Introduction to Electromagnetism
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Module - 08
Lecture - 67
Solution Assignment – 1
Problems 4-9

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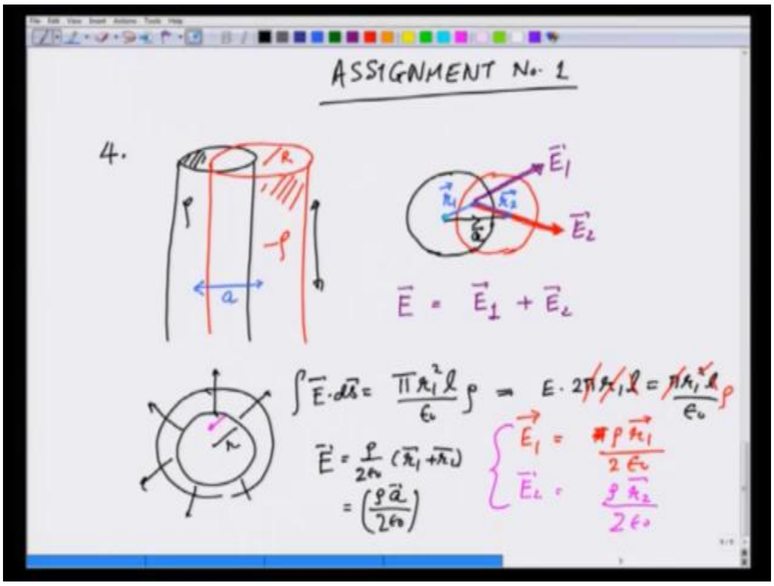

We have been assignment number 1, and this tutorial we will solve from problem 4 to problem number 9. Problem number 4 states, if we have 2 infinitely long overlapping cylinders. So, let us make 2 infinitely long overlapping cylinders of each of radius capital R. So, this is of radius capital R, one cylinder carries charge density rho. So, let this black one carry charge density rho, the red one carry charge density minus rho. The distance between the axis of the 2 cylinders is a, and a is less than 2 r and therefore there is an overlap. So, this distance is a. The magnitude of the electric field in the overlapping region is, this is what we wish to calculate. So, let me look at it from the top, if I look at it from the top here is one cylinder, and here is the other cylinder, and it is in this overlapping region that we wish to calculate the electric field.

So, a electric field in the overlapping region is going to be net field is going to be E, which is sum of electric field E 1, which is due to the positive charge plus E 2 which is due to the negative charge. So, let me show how this field looks in the overlapping region, the electric field due to E 1 due to the positive charge is going to be at any point is going to be along the vector, let me make this like this radially out. And the field due to red is going to be along the red vector shown, what about their magnitudes lets calculate those.

Let us take the distance from the centre of the positively charged cylinder to be R 1 and therefore this is vector R 1. And the distance from the negative charge to the centre of negatively charged cylinder, let that vector be R 2, and so field is going to be due to E 1 and E 2, hence their sum. Let us by calculate, let us calculate E 1 and E 2 by Gauss's law. So, if I look at positively charge cylinder at a distance r 1 by Gauss's law first by symmetry E is going to be only in the radial direction. And therefore, E dot d s comes out to be equal to charge enclosed which is going to be pi, if this is distance is r pi r 1 square, and if I take the length or the height of the cylindrical surface to be l divided by epsilon 0 times rho. And E is same all over r. So, this is going to be E times 2 pi r 1 l is equal to pi r 1 square l over epsilon 0. Probably you know this result from your twelfth grade, but any well as do it. So, pi cancels from both sides r 1 one of the r 1 cancels from both sides, l cancels from both sides, and therefore E 1 magnitude is equal to pi; pi has canceled is equal to there's a rho also sorry, is equal to r 1 rho over 2 epsilon 0. And for the vectors since its in radial direction, I can write E vector to be equal to r 1 vector rho over 2 epsilon 0.

In the same manner for the negative charge, the field is going to be in the opposite direction magnitude is going to be the same, and therefore 2 is going to be rho r 2 over 2 epsilon 0. I add the 2 and I get E is equal to rho over 2 epsilon 0 r 1 plus r 2, but what is r 1 plus r 2? Let me go to this diagram showing 2 overlapping cylinders r 1 plus r 2 is this vector a, the vector form the positively charged cylinder to negatively charged cylinder. And therefore, this is equal to rho a over 2 epsilon 0. This is a fantastic result that in the overlapping region, I have constant electric field irrespective of which point I am at and its magnitude is rho a over 2 epsilon 0.

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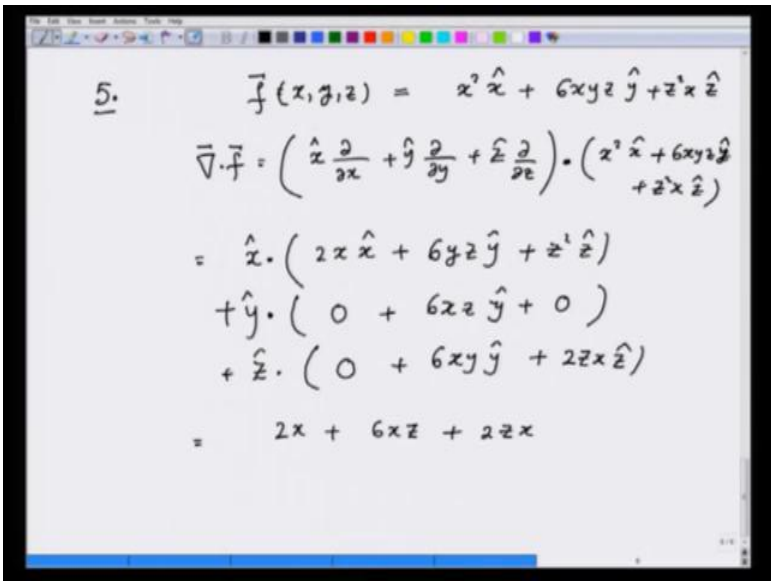

Next, I solve problem number 5 which says the divergence of vector field described by the function f is as a function of x, y, and z is equal to x square x unit vector plus 6 x y z y unit vector plus z square x z unit vector, what is the divergence? We wish to calculate its divergence of f which is nothing but partial derivative with respect to x in the x direction plus y partial y plus z partial z dotted with x square x plus 6 x y z y plus z square x z, which I can write as I will do it once in full glory, and then you can see that it makes sense only to take the x partial derivative of x component, y partial derivative of y component z partial derivative of z component, but let us do it fully for the time being.

So, we will do x dotted with d by d x will give me 2 x x unit direction plus 6 y z in y direction plus z square in z direction plus y unit vector dotted with first term gives me 0 in the x direction, second term gives me plus d x z in the y direction plus 0 again plus z unit vector dotted with first term gives me 0, second term gives me 6 x y in the y direction plus 2 z x in the z direction. I take the dot product and the final answer that we get is 2 x, because x dot y is 0 x dot z is 0 plus d x z plus 2 z x, which is effectively taking z derivative of the z component, y derivative of the y component, x derivative of x component adding them up.

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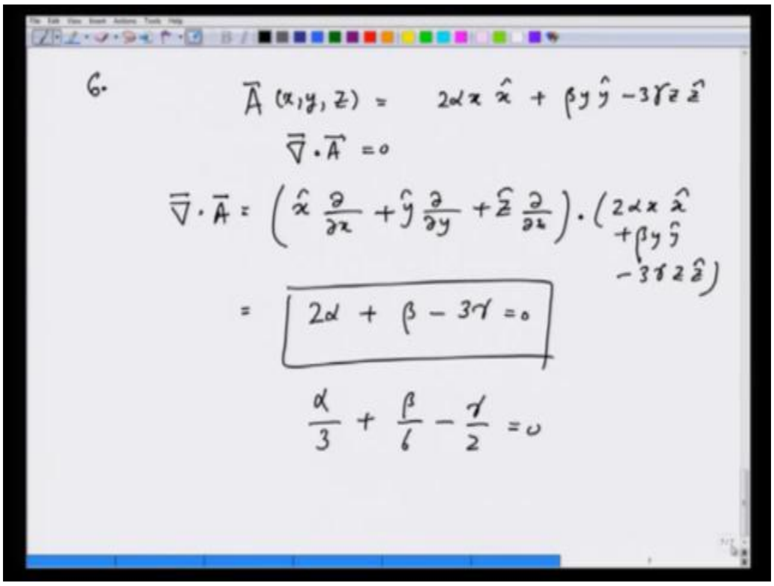

Problem number 6, which says if the divergence of the vector field A x y z which is given as 2 alpha x in the x direction plus beta y in the y direction minus 3 gamma z in the z direction, if its divergence is to be 0 then what is the relationship between alpha, beta, and gamma. So, let us take divergence of A, which is x unit vector partial with respect to x plus y unit vector partial with respect to y plus z unit vector partial with respect to z dotted with 2 alpha x x plus beta y y minus 3 gamma z z, which is equal to take the partial derivatives with respect to x of x component only. So, this becomes 2 alpha plus beta minus 3 gamma is equal to 0. So, the relationship is this, if the divergence has to be 0 or in terms of the answer which I have given in your assignment is alpha over 3 plus beta over d minus gamma over 2 is equal to 0.

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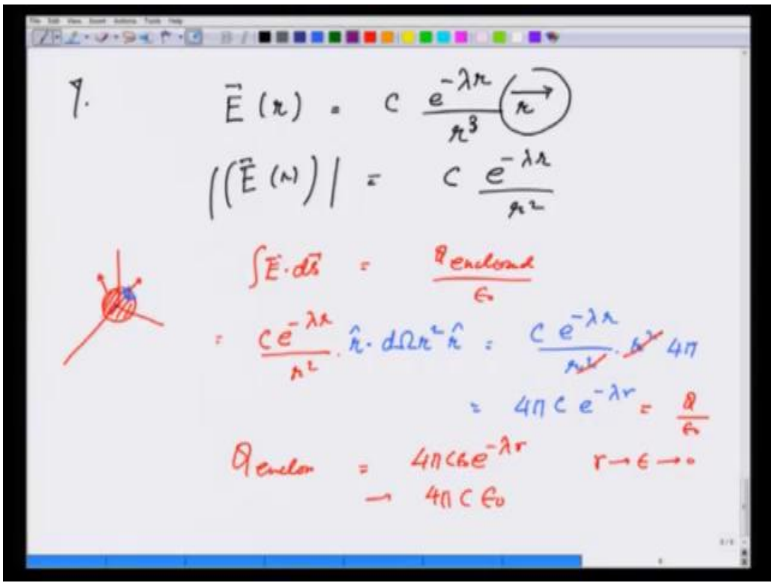

Problem number 7: electric field for a charge distribution is given as you have been given an electric field for a charge distribution as in terms of r which is a spherical distance spherical polar coordinates are used C e raise to minus lambda r divided by r cubed r vector; that means, the dependence if I would calculate the dependence of r, in terms of r it will be only C e raise to minus lambda r over r square, and it is in the radial direction r square comes, because this r vector has a magnitude r. What we want to find now, if we take a very small sphere of radius epsilon around the origin, the charge enclosed by this sphere in the limit of epsilon going to 0 is given by.

So, we will apply Gauss's law which says at E dot d s is equal to Q enclosed by epsilon 0 and what we are doing is we are taking the origin, and taking a very small sphere here. d s is going to be all radial. So, I can write E dot d s as C e raise to minus lambda r over r square times r unit vector dot d s which is going to be 4 pi r square all in the radial direction. If you really like to write it more properly, I would take r unit vector dotted with d omega r square which is really the small element of area here r unit vector which then comes out to be C e raise to minus lambda r over r square times r square, and since it does not depend on phi and theta, I can write this as 4 pi. So, this is actually 4 pi C e raise to lambda r, r square cancels which is equal to Q enclosed over epsilon 0.

So, Q enclosed is equal to 4 pi C e raise to minus lambda r times epsilon 0. If r is very small r going to epsilon which is going to 0, this goes 4 pi C epsilon 0. What it means is that there is a point charge at the origin, because even if I make the radius goes smaller and smaller in the limit going to 0, it still encloses a finite charge 4 pi C epsilon 0; that means, there is a point charge of basically extent 0 at the origin.

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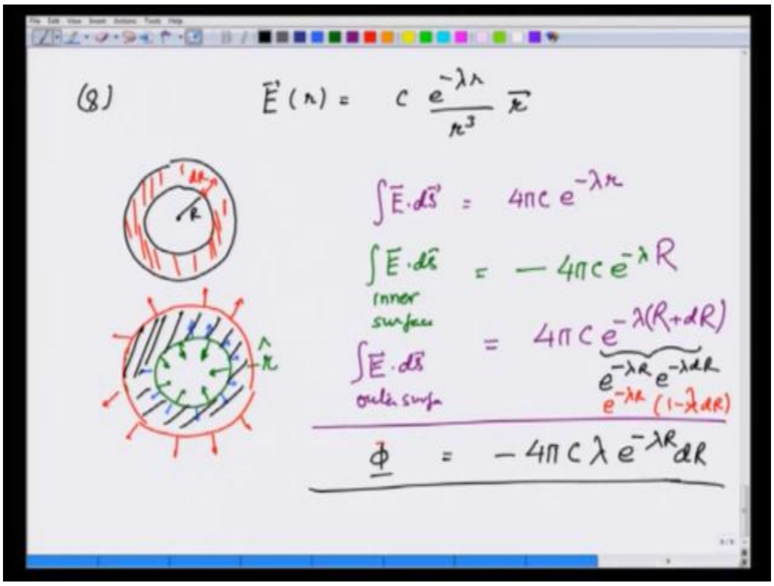

Problem number 8, for the electric field we are again back to this electric field E r which is equal to C e raise to minus lambda r over r cubed vector r. The flux through a small radius shell of radius flux through a shell not small flux through a shell of radius capital R infinitesimal thickness d r. So, we are taking a shell of radius R, and thickness d r, the flux through this will be how much.

Notice in this case when I am talking about this volume which I am shading with red, there are 2 surfaces. So, the net flux is going to be flux through the inner surface and flux through the outer surface. In the inner surface if I look at the inner surface the area vector is pointing invert, because area vector is always taken to be going out of the volume. So, therefore, the area vector is actually in negative r unit vector direction. On the outer surface on the other hand which I will make by red the area vector is in the positive r direction. I have already calculated in the previous problem, when I did E dot d s taking s the elemental vector to be area vector to be pointing out of this sphere, this came out to be 4 pi C e raise to minus lambda r times, that is it for d 4 pi C e raise to minus lambda r.

So, for the inner surface let me write it with green, since d s vector is pointing in the opposite direction. So, E dot d s for the inner surface is going to be minus 4 pi C e raise to minus lambda r, because the vector unit vector the electric field vector is going out, going the way that the blue lines are showing, and the d s vector is in the exactly 180 degrees opposite to it. And E dot d s for the outer surface is going to be equal to plus 4 pi c e raise to minus lambda r plus d r. I am going to replace this small r now by capital R. So, I will erase this small r replace this by capital R, remove this small r replace this by capital R, remove this small r and replace this by capital R, because we are doing it at radius capital R. And I add the two. So, the net flux is going to be since d r is infinitesimal, I can write this exponential as E raise to minus lambda r times e raise to minus lambda d r. So far is exact and then I am going to expand it further as e raise to minus lambda r 1 minus lambda d r, and then higher order terms. So, to the first order the flux is going to be equal to 4 pi C e raise to minus lambda r cancels, and I get minus 4 pi C lambda e raise to minus lambda r d r, that is a flux through this shell of radius r and thickness d r, that is the answer.

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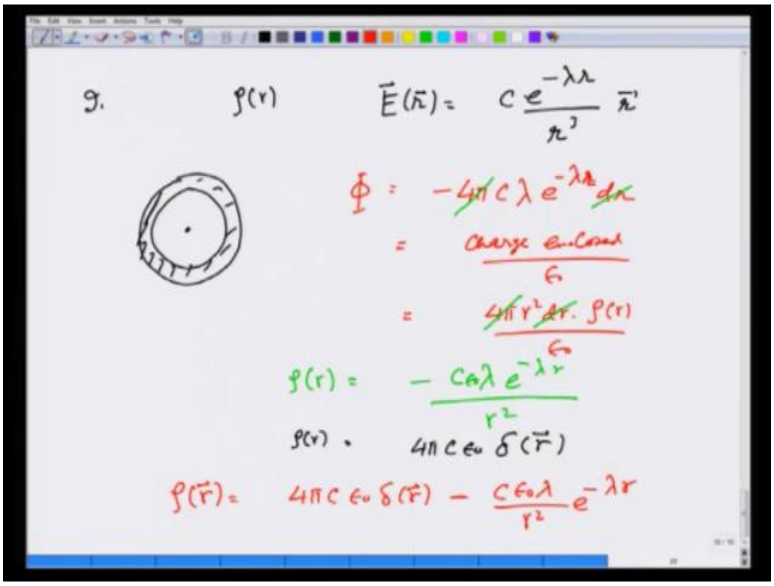

Problem number 9: the charge density rho r for the electric field E r which is given as C e raise to minus lambda r over r cubed vector r, we want to find a charge density by Gauss's law. Now, I can use the divergence theorem directly divergence taking divergence of E directly, but I am not going to do that. Let us look at the shell what is the charge density near the shell. We have already calculated the flux through this. So, flux through this shell is nothing but minus 4 pi C lambda e raise to minus lambda r d r, and this is supposed to be charge enclosed divided by epsilon 0. Charge enclosed is going to be the volume of the shell which is 4 pi r square d r times rho r divided by epsilon 0. Look at the 2 sides and you can cancel this delta r, we do not cancel d r, it should actually be returns as delta r. So, we cancel that this 4 pi gets canceled. So, you end up getting rho r is equal to minus E lambda e raise to minus lambda r, there is an epsilon 0 divided by r square, that is a charge density when you are away from r equal to 0.

I had already pointed out in problem number 8, there are problem number 7 that there is a point charge at the center right, because as you go to at the center the electric field becomes more like 1 over r square, because E raise to minus lambda r becomes 1. And therefore, there's an additional density additional charge, there which the magnitude was 4 pi C epsilon 0 and charge density therefore is going to be delta r, because this is a point charge. So, the net charge density of this electric field is going to be 4 pi c epsilon 0 delta r minus C epsilon 0 lambda over r square e raise to minus lambda r, and that is your answer.