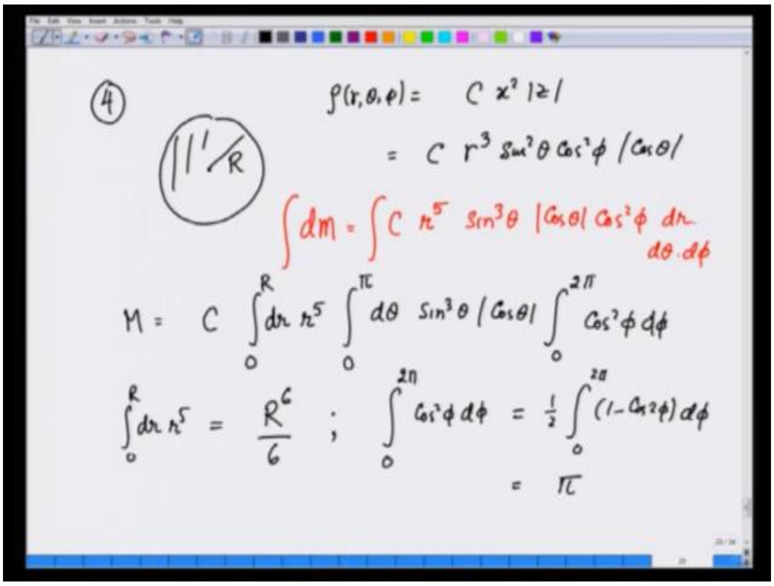

Question number 4, then ask mass of the sphere of radius r centered around the with density distribution C x square mod z is. So, now, we are asking, if I have a sphere centered at the origin of radius r with this mass density rho r theta and phi is equal to C x square mod z which we have already written as C r cubed sin square theta cosine square phi modulus of cosine theta. What is the total mass of a sphere of radius R centered at the origin. We have already calculated small mass element of is small volume element here, which is given as we have written in previous page, I am going to copy it from here. C r raise to 5 sin cube theta modulus cosine of theta cosine square phi d R d theta d phi. And therefore, the total mass is going to be an integral of this, now let me write this integral separately. So, total mass m is going to be given as, C is a constant which comes out and integral d r r raise to phi goes from 0 to R integral d theta sin cube theta modulus of cosine of theta theta goes from 0 to pi an integral 0 to 2 pi cosine square, phi that is the answer. Now I can do these 3 integral separately.

So, 0 to R d r r raise to phi gives me, R raise to 6 over 6, similarly the integral over 5 is cosine square phi, I forgot to write d phi here d phi 0 to 2 pi is nothing but 1 half integral 0 to 2 pi 1 minus cosine 2 phi d phi which gives me nothing but pi which gives me.

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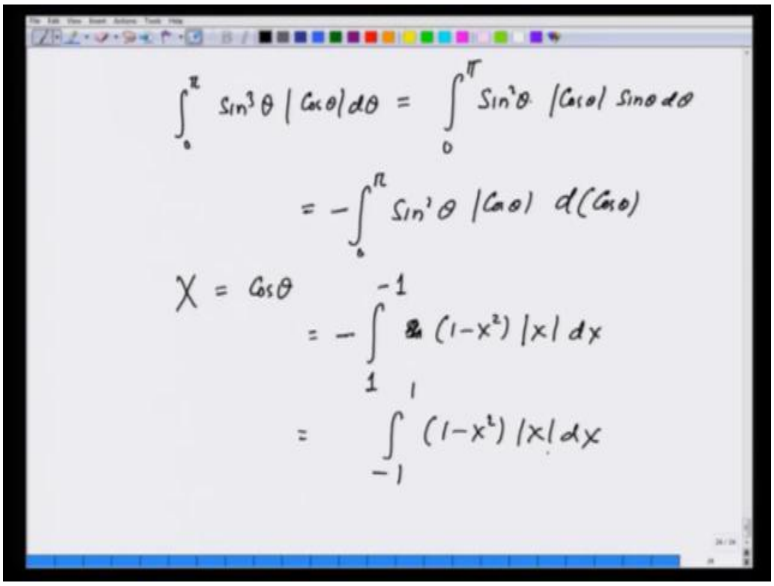

Final integral this d theta integral, let me do that in the next page which is 0 to pi sin cube theta modulus of cosine theta d theta, which I can write it as to make life easy 0 t pi sin square theta modulus cosine theta cos theta, sorry sin theta d theta which is nothing but integral 0 2 pi sin square theta modulus cosine of theta d of cosine of theta with the minus sign in outside. Let us now define x as cosine of theta, then I have this integral equal to minus 1 2 minus 1 sin square theta will be become 1 minus x square modulus of x d x which is nothing but minus 1 to 1 1 minus x square mode x d x.

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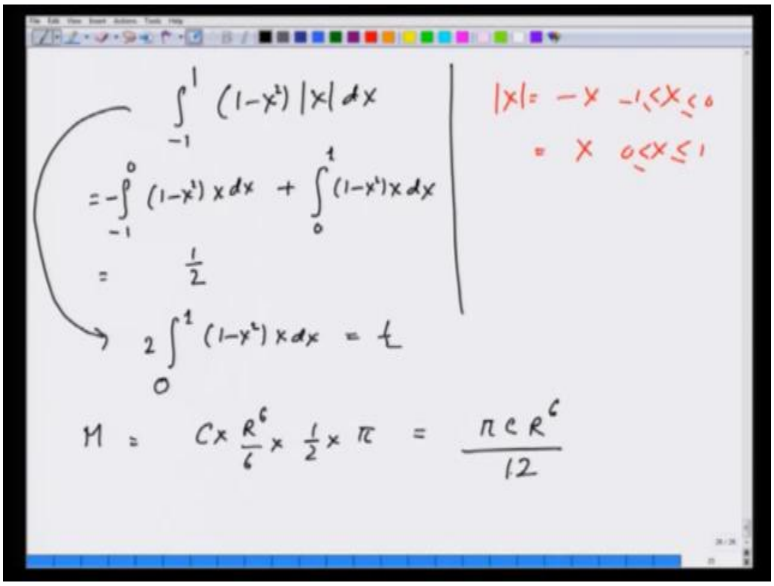

Let us do this integral. So, I have minus 1 to 1 1 minus x square mod x d x; the mod x on the side mode x is equal to minus x for x between minus 1 and 0, and is equal to x for x between 0 and 1. So, I can write this equal to this integral therefore, becomes equal to minus 1 to 0 1 minus x square times x with the minus sign outside d x plus 0 to 1 1 minus x square the x d x. These are very easy integral to perform and the final answer they give you, when you do all the integral comes out to be 1 half. Another way of looking at it is since mod x is even about x is equal 0, I could have written this as 2 times 0 to 1 1 minus x square x d x which will give me 1 half. So, the total mass, therefore if I collect all term is going to C times R raise to 6 over 6 times 1 half times pi, which comes out to be pi C R raise to 6 over 12, and that is your answer.