Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 66
Solution Assignment – 1
Problems 1 - 3

In this tutorial I will be solving the first assignment.

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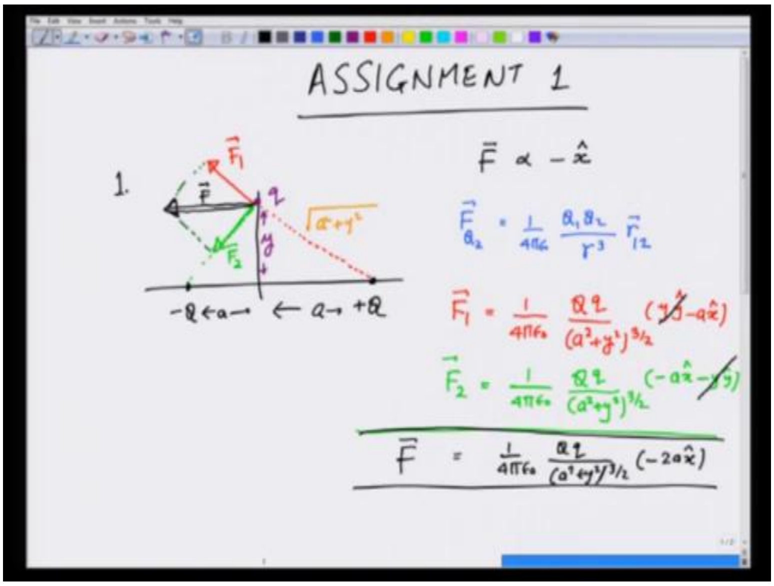

So, this is assignment 1, the first problem of the assignment says, there are two charges plus Q and minus Q placed at a distance a from the origin on the x axis and we put a third charge Q on the y axis at a distance y and we wish to calculate the force on the charge q which is on the y axis. So, let us look at it physically, the force on charge small q due to the positive charge will be along the line from capital Q to small q in the direction shown by red.

Similarly, the force by minus Q on small q will be along the line from small q to capital Q along the line shown by green and the net force is going to be a combination or sum of these two forces. Let me call these force as F 1 and F 2 respectively and you can see because of the distances involved, the magnitude of F 1 and F 2 is going to be the same. And therefore, the net force if you just do the vector sum is going to be in the direction towards the left shown by this black arrow, this will be the net force.

So, we already anticipate geometrically that the force F is going to be proportional to minus x or in the negative x direction. Let us now do it mathematically recall that when we wrote Coulombs law, the force F between two charges Q 1 and Q 2 was nothing but, Q 1 Q 2 over r cubed and vector r 1 over 4 pi Epsilon 0. Where, if I am calculating force on Q 2 it is going to be r is going to be from 1 to 2 and this is force on Q 2. If the two charges are negative, then you can see the force in the opposite direction.

Now, let us calculate F 1, F 1 therefore, is going to be 1 over Epsilon 0 magnitude Q q over r cubed. The distance between the charges is shown in this yellow brown here is square root of a square plus y square and therefore, r cubed is going to be a square plus y square raise to 3 by 2. And the vector from plus Q to small q shown by the red and that vector is going to be y and the y unit direction minus a x, that is the force F 1 on small q due to the positive charge plus Q.

I can similarly now calculate F 2, F 2 is going to be 1 over 4 pi Epsilon 0 and I will just write the magnitude capital Q small q over the distances involved are the same. So, I am again going to have a square plus y square raise to 3 by 2. However, now the force is in the direction of green vector shown which is from small q to minus Q it is from y point y j or the point on the y axis to a point on minus x axis and so therefore, this is going to be minus a x minus y y, that is a vector from small q to minus capital Q charge that is the vector.

So, the net force is going to be sum of these two vectors, if I sum these two the net force F therefore, comes out to be notice that the terms involving y cancel out and I get equal to 1 over 4 pi Epsilon 0 Q q over a square plus y square raise to 3 by 2 times minus 2 a x, you can see that the net force is in minus x unit direction, so that is problem 1 done.

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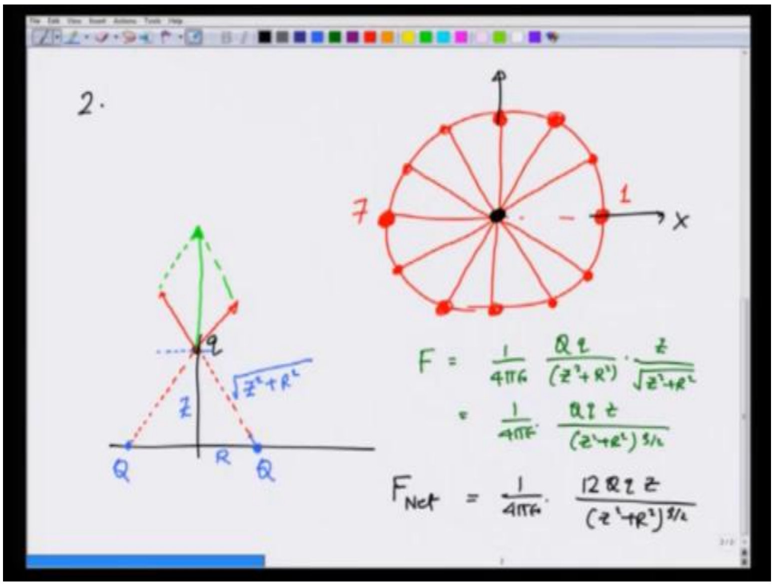

Let us come to problem number 2 which asks 12 charges of the same sign and magnitude capital Q are placed on a ring of radius R at equal distances. So, if I take this ring of radius R there are 12 charges placed on it at equal distances. So, we can make it like this 1, 2, 3, 4, 5, 6, 7th will be somewhere here 8th, 9th, 10th, 11th, 12th they are all separated by 30 degrees, third one will be here. So, let me make it cleanly 4th, 5th, 6th, 7th.

So, let me make it again, if I have this ring I will start from charge 1 at 30 degrees there is one more charge 2, 60 degrees there will be one more 3, 4th here 5th at 120, 6 at 150, 7th at 180, 8th 210, 9 240 at 10 270, 11th at 300 and 12th at 330, so there are these 12 charges. Notice that for each positive charge here, there is a charge on the opposite side, for each charge there is a charge on the opposite side, for each charge here there is a charge in the opposite charge.

So, that if I look at the force which is to be calculated, so this is the x, y plane. So, let us write this x and y plane I want to calculate the force on a charge small q kept on the z axis. Let us say I take on this red circle charge number 1 and charge number 7, you will notice that the force due to these two charges is going to be as in the arrow shown, this is going to be repulsive. And the distances are the same, now let us write the distances this charge is at a distance this plus Q capital Q is at a distance R, this is height said and therefore, this distance diagonal distance is z square plus R square.

So, these two forces shown be red arrows are going to have the same magnitude and therefore, the horizontal components are going to be cancelling and the net force let me erase this net force therefore, if I take a vector sum is going to be in the z direction. So, what I should be doing, so geometrically I have argued that the net force will be in the z direction. What I will do is, I will calculate the z component of each force due to each charge capital Q and add them up.

So, now, therefore, force due to one of the charges force is going to be 1 over 4 pi Epsilon 0 Q q over z square plus r square that is the force which is shown by the red arrow and it is vertical component is going to be z over square root of z square plus R square, which is equal to 1 over 4 pi Epsilon 0 Q q z over z square plus R square raise to 3 by 2. And I am calculating force due to 12 charges and therefore, the net force is going to be F, net is going to be 12 times that much. So, 1 over 4 pi Epsilon 0 12 Q q z over z square plus R square raise to 3 by 2.

Notice as z goes to 0; that means, if I put this charge Q at the center I am showing it in the big red circle, then the forces from opposite sides cancel each other and net force is going to be 0 which is indeed the answer. The horizontal components all cancel, because of the pair wise coming of this capital Q.

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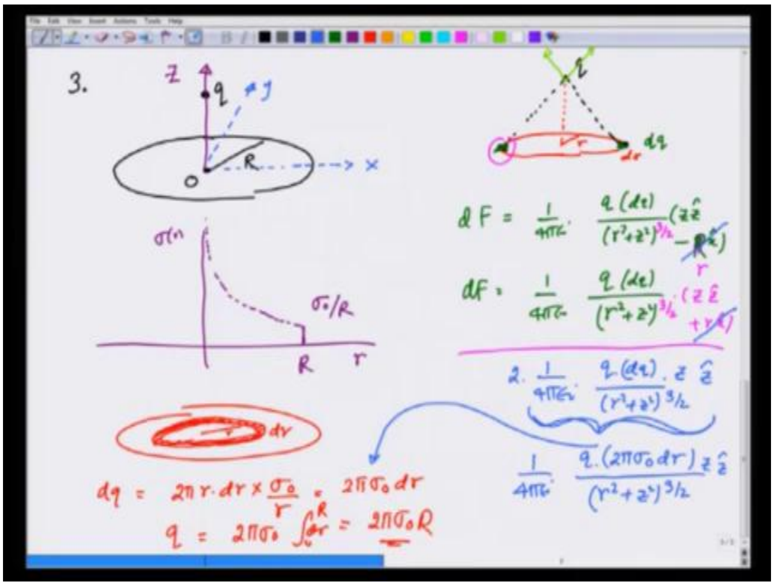

Problem number 3, a thin disc of radius R is placed in the x, y plane with the center at the origin. So, we are taking about a thin disc of radius R placed with the center at the origin and it is in the x, y plane. So, I can make this plane x and y plane and this is a z axis, the disc carries a surface charge sigma equals sigma naught over R. So, if I would plot this r versus sigma r, what it says is it carries sigma naught over r. So, that r equal to 0 is going to be infinity and then, it decays as 1 over R and at R equal to capital R it is going to be sigma naught over capital R.

So, as you go further and further out this charge density is decreasing. However, the net charge is going to be finite and you can see that very easily. The net charge is going to be suppose I take this disc, this is on the side I will calculate the force little later and I take a small ring here shown by this red filled circle of radius small r and thickness d r. Then, the area of this ring is 2 pi r d r and the charges carries that is d q is going to be 2 pi r d r the area times the surface charge density sigma naught over r which is equal to 2 pi sigma naught d r.

And therefore, the total charge carried by this is going to be integral of this 2 pi sigma 0 is a constant integrate d r from 0 to r it comes out to be 2 pi sigma naught R. So, although the charge density is blowing up at r equal to 0, the net charges is still finite, because of the particular dependence that this sigma has. Now, what the problem wants to know is, if a point charge q is placed on the axis of the ring at height z. So, what we are doing is, we are putting a charge small q here on the z axis shown by black on the top figure.

What is the force on this? The force on the charge is going to be, again if I take a small ring as I did in calculating the total charge, if I take a small ring of radius r and thickness d r and calculate the force due to this on this small charge q. Again notice that for each charge on one side, small charge on one side there is an equal charge on the other side. And therefore, these charges come in pairs and if I calculate the net force here on q is going to be like this as shown by green arrows and their horizontal component will cancel. You could do it vectorially also and let me show that.

If I take a small charge let us say on the x axis and this charge let say it is small d q. Then, the force due to this d q is going to be d F is going to be equal to 1 over 4 pi Epsilon 0 q times d q over r square plus z square and this is going to be from the vector here, where d q is 2 the charge where q is which is going to be z in the z direction minus x x and this x value is small R.

A similar force exists due to the charge on the other side and that force is going to be 1 over 4 pi Epsilon 0 q d q over r square plus z square times and since this is we are taking a vector on top this should be 3 by 2, here also it is 3 by 2 times z z minus the position of this charge on the left which is minus r x. So, this will going to be plus r x you can see if you add the 2 the x components cancel let me show that by different color, the x components cancel therefore, the net component is z.

So, all I am going to do is calculate the z component and what is the z component, you add these two charges it comes out to be 1 over 4 pi Epsilon 0 q d q over r square plus z square raise to 3 by 2 times z in the z direction that is due to one of the two charges then times two. So, what we see is it only z component is contributing, so if I take the contribution z contribution for each small d q and integrate it I get the total force due to this ring.

And that now you can see very easily is going to be 1 over 4 pi Epsilon 0 q times d q integrated over; that means, the charge over this entire ring which is going to be 2 pi sigma 0 d r I am taking this from here which we calculated earlier divided by r square plus z square raise to 3 by 2 in the z direction. This is the force due to this ring that I took of radius r the net force therefore, is going to be integration of this from r equal to 0 to capital R.

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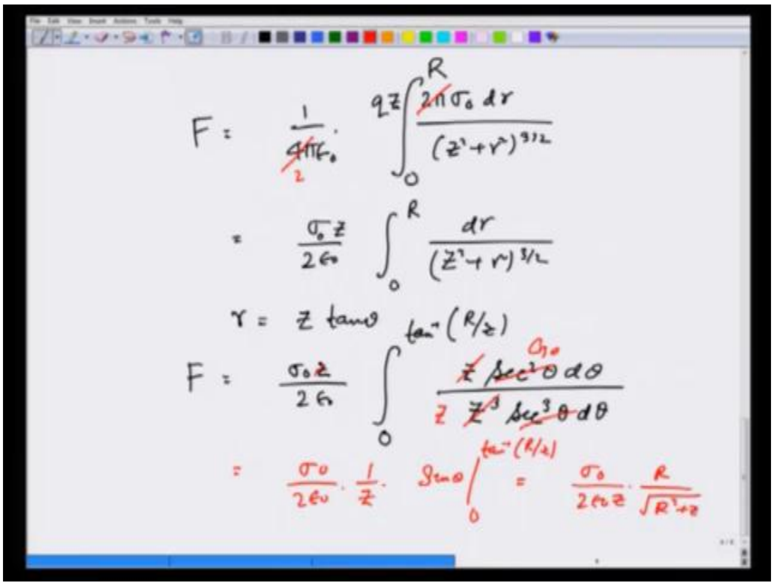

And therefore, the net force F is equal to 1 over 4 pi Epsilon 0 q then I have let us see what do I have, I have 2 pi sigma naught d r z is here divided by z square plus r square raise to 3 by 2 integrated from 0 to capital R, this comes out to be sigma naught over 2 Epsilon 0. Because, this 2 pi cancels here gives you 2 sigma naught z integration 0 to R d r over z square plus r square raise to 3 by 2 take r equals z tangent theta and therefore, the net force magnitude becomes sigma 0 z over 2 Epsilon 0 integration theta equals 0 to tan inverse capital R over z d r will become z secant square theta d theta over z cubed or this will become z cubed secant cubed theta d theta.

What we get is, we cancel z you get 1 over z and this therefore, becomes sigma naught over 2 Epsilon 0 1 over z this gives me cosine of theta sin theta going from 0 to tan inverse R over z which is equal to sigma naught over 2 Epsilon 0 z R over square root of R square plus z square. In the next tutorial we will solve the next set of problems that is from problem number 4 to 9.